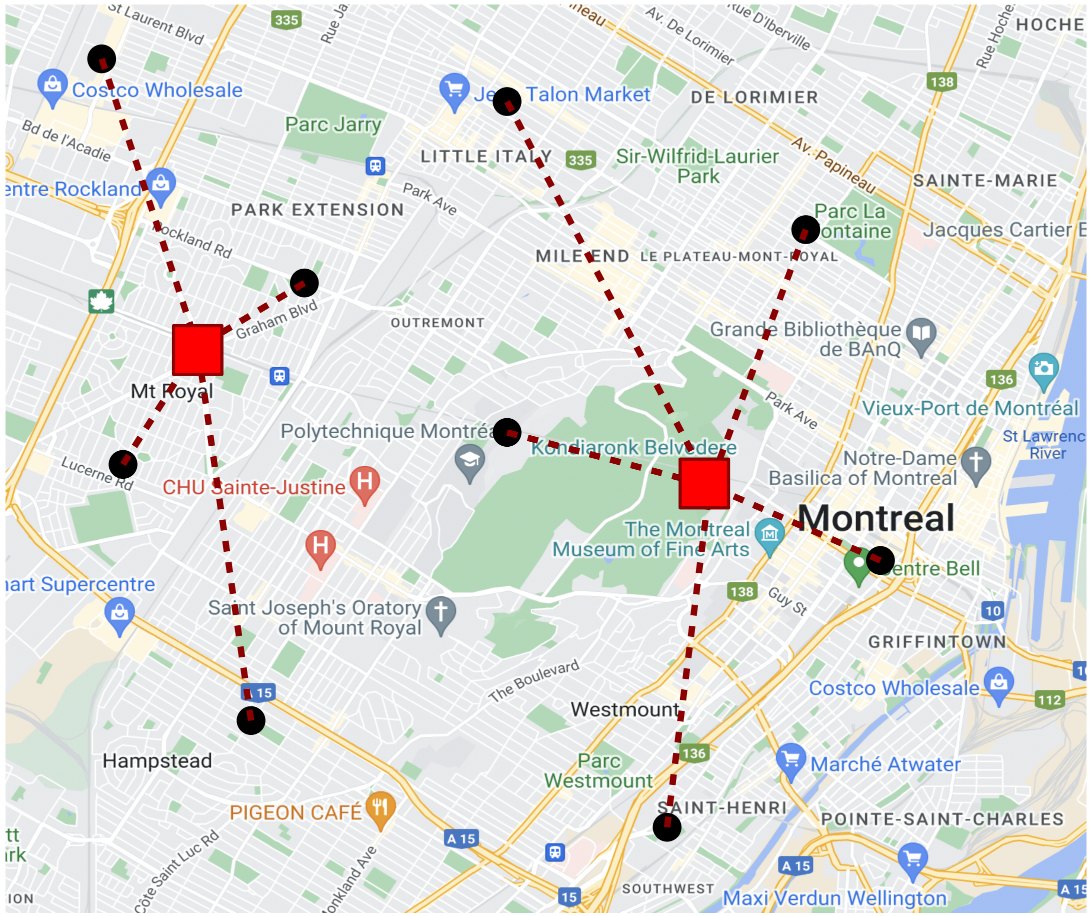
Parallel Line Centers with Guaranteed Separation

