Improved Algorithms for Burning Planar Point Sets

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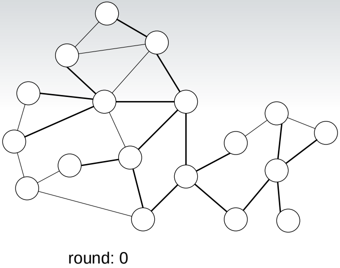
Graph Burning Problem

- Given an undirected graph G, the goal is to **burn** in a minimum number of **rounds** [Bonato et al., 2014].



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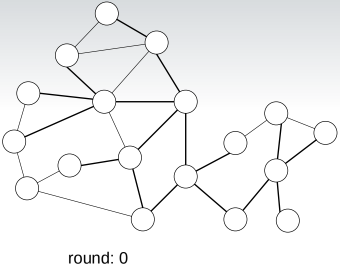

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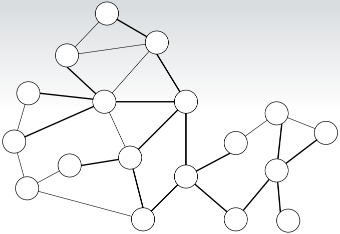

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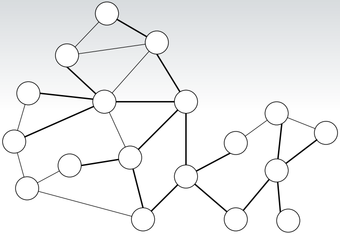

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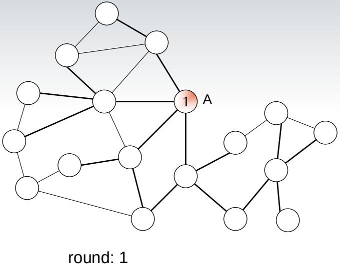

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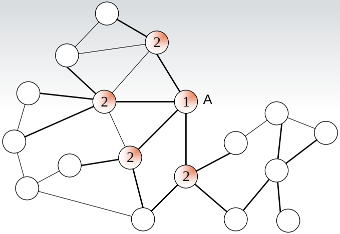

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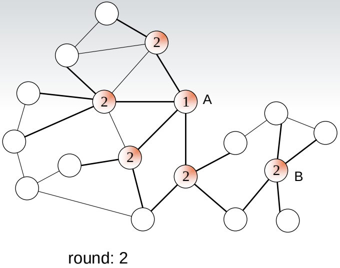

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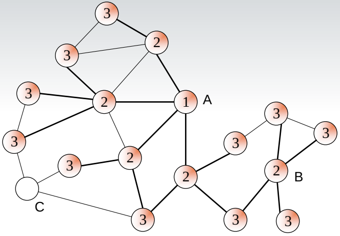

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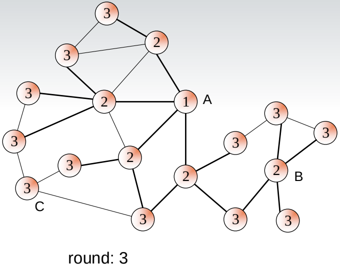

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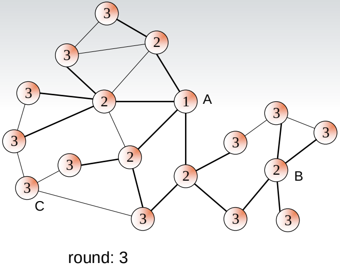

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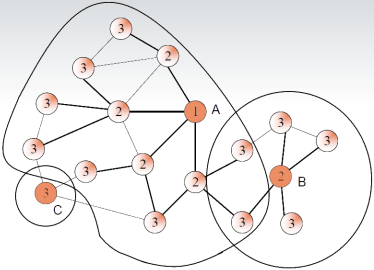

- Decision problem:
    - Can we burn G in k rounds?
    - Equivalently, can we cover the graph with "disks" of radii $0, 1, 2, \ldots, k-1$?

