City Guarding with Cameras of Bounded Field of View

Guarding Free Space with Vertex Half Guards

Ahmad Biniaz
Mohammad Hashemi

University of Windsor

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Agenda

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   - Problem Restrictions
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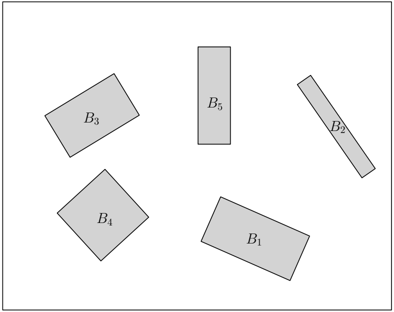
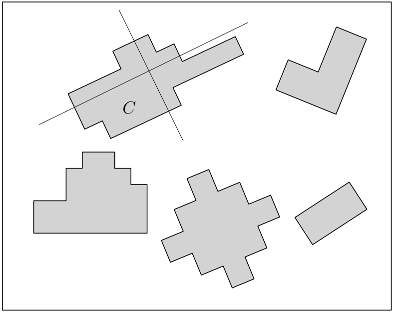
   - Lemmas and Theorem
3. Guarding Orthogonally Convex Polygons
   - Definition
   - Problem Restrictions
   - Conjecture
   - Theorem
4. Remarks

Addressed Problems

The **two** problems we addressed are related to the **city guarding** and the **art gallery** problems.

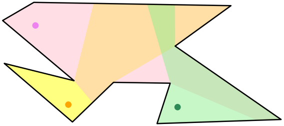


Arbitrary-oriented buildings (aerial view).

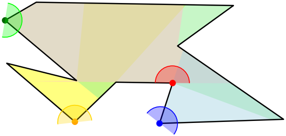


Orthogonally convex polygons in the plane.

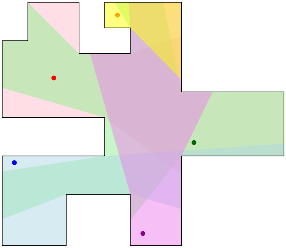
Literature and Related Works - Art Gallery Problem



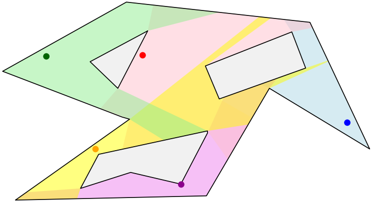
Point guards (360°).



Vertex guards (180°).

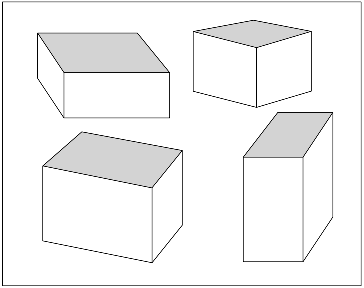


Orthogonal polygon & point guards (360°).

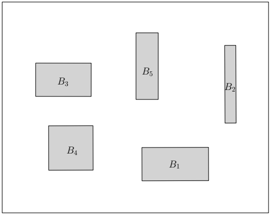


Polygon with holes & point guards (360°).

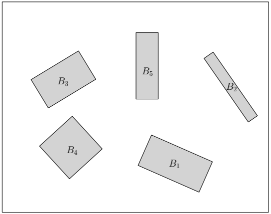
Literature and Related Works - City Guarding



A city with rectangular buildings.



Axis-aligned buildings (aerial view).

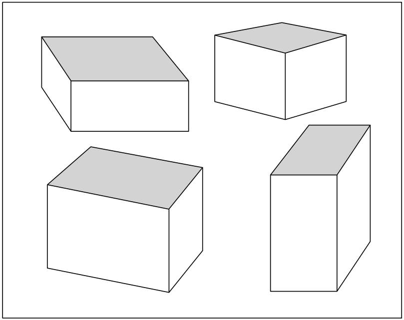


Arbitrary-oriented buildings (aerial view).

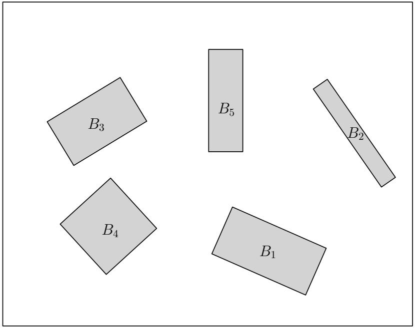
Problem 1: City Guarding

1. Given a city with k **arbitrary-oriented rectangular-base** buildings, the task is to guard the **free space** (the ground, walls, roofs, and the sky) by placing cameras with $\mathbf{180^\circ}$ field of view **only on the top corners** of the buildings.
   - We proved that $\mathbf{3k+1}$ such cameras are always **sufficient**, which answers *Daescu and Malik's conjecture (CCCG, 2020)*.

