Dynamic Schnyder Woods

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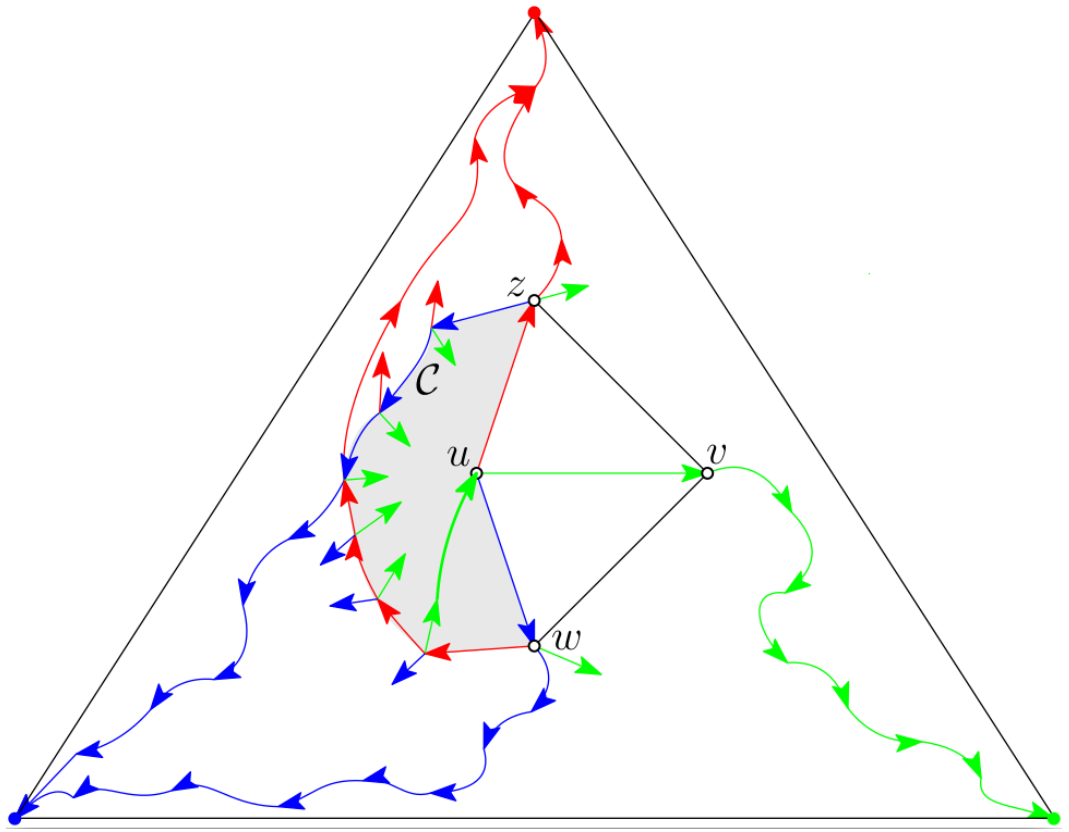
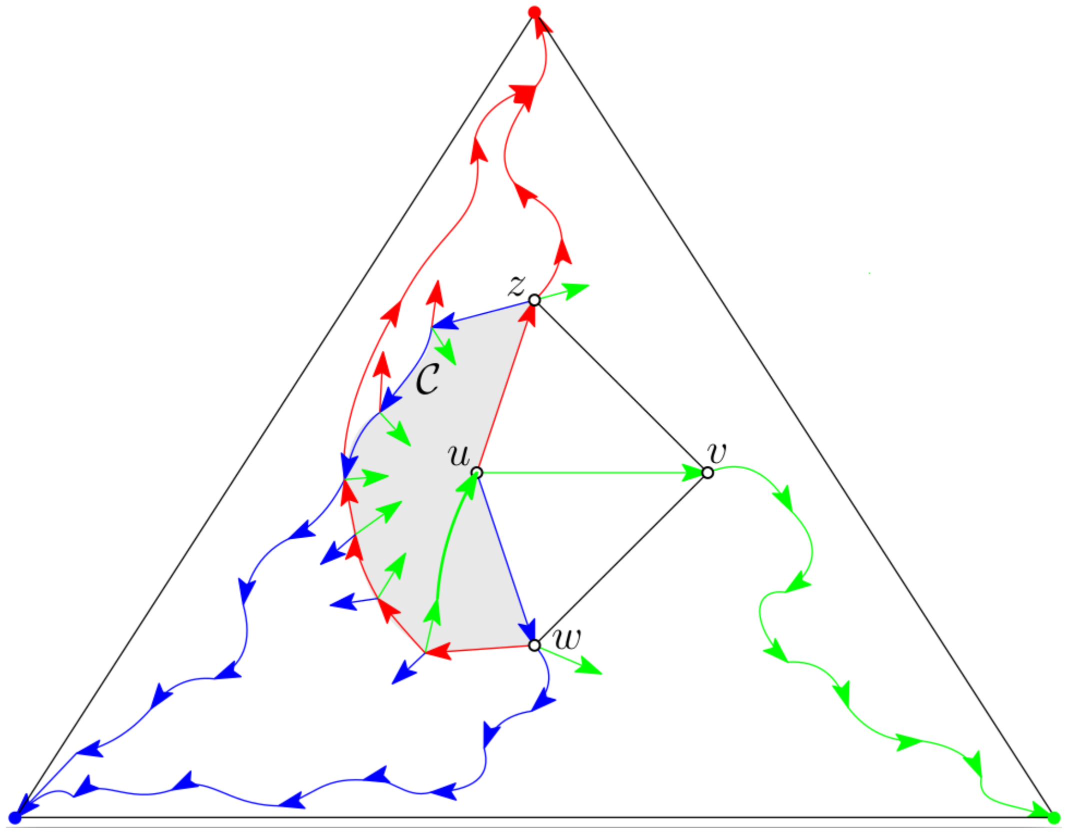
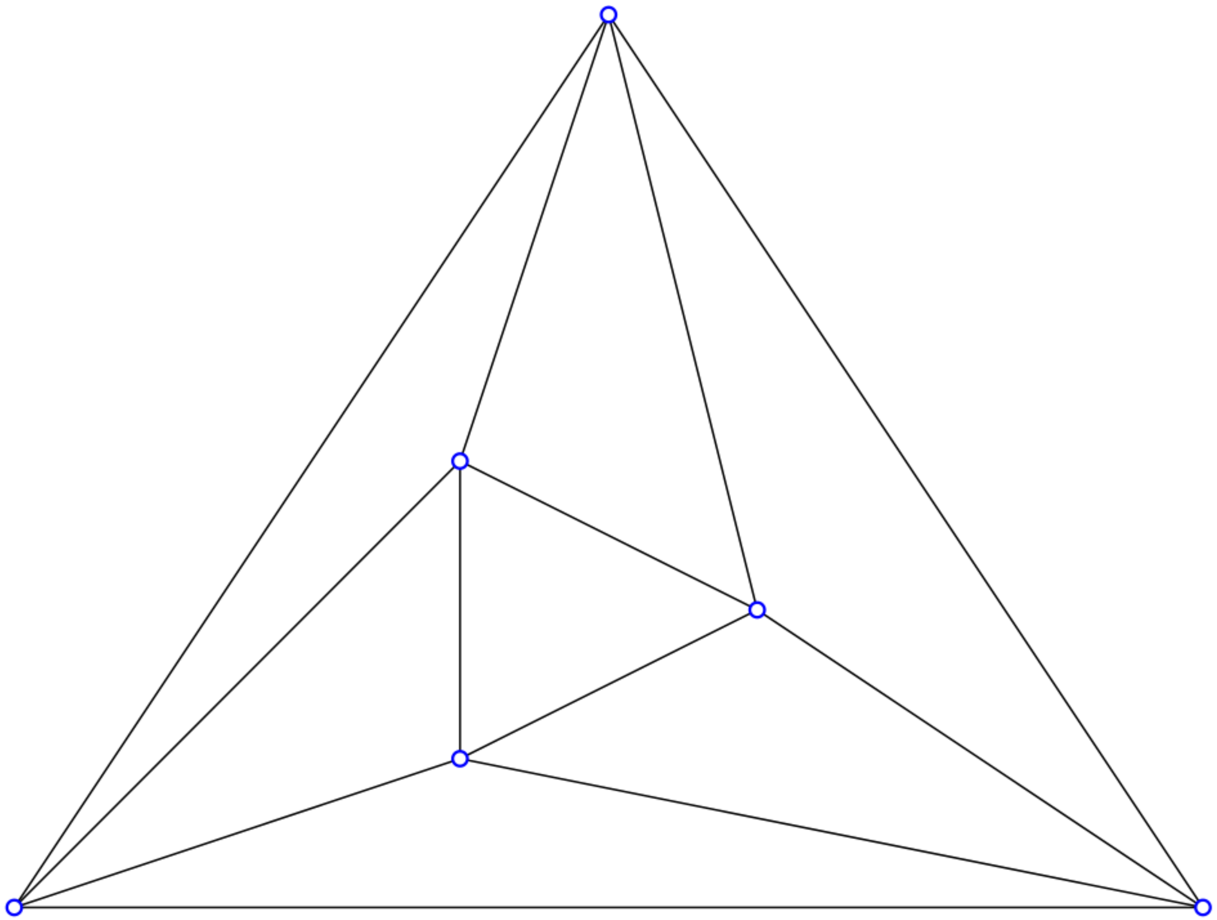