Burning Paths

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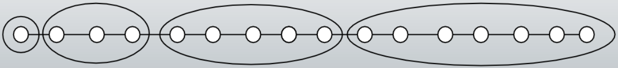

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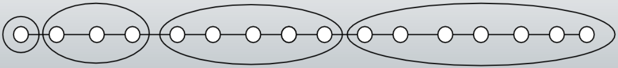

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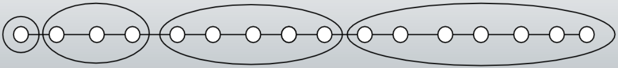

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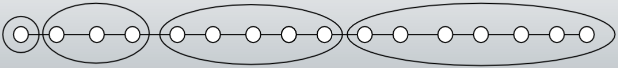
- **The burning graph conjecture:** The burning number of any connected graph is at most $\lceil \sqrt{n} \rceil$ [Bonato et al. 2014].
  - The upper bound for the burning number of any connected graph is improved a few times: from $2\sqrt{n}$ [Bonato et al., 2014], to $\frac{\sqrt{6}}{2}\sqrt{n} \approx 1.22\sqrt{n}$ [Land and Lu, 2016] to $\frac{2}{\sqrt{3}}\sqrt{n} + O(1) \approx 1.15\sqrt{n} + O(1)$ [Bonato and S.K., 2021], to $\frac{2}{\sqrt{3}}\sqrt{n} + 1 \approx 1.15\sqrt{n}$ [Bastide et al. 2022], to $\sqrt{n} + o(\sqrt{n})$ [Norin and Turcotte, 2023].



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  - The problem is more "interesting" when the underlying graphs are sparse.
- The problem is APX-hard [Mondal et al., 2021].

Approximation Algorithms

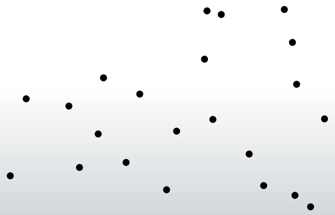
Graph family	Apx. Factor	
general graphs	3	[Bessy et al., 2018], [Bonato and S.K., 2019]
forests of disjoint paths	$1 + \epsilon$ (FPTAS)	[Bonato and S.K., 2019]
graphs of bounded treewidth	$1 + \epsilon$ (PTAS)	[Lieskovský and Sgall, 2022]
graphs of bounded path-length	$1 + o(1)$	[S.K. et al., 2020]
graphs of bounded tree-length	$2 + o(1)$	[S.K. et al., 2020]

- There are also probabilistic models for graph burning [Pralat, 2014, Mitsche et al., 2017]

Geometric Burning

Anywhere Burning

- The input is a set P of points in the Euclidean plane.
- In the **anywhere burning** problem, at each round:
  - A fire may start at **any** point in the plane.
  - The existing fire extends to all points within distance 1.
- The objective is to select starting points (centers) in a way to minimize the number of rounds to burn all points in P.



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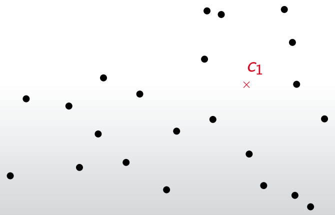

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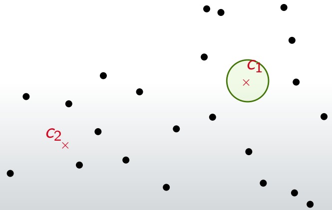

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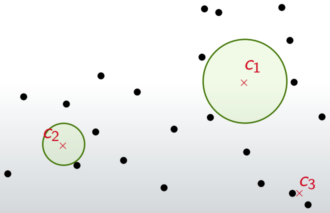

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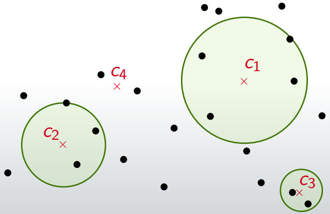

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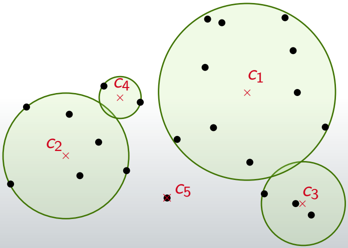

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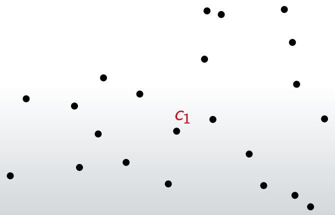

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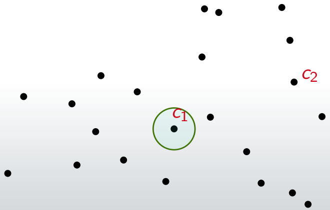

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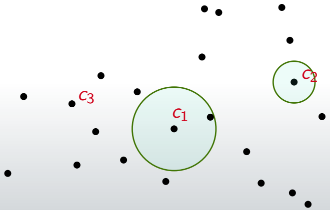