A Glimpse of City Guarding



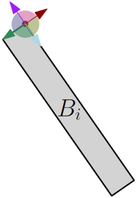
A city with rectangular buildings.



Arbitrary-oriented buildings (aerial view).

City Guarding - Problem Restrictions

- Buildings' base shape: **rectangular**
- Buildings' orientation: **arbitrary**
- Cameras' horizontal field of view: **180°**
- Cameras' range of vision: **infinite**
- Place of cameras: only on the **top corners** of the buildings
- Direction of cameras: **orthogonal to the walls**
- Goal: Guarding the **city** (roofs, ground, walls, and whole aerial space)



Four possible directions a camera can have.

City Guarding - Conjecture

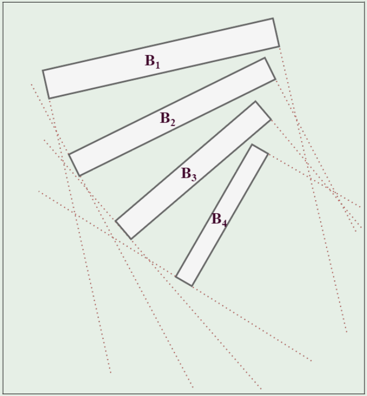
Conjecture (Daescu and Malik, CCCG, 2020)

$3k + 1$ vertex guards are sometimes necessary to guard a city with k vertical buildings with rectangular bases, where guards are placed only at the top vertices of the buildings. (They conjecture the bounds are tight.)

- They proved **necessity** bound and **conjectured** that it is **tight**.
- Their example for proving the lowerbound is given in the next slide.

Necessity bound example

Example



A city with $k = 4$ buildings that needs $3k + 1 = 13$ guards.

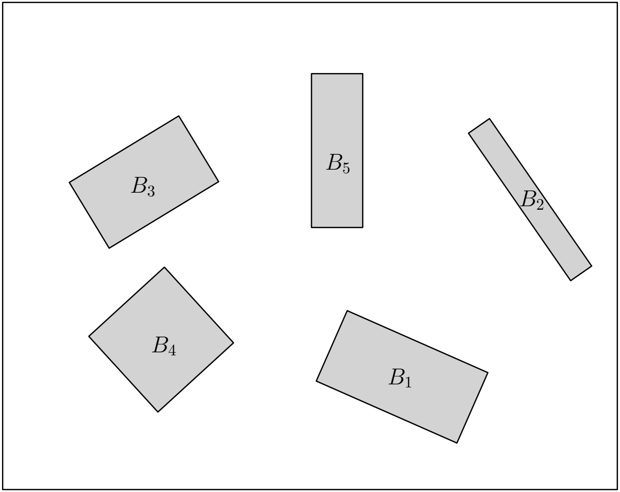
Lemma 1

Lemma 1 (Daescu and Malik, CCCG, 2020)

If in a city the roofs, walls, and the ground are guarded by a set of cameras, then every point in the aerial space of the city is visible by a camera.

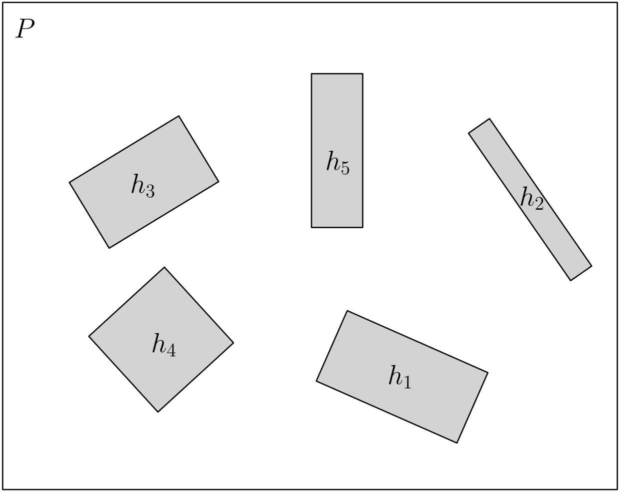
- Therefore, a guarding of **roofs, walls, and the ground** is a guarding of **the whole city's aerial space**.
- We focus on the guarding of the **ground and walls** and place the cameras in a way guarding the **roofs** too.