Outline

- **Introduction:**
  - Schnyder woods
  - Flips
  - Schnyder Woods and Flips
- **Our work:**
  - Results on Flips in Schnyder woods
  - Results on Dynamic Schnyder woods
- **Conclusions**

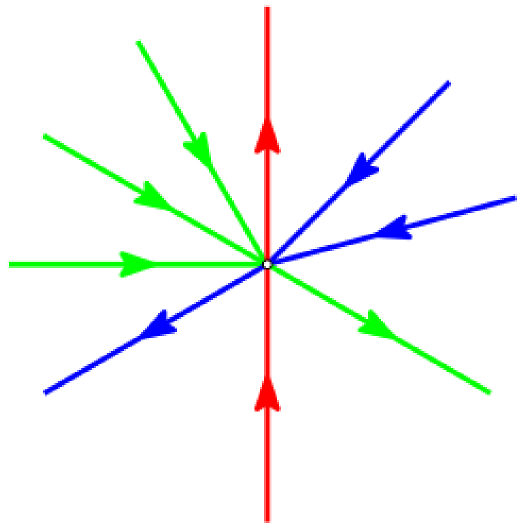


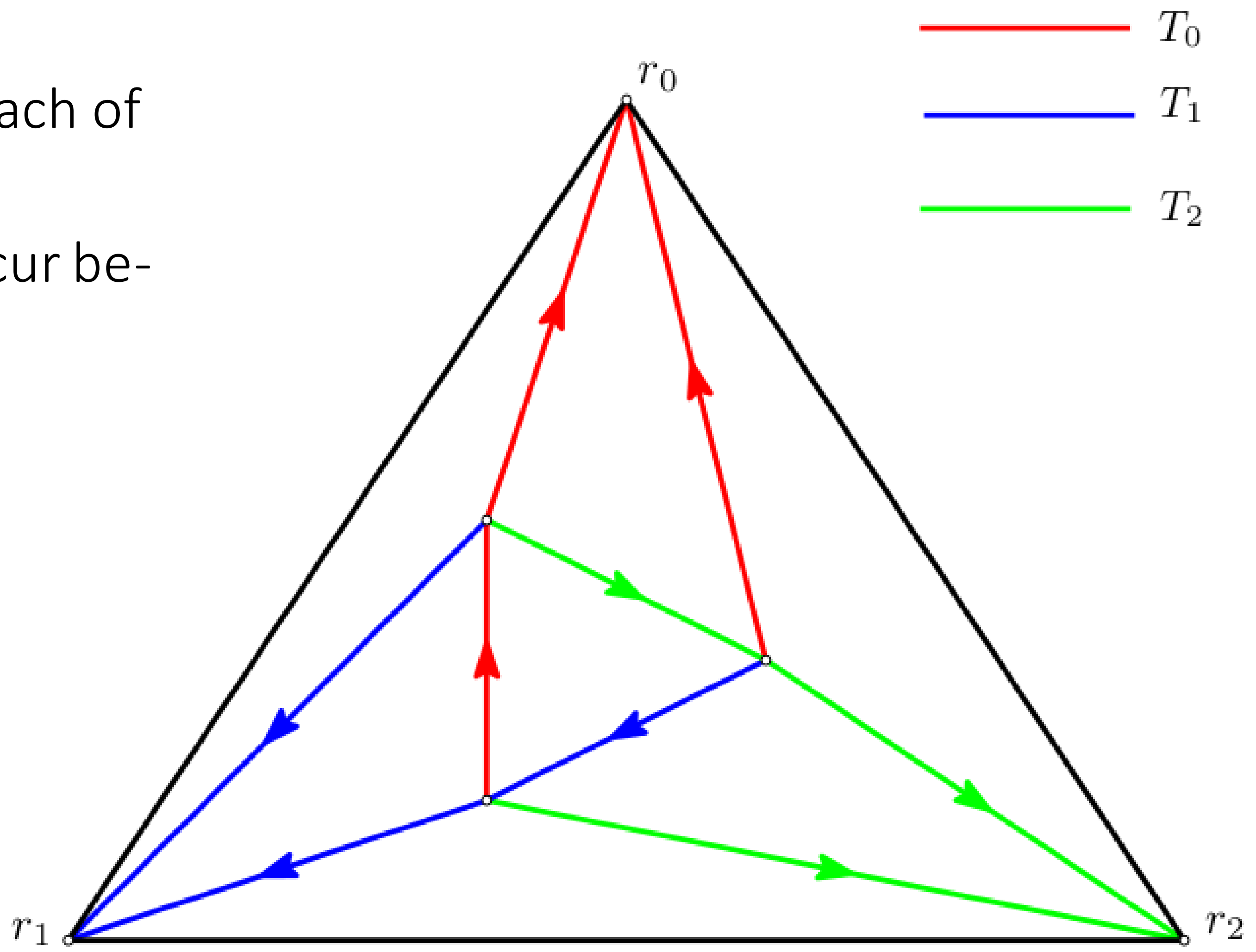
Triangulation



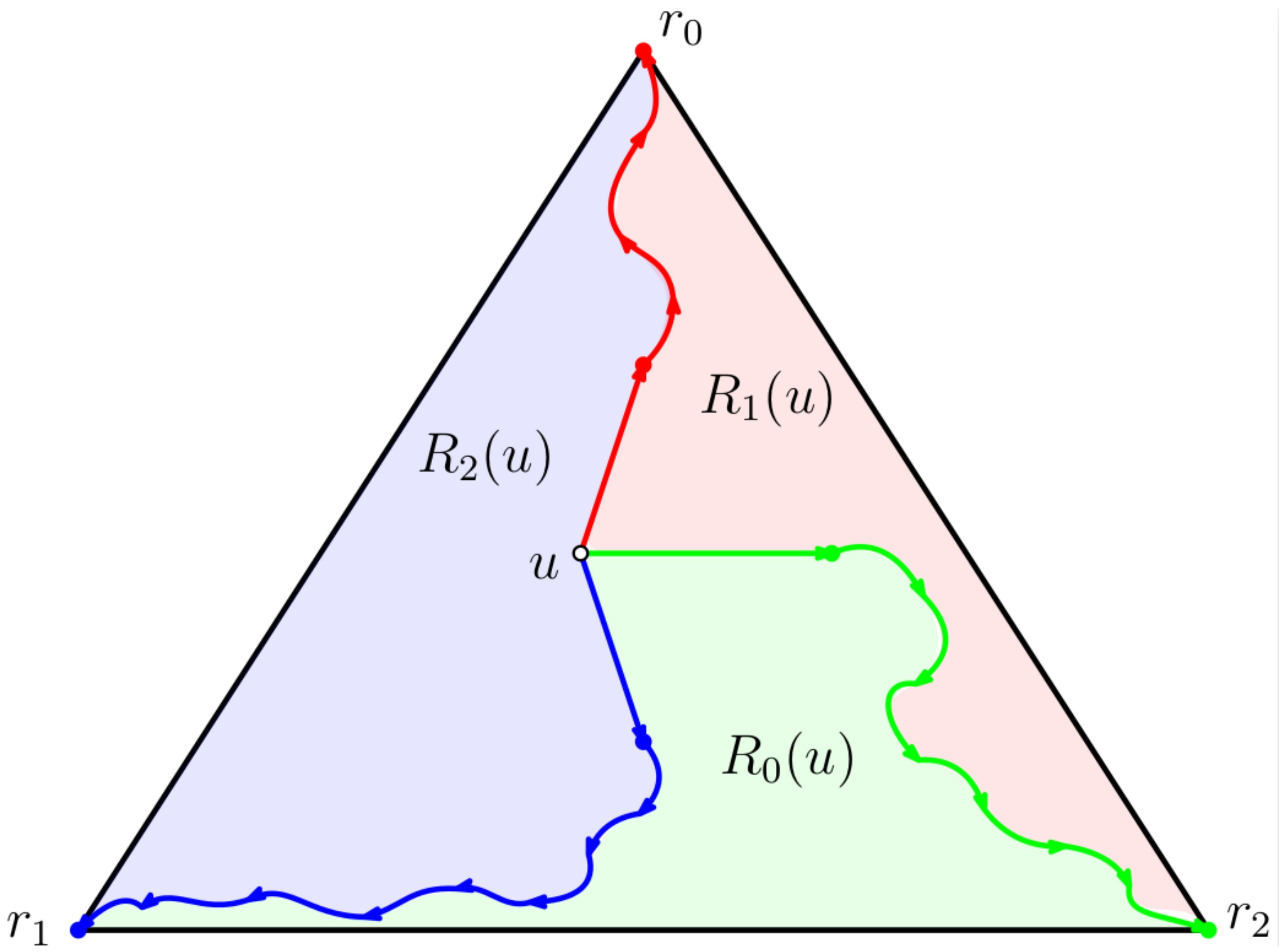
Schnyder Woods

- Vertex u has out-degree exactly one in each of T_0, T_1 and T_2 in counter-clockwise order.
- All incoming edges of T_i adjacent to u occur between the outgoing edge of T_j and T_k for distinct i, j, k mod 3