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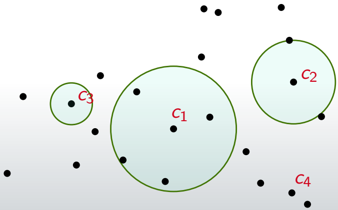

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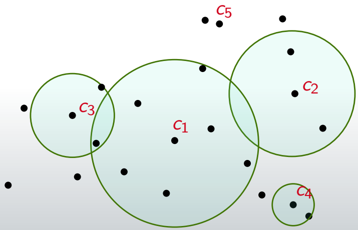

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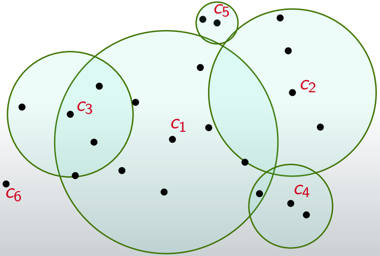
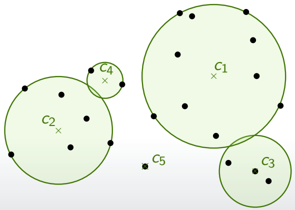


Geometric Burning Problems

- In geometric burning problems, the goal is to minimize k such that disks of distinct radii from $\{0, 1, 2, \ldots, k\}$ cover the input set P.
  - In anywhere burning, the disks can be centred anywhere in the plane.
  - In point burning, the disks must be centred at points in P.



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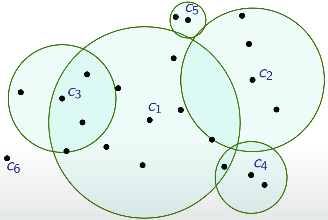

Point burning

Results

- Both problems are NP-hard [Keil et al., 2022].
- For anywhere burning, the best existing approximation ratio has improved from $2+\epsilon$ [Keil et al., 2022] to $1.92188+\epsilon$ [Gokhale et al., 2023].

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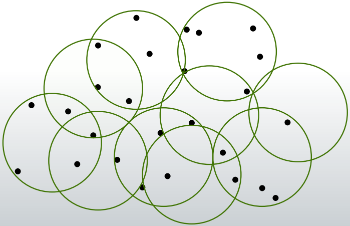
- The input is a given set of P points and a set of disks of uniform radii r.



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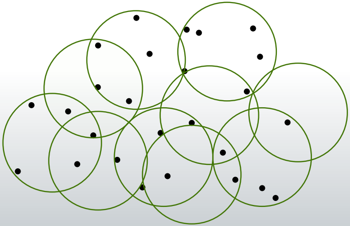

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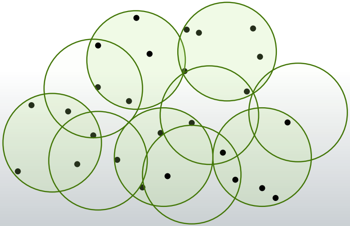

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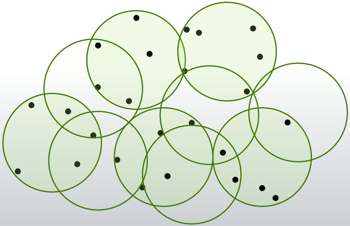

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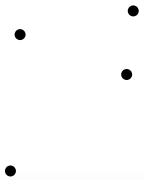

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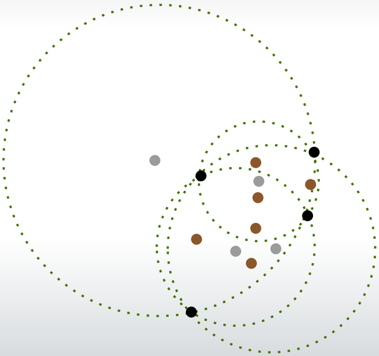
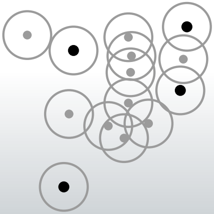


Anywhere Burning

- Use the DUDC PTAS of [Mustafa and Ray, 2010] to find the smallest value k^* so that P can be covered with k^* disks of radius k^* centered at points in C.