Vertical Projection of the Buildings



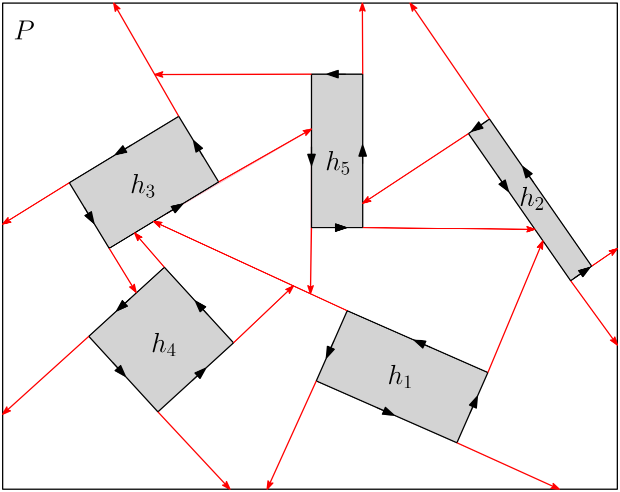
Buildings (vertically projected to the plane).

The Problem in 2D



Polygon P (city's boundary) with holes (buildings).

Extension of Holes' Sides



Extensions divide the free space of P into regions $R_1, R_2, \ldots$

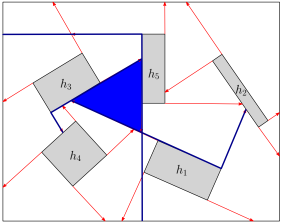
Lemma 2

Lemma 2 (Regions' Convexness)

Each region R_i is convex.

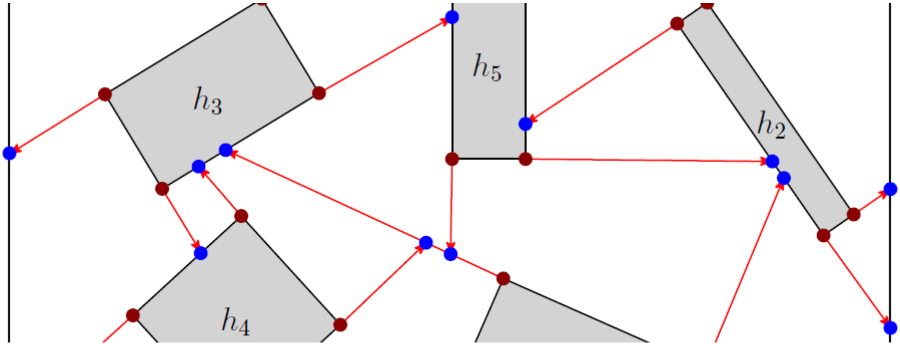
Proof.

The extensions of each two adjacent sides of a hole create a quadrant. Each region R_i is the intersection of a set of quadrants. Since quadrants are convex, each region is convex as well. □



Intersection of some quadrants (the blue region).

Intersections of Extended Sides



A cut of polygon P focusing on the holes' corners (red points) and extensions' intersections (blue points).

Lemma 3

Lemma 3 (Number of the Regions)

The number of regions $R_1, R_2, \ldots$ is $3k + 1$.

We define a **planar** graph $G = (V, E)$:

$$V = \{Holes' \ Corners \ \cup \ Extensions' \ Intersection \ Points\}$$

$$= \{Corner \ Vertices \ \cup \ Intersection \ Vertices\}$$

$$E = \{Segmented \ Holes' \ Sides \ \cup \ Segmented \ Sides' \ Extensions \cup$$

$$Segmented \ Boundary \ of \ P\}$$

Observation

Each hole has 4 corners and 4 sides and each extension defines an intersection vertex: $|Corner \ Vertices| = |Intersection \ Vertices| = 4k$

$$|V| = 4k + 4k = 8k$$

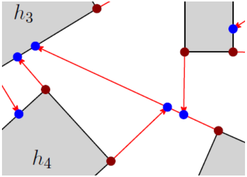
Claim

G is 3-regular.

Proof.

- Each corner vertex is incident to two sides of a hole and one extension.
- Each intersection vertex is incident to an extension and two segments from the intersected segment.

Therefore, G is 3-regular. □



A cut of polygon P focusing on vertices' degrees.

Lemma 3 (Continue)

Proof of Lemma 3.