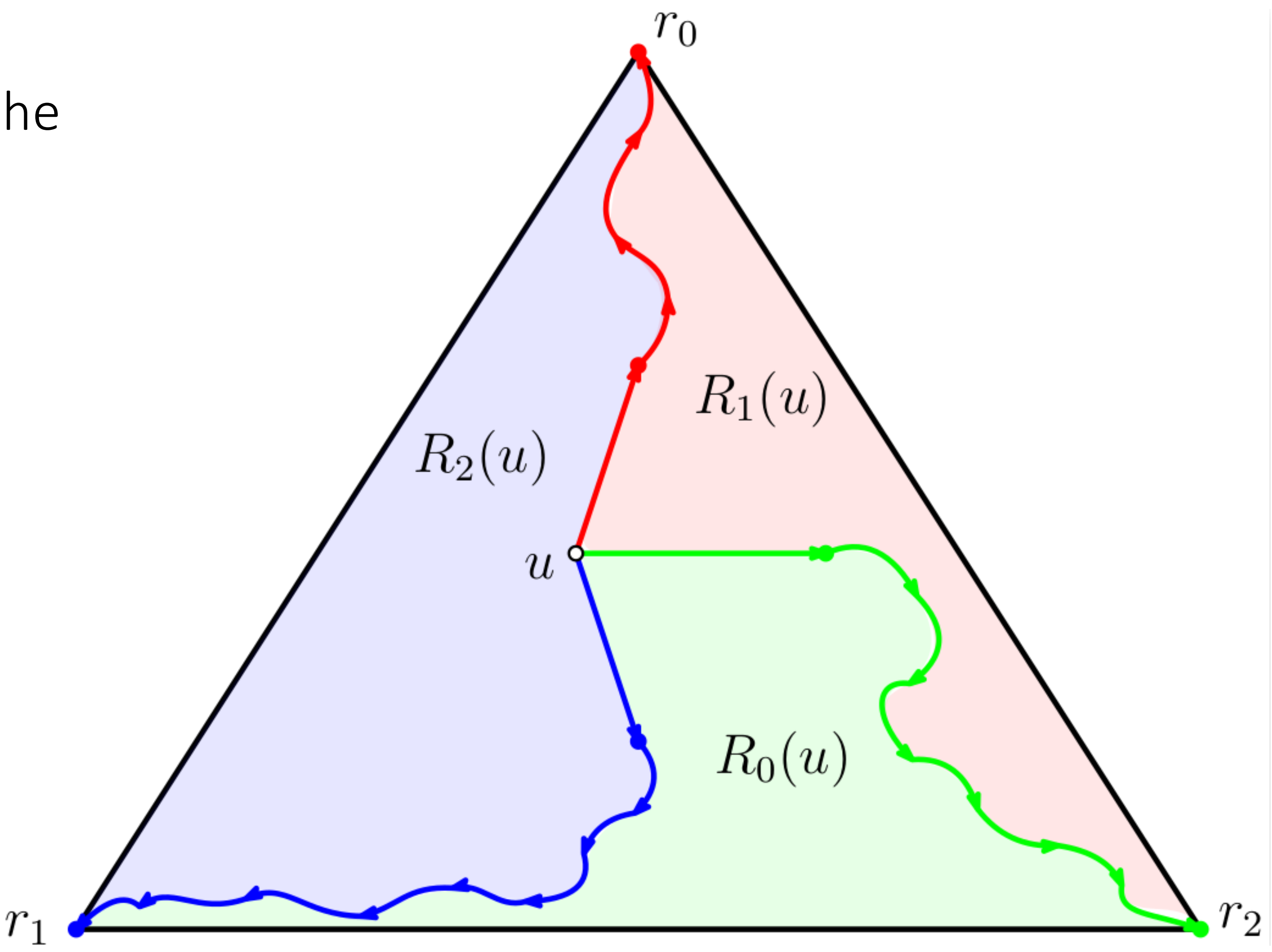
Schnyder Woods



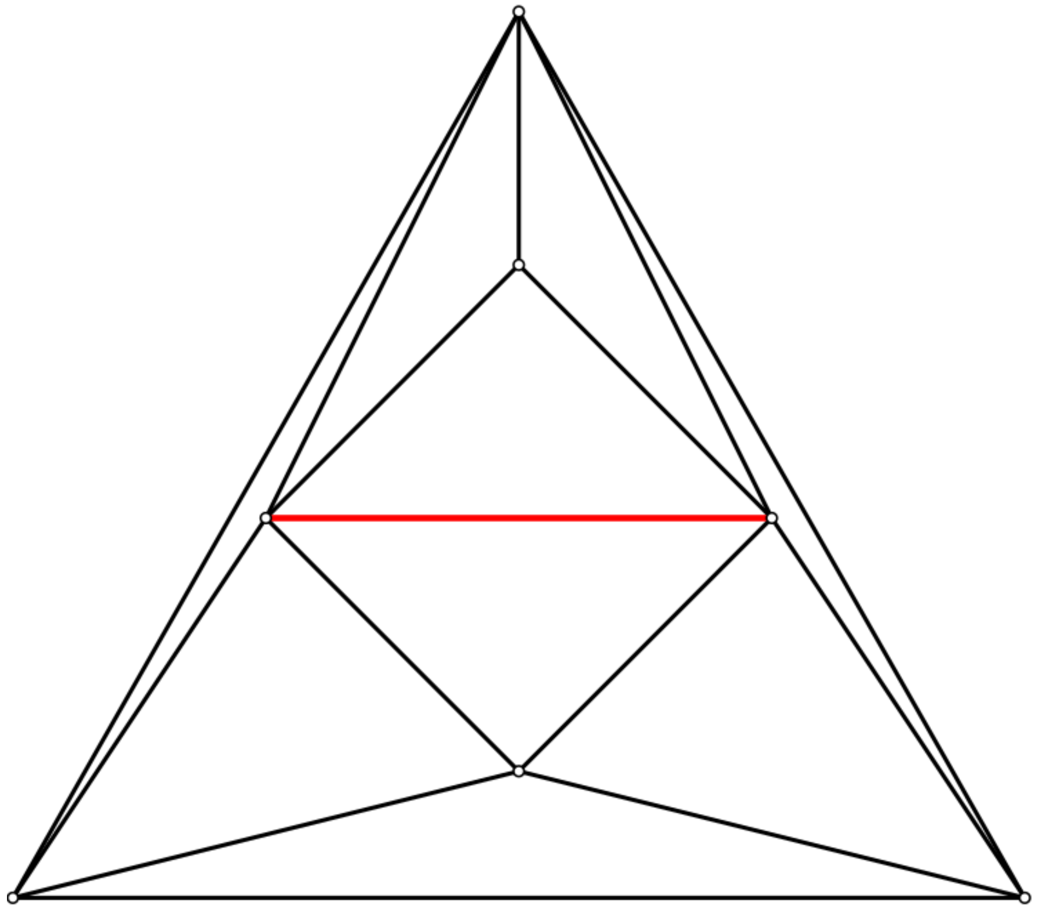
Schnyder Woods

Barycentric coordinates defined by the function:

$$f(u) = \frac{1}{n-1}(|R_0(u)|, |R_1(u)|, |R_2(u)|)$$