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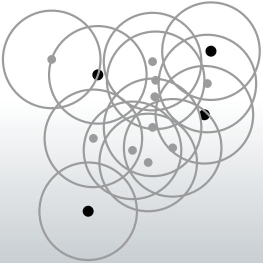

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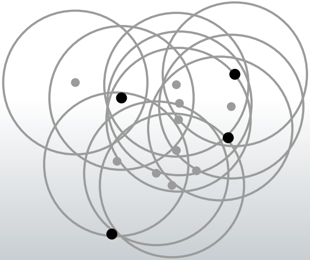

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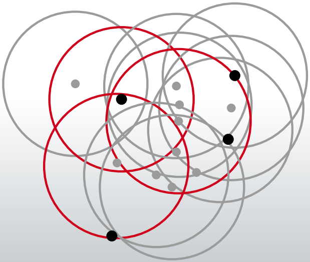

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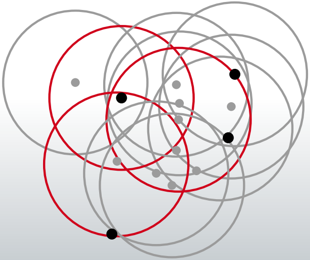
- Use the DUDC PTAS of [Mustafa and Ray, 2010] to find the smallest value k^* so that P can be covered with k^* disks of radius k^* centered at points in C.
  - It is not possible to cover P with $(k^* - 1)/(1 + \epsilon)$ disks of radius $k^* - 1$.



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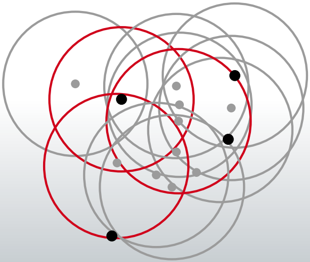

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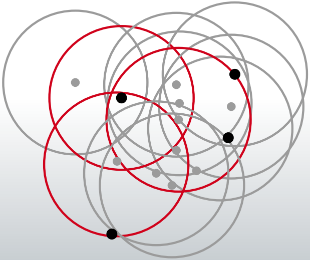
  - **Burning P takes at least $(k^* - 1)/(1 + \epsilon)$ rounds!**



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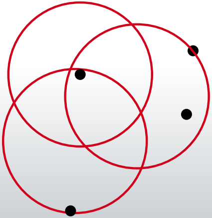
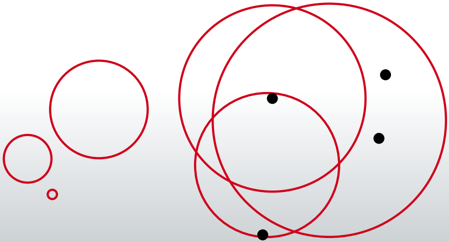


Anywhere Burning

- **Burning P takes at least $(k^* - 1)/(1 + \epsilon)$ rounds!**
- It is possible to cover P with k^* disks of radius k^*.
- So, it is possible to cover P with disks of distinct radii in $\{0, 1, \ldots, k^*, k^* + 1, \ldots, 2k^* - 1\}$.



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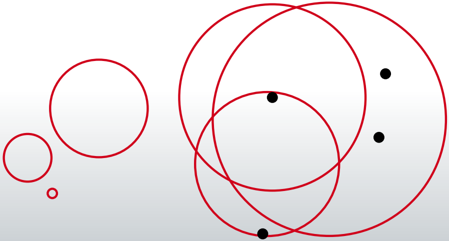
- **It is possible to burning P within $2k^*$ rounds.**



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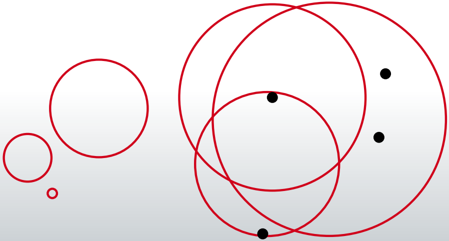
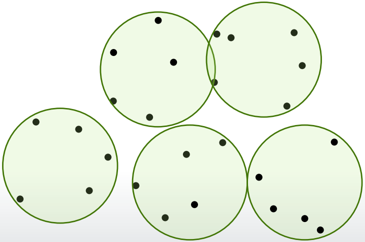
- **It is possible to burning P within $2k^*$ rounds.**
- We get an approximation factor is $2 + \epsilon'$.



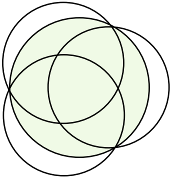
Anywhere Burning

- **Summary so far:** Minimize γ s.t. one can cover k^* disks of uniform radius k^* with disks of distinct radii $\{0, 1, \ldots, \gamma k^* - 1\}$.
  - This ensures a competitive ratio of $(1 + \epsilon)\gamma$.