Knowing G is 3-regular and $|V| = 8k$:

$$\sum_{i=1}^{|V|} deg(v_i) = 2|E| => 3|V| = 2|E| => |E| = \frac{3(8k)}{2} = 12k$$

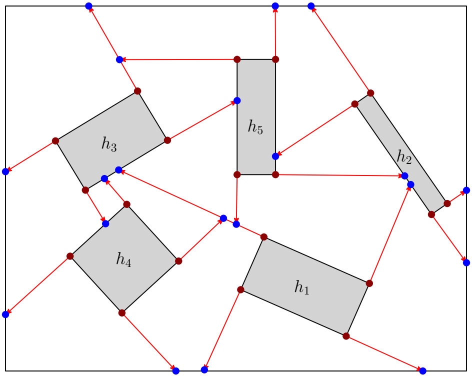
F = faces of G (**outerface**, **holes**, and **regions** $R_1, R_2, \ldots$).

Using Euler's formula for connected planar graphs:

$$|F| = |E| - |V| + 2 = 12k - 8k + 2 = 4k + 2.$$

Excluding the **outerface** and the ***k* holes**, the number of regions $R_1, R_2, \ldots$ is $3k + 1$.

□



Planer 3-regular graph G.

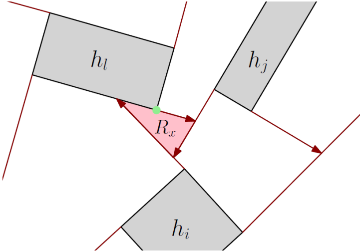
Lemma 4

Each region R_i contains a corner of a hole on its boundary.

Lemma 4 (Continue)

Proof of Lemma 4.

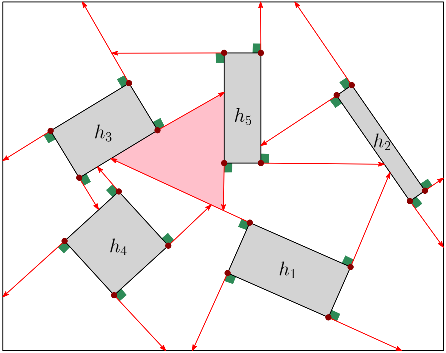
Each region R_i is built up by **extending sides**. Consider the **last** extension that **closes** the boundary of region R_i. This **extension** (the entire directed line segment) is a part of the boundary of R_i. The **initial point** of this directed line segment is a **hole's corner**. □



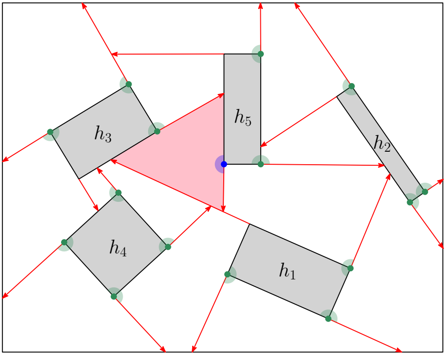
A worst-case region R_x. Order of extending sides: h_i, h_j, h_l.

Contact: Mohammad - smh10.mh@gmail.com

Good and Bad Regions, Corner and Edge Guards

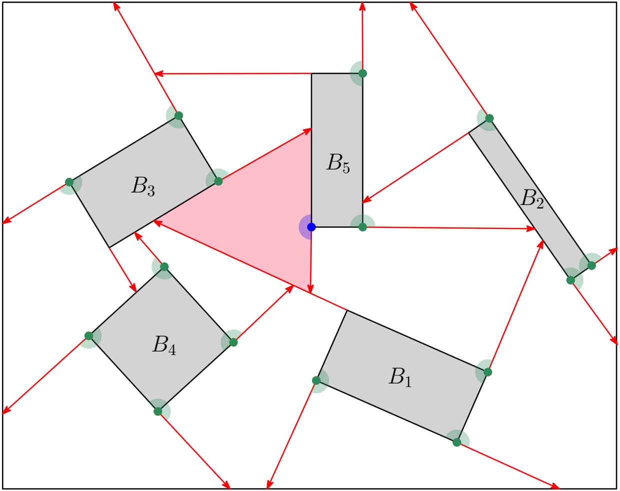


Good regions (white regions with green 90° corner(s) on their boundary) and bad regions (in pink).



One corner guard (in green) per good region, one edge guard (in blue) per bad region.

Camera Placement



Cameras looking toward the interior of the regions they guard, perpendicular to the regions' boundary.

Theorem 5

Theorem 5

Given k arbitrary-oriented rectangular-base buildings, we can guard the entire space (the ground, walls, roofs, and the sky) with at most $3k + 1$ cameras of 180° field of view that are placed at top corners of buildings orthogonal to a wall. The bound $3k + 1$ is the best achievable.

Problem 2: Guarding orthogonally convex polygons

2. Given k **orthogonally convex polygons** of total m vertices in the plane, the task is to guard the **free space (everywhere except for the polygons)** by placing guards with **180°** field of view **only on the corners** of the polygons.
   - We proved that $\frac{m}{2} + k + 1$ such guards are always **sufficient**, which answers another *conjecture of Daescu and Malik (Theoretical Computer Science, 2021)*.