Diagonal flip



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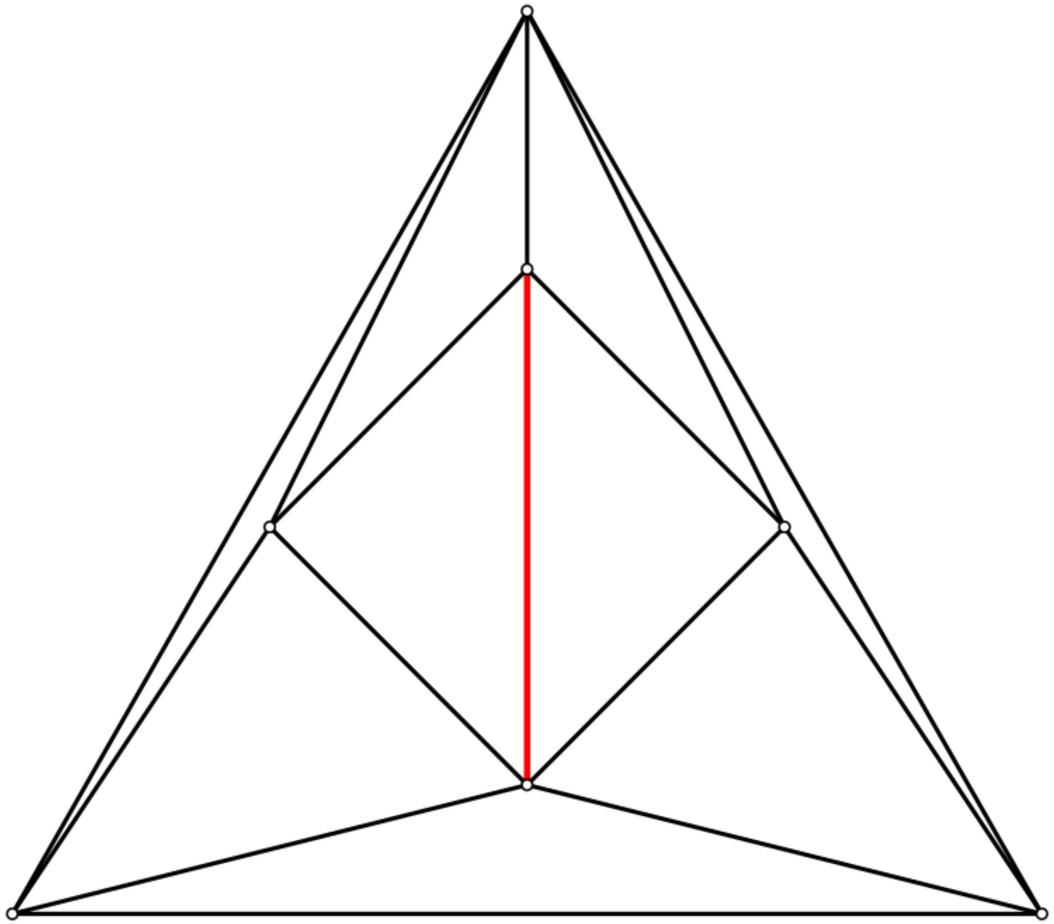
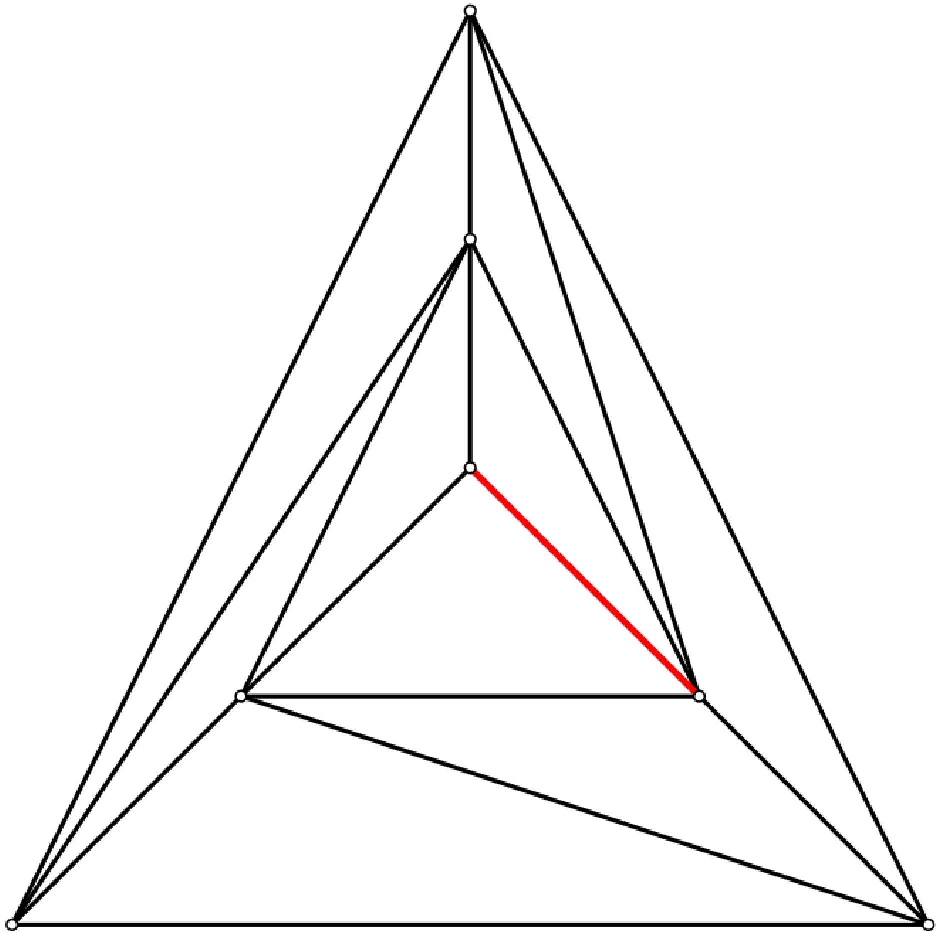
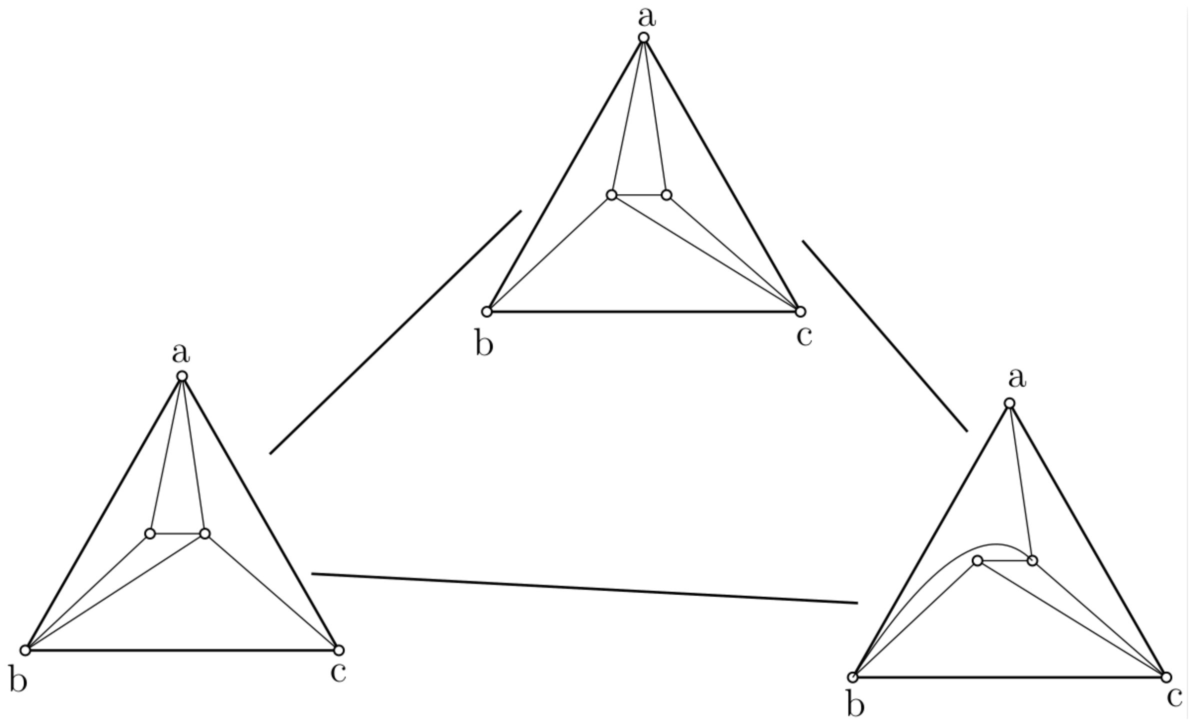