Anywhere Burning

- **Disk Covering problem:** $\gamma(i)$ is the minimum value s.t. i disks of radius $\gamma(i)$ can cover a unit disk.[1]



$$\gamma(3) = \sqrt{3}/2$$

[1] E. Friedman. Circles covering circles. `https://erich-friedman.github.io/packing/circovcir/`.

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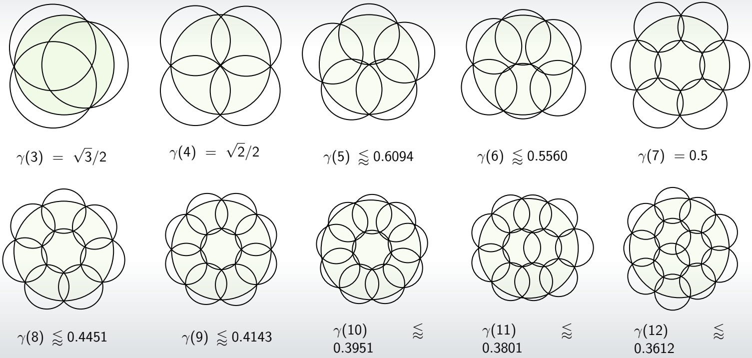

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Anywhere Burning

- It is possible to cover k^* disks of radius k^* with disks of distinct radii from $\{0, 1, \ldots, \lfloor 1.865k^* \rfloor - 1\}$.

i	radius range	covered disks
0	$\geq k^*$	$\lfloor 0.865k^* \rfloor$
3	$[\sqrt{3}k^*/2, k^*)$	$\lfloor (2-\sqrt{3})k^*/6 \rfloor$
4	$[\sqrt{2}k^*/2, \sqrt{3}k^*/2)$	$\lfloor (\sqrt{3}-\sqrt{2})k^*/8 \rfloor$
5	$[0.6094k^*, \sqrt{2}k^*/2)$	$\lfloor (\sqrt{2}/2 - 0.6094)k^*/5 \rfloor$
6	$[0.5560k^*, 0.6094k^*)$	$\lfloor 0.0534k^*/6 \rfloor$
7	$[0.5k^*, 0.5560k^*)$	$\lfloor 0.0560k^*/7 \rfloor$
8	$[0.4451k^*, 0.5k^*)$	$\lfloor 0.0549k^*/8 \rfloor$
9	$[0.4143k^*, 0.4451k^*)$	$\lfloor 0.0308k^*/9 \rfloor$
10	$[0.3950, 0.4143)$	$\lfloor 0.0193k^*/10 \rfloor$
11	$[0.3801, 0.3950)$	$\lfloor 0.0149k^*/11 \rfloor$
12	$[0.3612, 0.3801)$	$\lfloor 0.0189k^*/12 \rfloor$
sum	–	$> 1.0009k^* - 11 > k^*$

Anywhere Burning

- It is possible to cover k^* disks of radius k^* with disks of distinct radii from $\{0, 1, \ldots, \lfloor 1.865k^* \rfloor - 1\}$.

Theorem

There is an anywhere burning algorithm with an approximation factor of 1.865.

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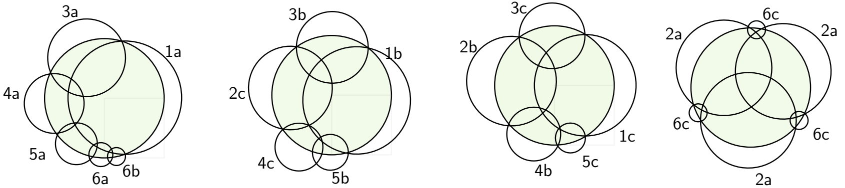
Anywhere Burning

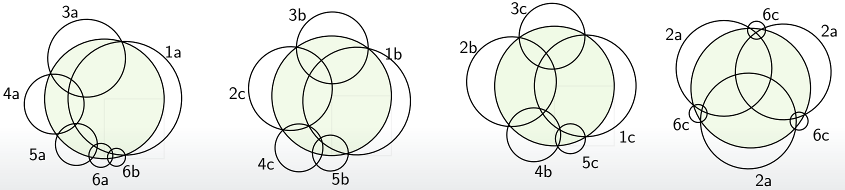
- Instead of using disks of the same class to cover a disk of radius k^*, apply a "mix-and-match approach".