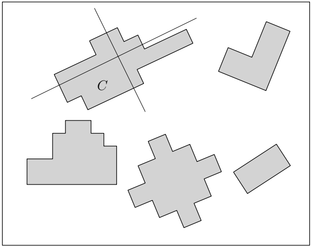
A Glimpse of Guarding Orthogonally Convex Polygons

Definition

Orthogonal Polygon: a polygon whose edges are orthogonal (axis-aligned).

Definition

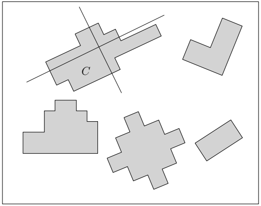
Orthogonally Convex Polygon: An orthogonal polygon whose intersection with any orthogonal line segment is either empty or a single line segment.



Orthogonally convex polygons in the plane.

Guarding Orthogonally Convex Polygons - Problem Restrictions

- Polygons' orientation: **arbitrary**
- Polygons' type: **orthogonally convex**
- Guards' horizontal field of view: **180°**
- Guards' range of vision: **infinite**
- Place of Guards: only on the **corners** of the polygons
- Direction of guards: **orthogonal to the sides**
- Goal: Guarding the **free space** (everywhere except polygons' interior)



Orthogonally convex polygons in the plane.

Orthogonally Convex Polygons - Conjecture

Conjecture (Daescu and Malik, Theoretical Computer Science, 2021)

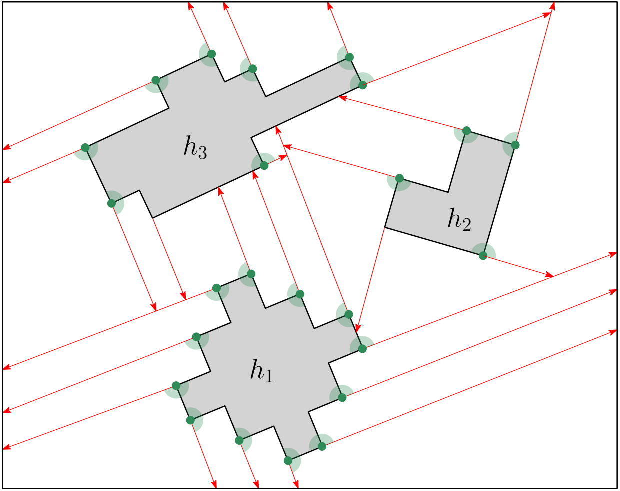
Given a set F of k pairwise disjoint orthogonal polygons in the plane, $\frac{m}{2} + k + 1$ vertex guards are sometimes necessary to guard the free space and the boundaries of the polygons in F.

- They proved **necessity** bound and **conjectured** that it is **tight**.
- Their example for proving the lowerbound is **similar** to the one for City Guarding with **orthogonally convex holes**.

Same Approach

1. Likewise, we extend sides **but only the sides which end at a convex corner**, and they divide the free space into some regions.
2. Similar to *Lemma 2*, the result regions are **convex**.
3. Like *Lemma 3*, the **number of these regions** is equal to $\frac{m}{2} + k + 1$:
    - We define 3-regular planar graph G and have total number of convex corners $c = \frac{m}{2} + 2k$.
    - Similarly, $|V| = 2c = m + 4k$.
    - So, $|E| = \frac{3m}{2} + 6k$ and $|F| = \frac{m}{2} + 2k + 2$.
4. Regions are **good and bad**, and the guard placement is in the same manner by **corner and edge guards**.

Guard Placement



Sides' extensions and guard placement of 3 polygons.

Theorem 6

Theorem 6

Given k pairwise disjoint arbitrary-oriented orthogonally convex polygons of total m vertices in the plane, we can guard the entire free space with at most $\frac{m}{2} + k + 1$ cameras of 180° field of view that are placed at the corners of the polygons orthogonal to a side. The bound $\frac{m}{2} + k + 1$ is the best achievable.

Contact: Mohammad - smh10.mh@gmail.com

Remarks

- **Generalization:** Same results for guarding cities with orthogonally convex bases buildings:
  - Rectangles are special orthogonally convex polygons (they only have convex corners): $m = 4k$

$$\frac{m}{2} + k + 1 = \frac{4k}{2} + k + 1 = 3k + 1$$

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Questions

- Any Questions?
- Contact: Mohammad Hashemi
  - hashem62@uwindsor.ca
  - smh10.mh@gmail.com
- Thank you for considering this presentation!