Not flippable

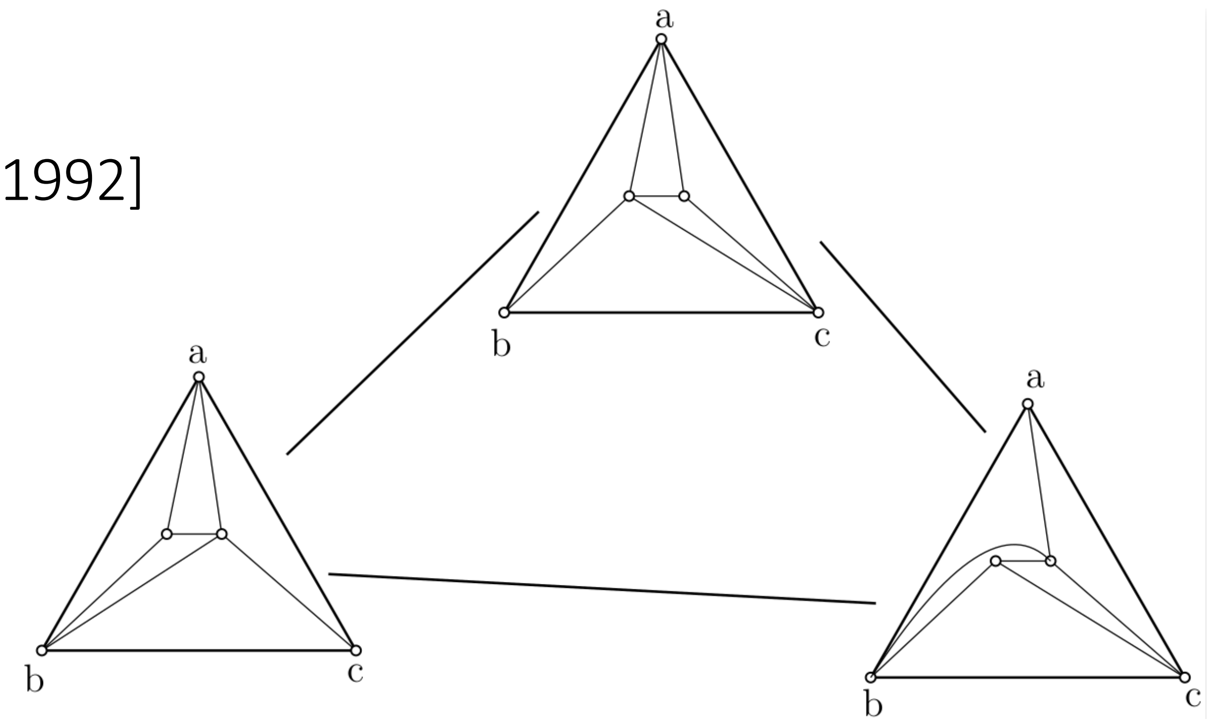


Flip graph of diagonal flips

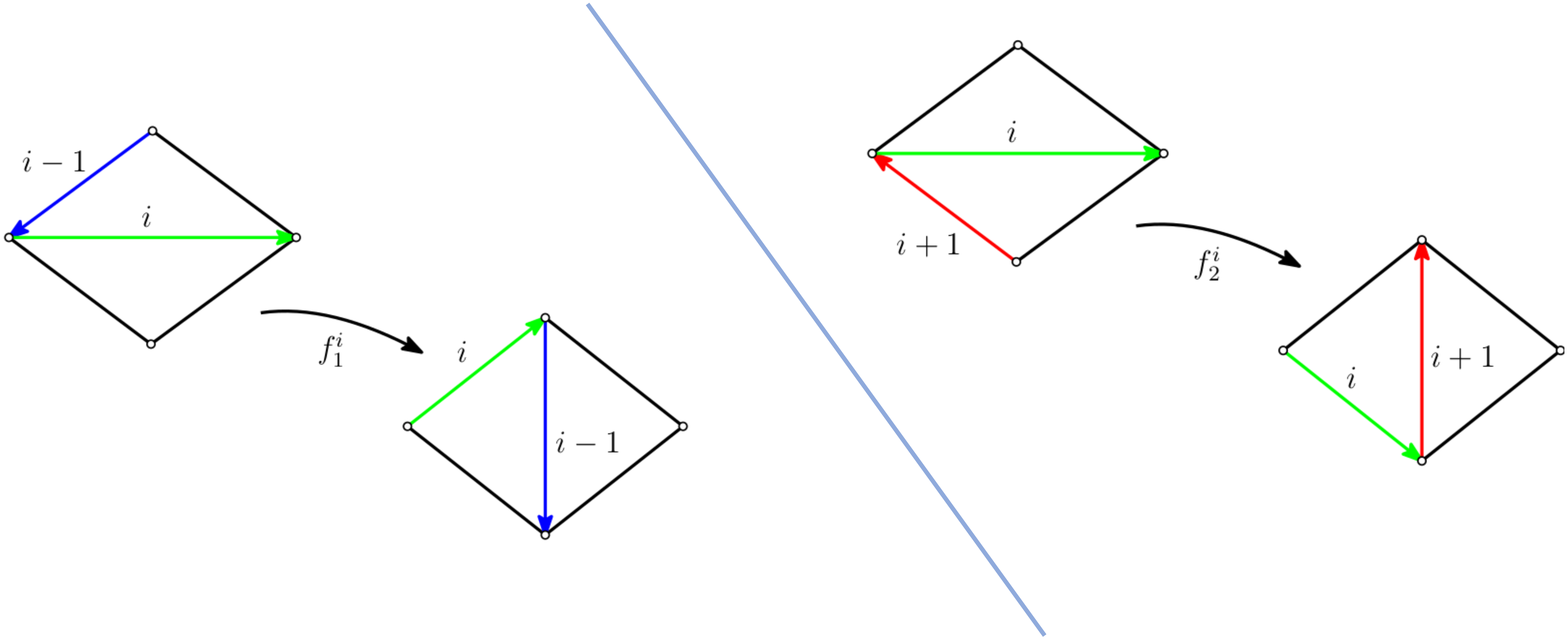


Flip graph of diagonal flips

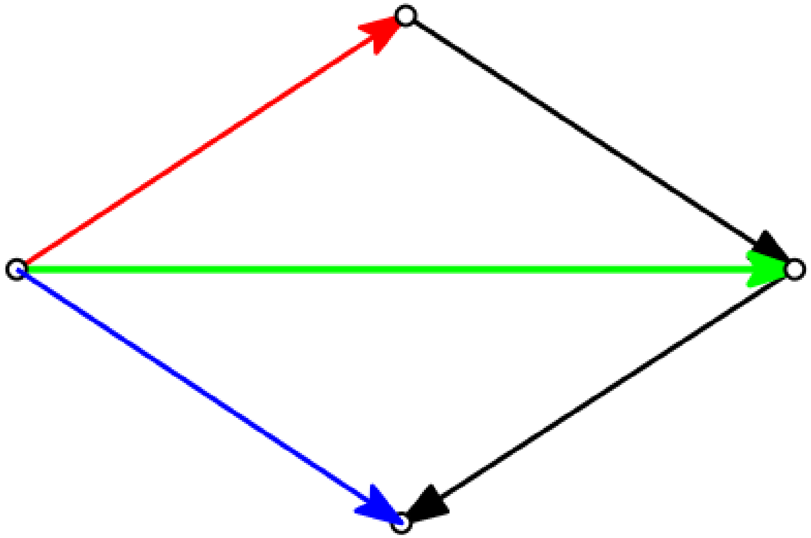
- T_n is connected [Wagner, 1936]
- T_n has diameter O(n) [Komuro et. al, 1992]



Colored flips



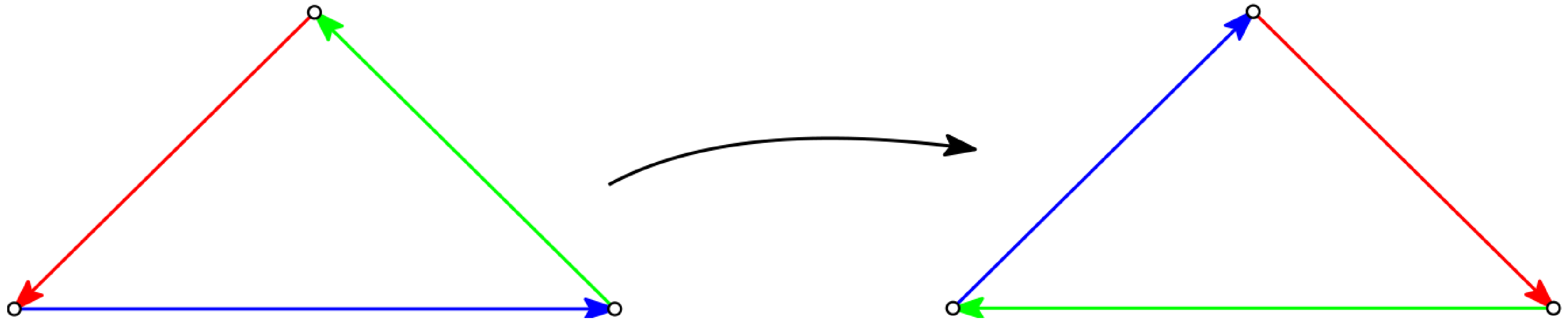
Not colored flippable



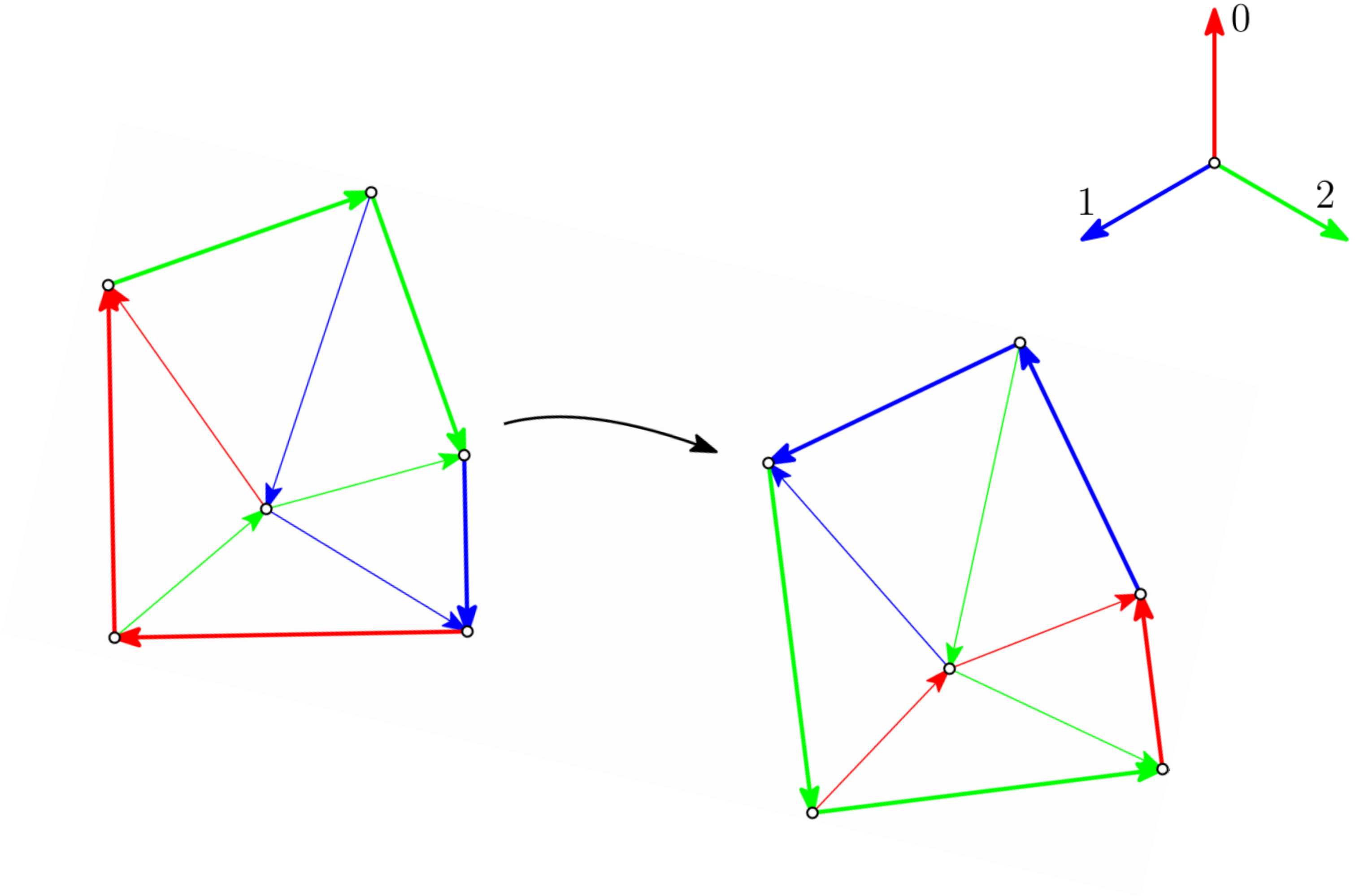
Flip graph of colored flips

- Bonichon et al. [2002] showed that the colored flip graph denoted R_n is connected.

Face flip



Cycle flip



Flip graph of cycle flips

- Note that a cycle flip transforms one Schnyder woods of a triangulation T into another Schnyder woods of the <u>same</u> triangulation T.

- Brehm showed that given a 4-connected triangulation T, the flip graph of face flips, denoted *R*(T), of the Schnyder woods of T is connected

- Brehm showed that a cycle flip can be obtained by a sequence of *m* face flips, where *m* is the number of face inside the cycle.