Class 1	Class 2	Class 3	Class 4	Class 5	Class 6
1a: $[0.95k^*, k^*]$	2a: $[0.8k^*, 0.85k^*)$	3a: $[0.65k^*, 0.7k^*)$	4a: $[0.5k^*, 0.55k^*)$	5a: $[0.35k^*, 0.4k^*)$	6a: $[0.2k^*, 0.25k^*)$
1b: $[0.9k^*, 0.95k^*]$	2b: $[0.75k^*, 0.8k^*)$	3b: $[0.6k^*, 0.65k^*)$	4b: $[0.45k^*, 0.5k^*)$	5b: $[0.3k^*, 0.35k^*)$	6b: $[0.15k^*, 0.2k^*)$
1c: $[0.85k^*, 0.9k^*]$	2c: $[0.7k^*, 0.75k^*)$	3c: $[0.55k^*, 0.6k^*)$	4c: $[0.4k^*, 0.45k^*)$	5c: $[0.25k^*, 0.3k^*)$	6c: $[0.1k^*, 0.15k^*)$

Anywhere Burning

- It is possible to burn k^* disks of radius k^* using disks of distinct radii from $\{0, 1, \ldots, \lfloor 11k^*/6 \rfloor - 1\}$.



Group 1 covering　Group 2 covering　Group 3 covering　Group 4 covering

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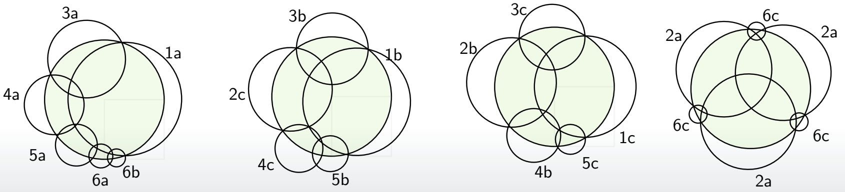
  - $0.05k^*$ disks are covered by each of Groups 1, 2, and 3 (summing to $0.15k^*$ covered disks), and $0.05k^*/3$ disks are covered by Group 3.
  - The remaining $11k^*/6 - 0.15k^* - 0.05k^*/3 = 5k^*/6$ disks are covered by $5k^*/6$ disk of radius $\geq k^*$.



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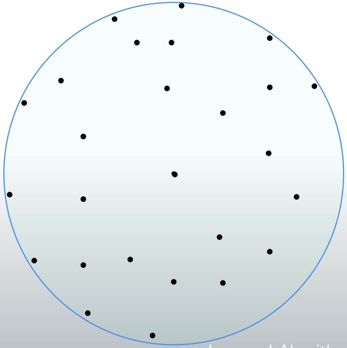
Theorem

There is an anywhere burning algorithm with an approximation factor of $11/6 + \epsilon = 1.8\overline{3} + \epsilon$

Point Burning

- As before, we use the PTAS for DUDC to find a set U of k^* disks of radius k^* that cover all points.



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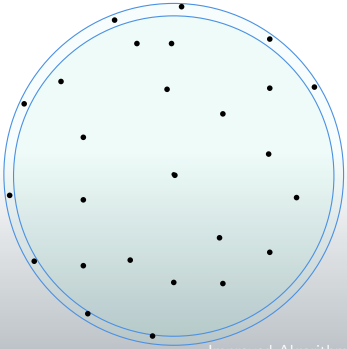

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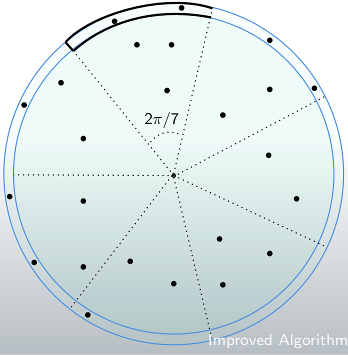
  - Partition the uncovered annulus into i regions and cover non-empty sectors with disks of class i.



Point Burning

- It is possible to burn k^* disks of radius k^* using disks of distinct radii from $\{0, 1, \ldots, \lfloor 1.944k^* \rfloor - 1\}$.

Theorem

There is a point burning algorithm with an approximation factor of $1.944 + \epsilon$

Summary

- **Anywhere burning:** We presented an algorithm with an approximation factor of $1.8\overline{3} + \epsilon$, improving the ratio $1.92188+\epsilon$ of [Gokhale et al., 2023].

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  - Is there any PTAS for anywhere/point burning problems?
  - Instead of fixing the spread factor and minimizing the number of rounds, fix the number of rounds and minimize the spread factor.

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