Our results

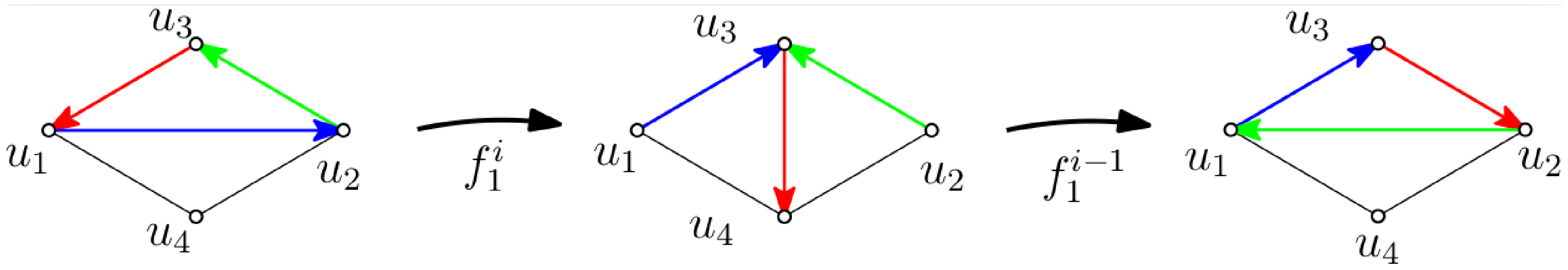
- For any triangulation T with diagonal flippable edge uv. There exists a R Schnyder wood of T where the oriented edge u→v in R is colored flippable.

- The diameter of R_n is O(n^2).

- A data structure is given to dynamically maintain a Schnyder wood over a sequence of colored flips which supports queries in O(log n) time per flip or query

Cycle flips and colored flips

- A face flip corresponds to exactly two colored flips



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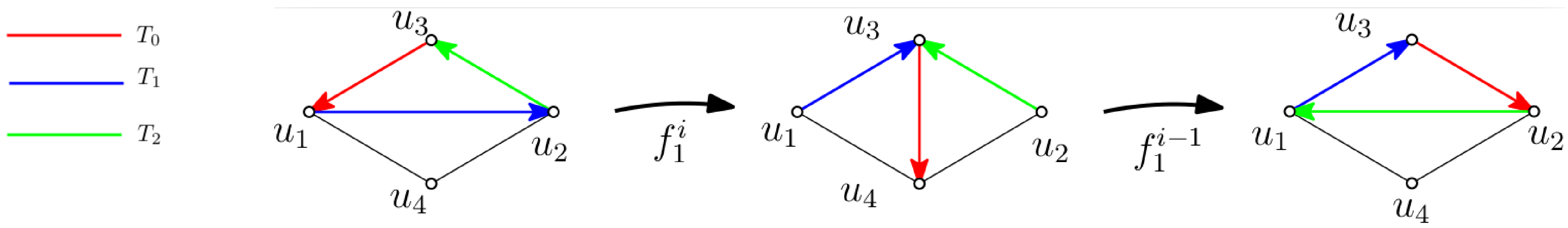

- A cycle flip corresponds to exactly $2m$ colored flips, where m is the number of interior faces in the corresponding cycle.

Diagonal flips and colored flips

- Let T be a triangulation with n vertices with diagonal flippable edge uv.
- Let R be a Schnyder wood of T

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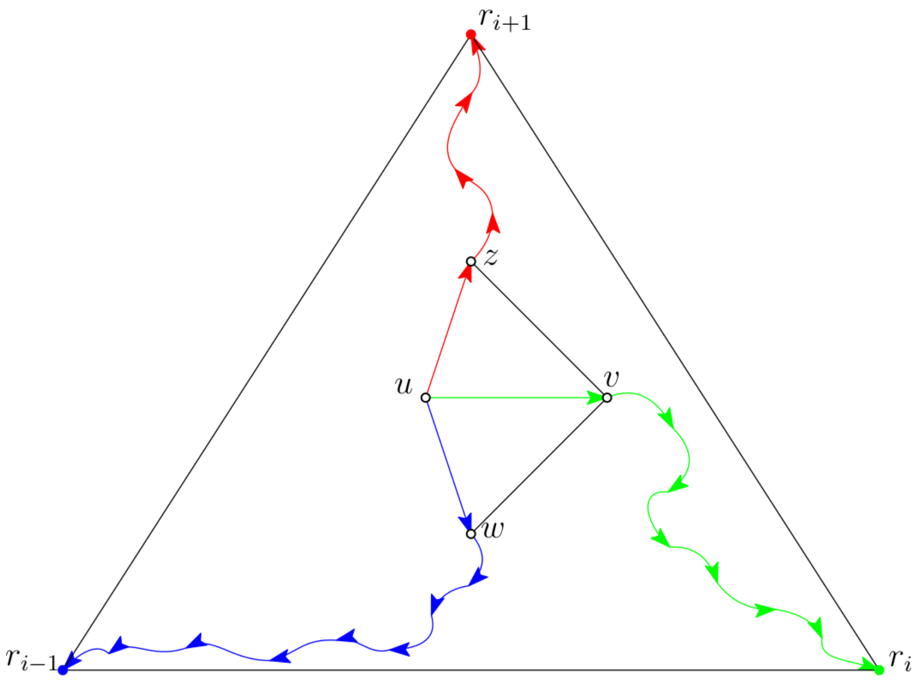

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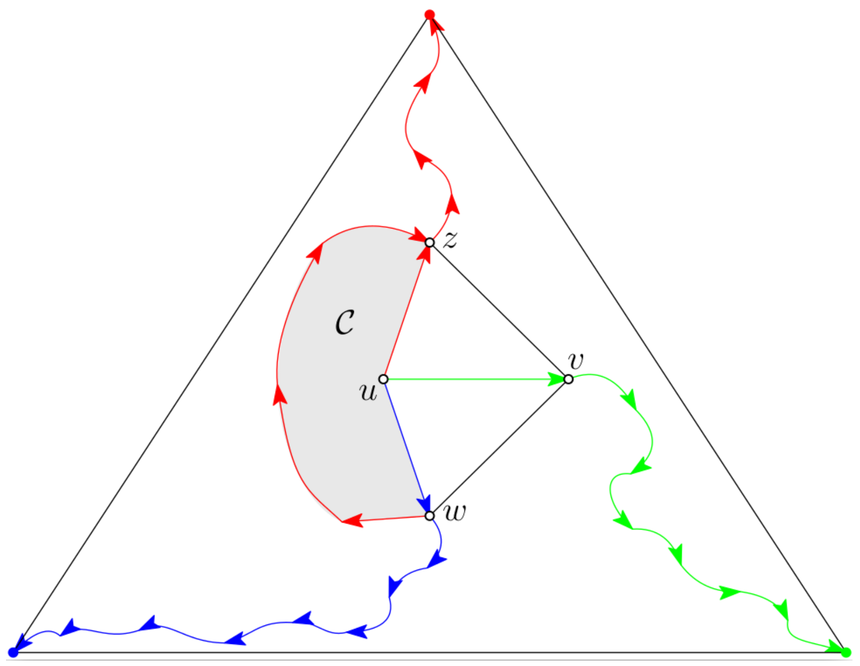

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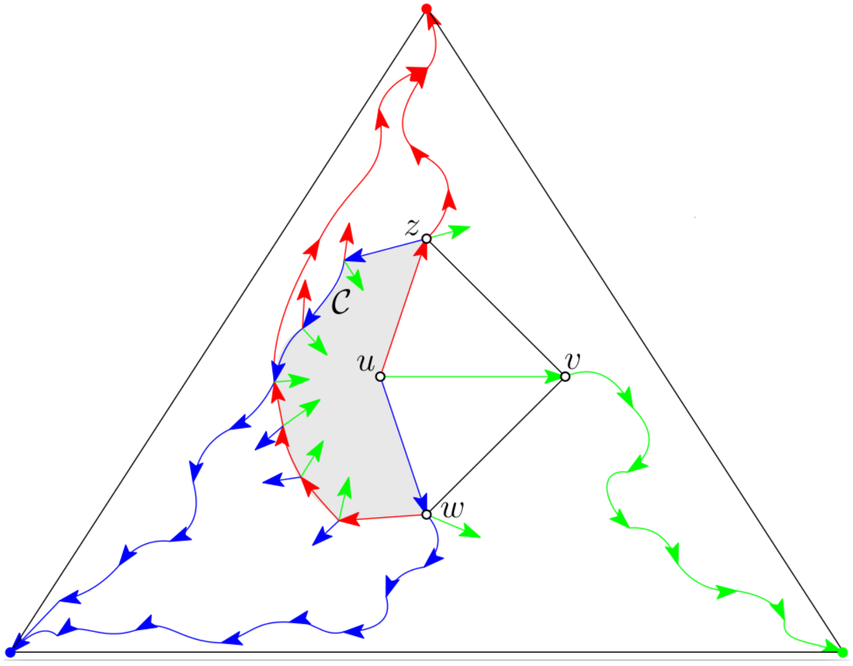

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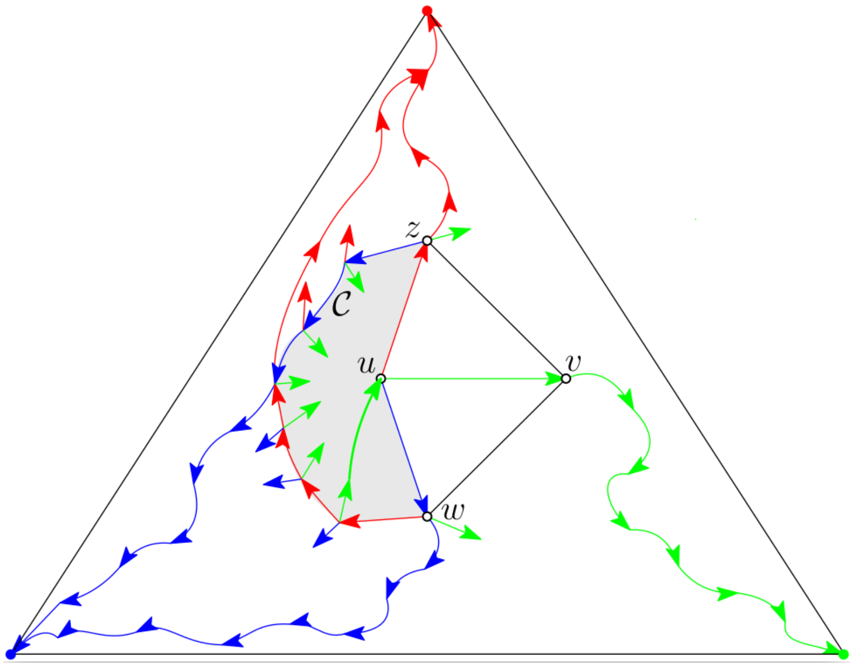

Theorem. *Let uv be a flippable edge in a triangulation T. Then, there exists a Schnyder wood R of T where the oriented edge* $u \rightarrow v$ *in R is colored flippable.*

Theorem. *The diameter of the colored flip graph of n vertices is* $O(n^2)$.

Dynamic Schnyder Woods

An Euler Tour tree(ETT) is a data structure [Henzinger and King, 1995]

- link(u, v): Add edge uv
- parent(u): Return parent of vertex u.
- cut(u): Delete edge uparent(u).
- cost(u): Return current cost in u
- *T*-updatecost(u, x): Add x to the cost of all vertices in subtree *T*(u).

The ETT supports these operations in worst case O(log n) time.

Dynamic Schnyder Woods

Consider data structure of a Schnyder wood as a ETT with trees T_0, T_1, T_2.

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- label(u, v): Return label of edge uv.

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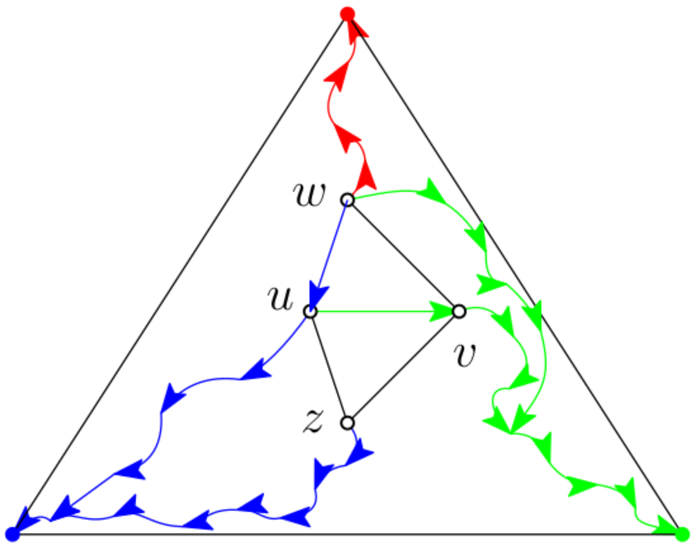
Consider data structure of a Schnyder wood as a ETT with trees T_0, T_1, T_2.

- label(u, v): Return label of edge uv.
- orientation(u, v): Return orientation of edge uv.

Dynamic Schnyder Woods

Consider data structure of a Schnyder wood as a ETT with trees T_0, T_1, T_2.

- label(u, v): Return label of edge uv.
- orientation(u, v): Return orientation of edge uv.
- coordinates(u): Return current barycentric coordinates of u.



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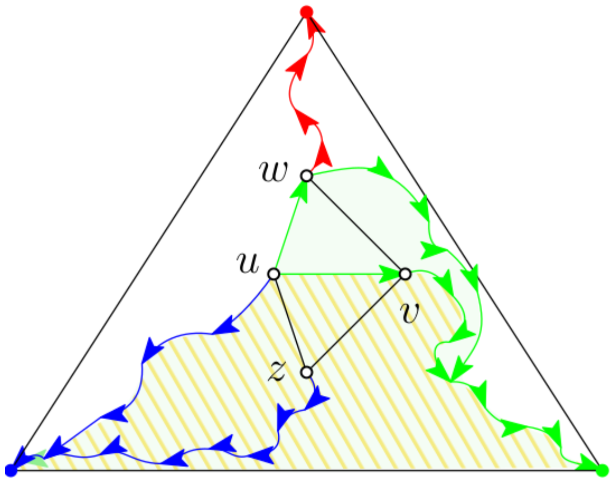

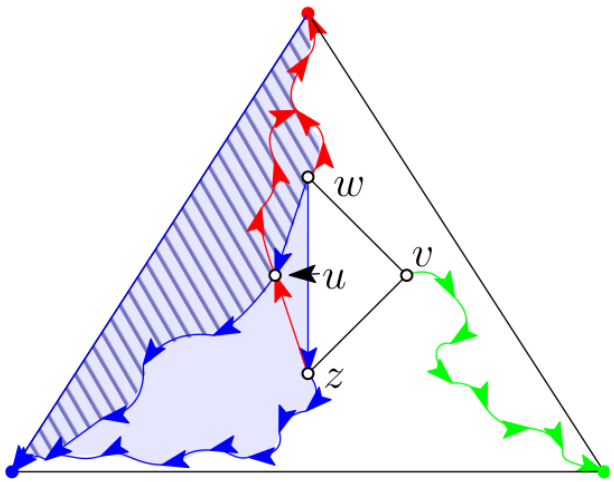

Dynamic Schnyder Woods

Consider data structure of a Schnyder wood as a ETT with trees T_0, T_1, T_2.

- label(u, v): Return label of edge uv.
- orientation(u, v): Return orientation of edge uv.
- coordinates(u): Return current barycentric coordinates of u.
- flip(u, v, w, z): Apply colored flip to edge u→v with respect to w→u.

Theorem. *A Schnyder wood of a triangulation can be maintained in amortized O(log n) per* flip. *Furthermore, queries* orientation, label coordinates *and* cost *can be obtained in O(log n) amortized time*

Summary

- The diameter of the colored flip graph is O(n^2)
- We present a data structure to dynamically maintain a Schnyder wood implicitly under colored flips while supporting queries to a corresponding straight line embedding over a sequence of colored flips in O(log n) amortized time per update or query.

Summary

- The diameter of the colored flip graph is O(n^2)
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Thanks!!! Merci!!