CCCG 2023
August 3

Chaeyoon Chung, Taehoon Ahn, Sang Won Bae, Hee-Kap Ahn

Facility-location problems



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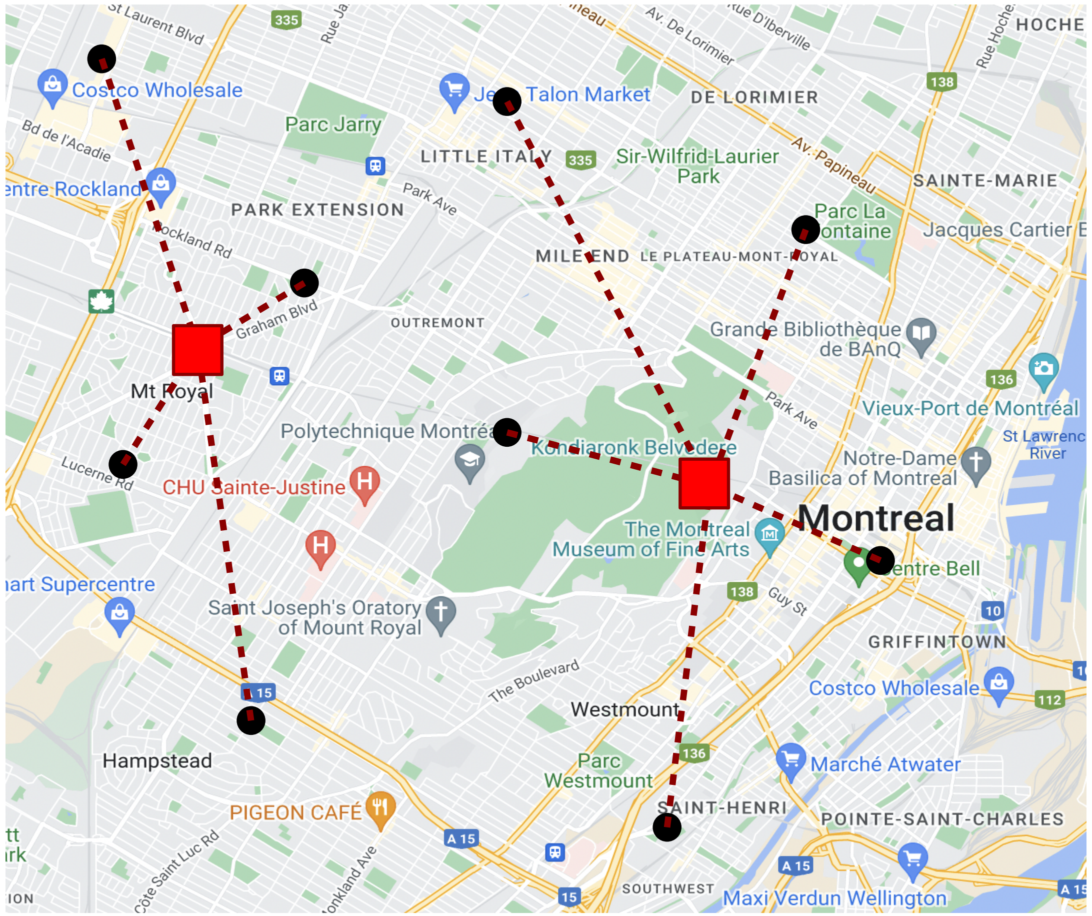
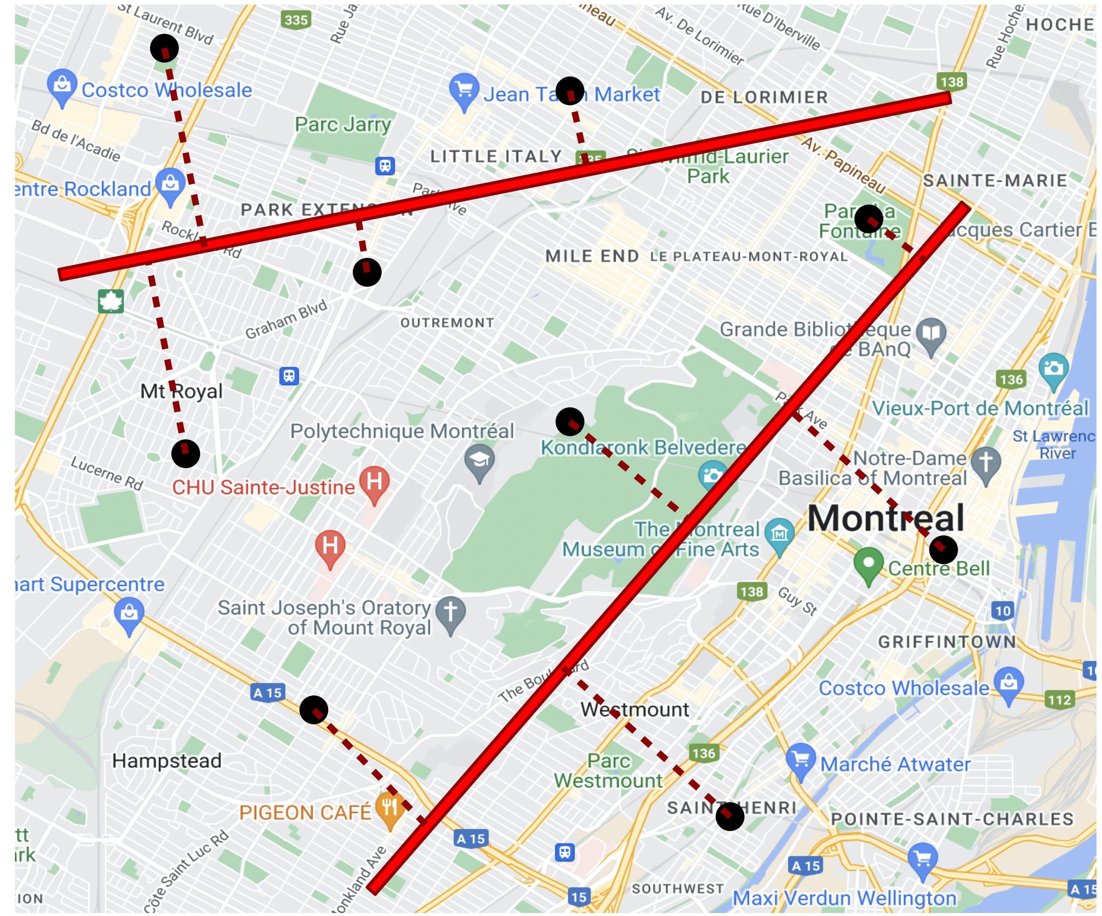
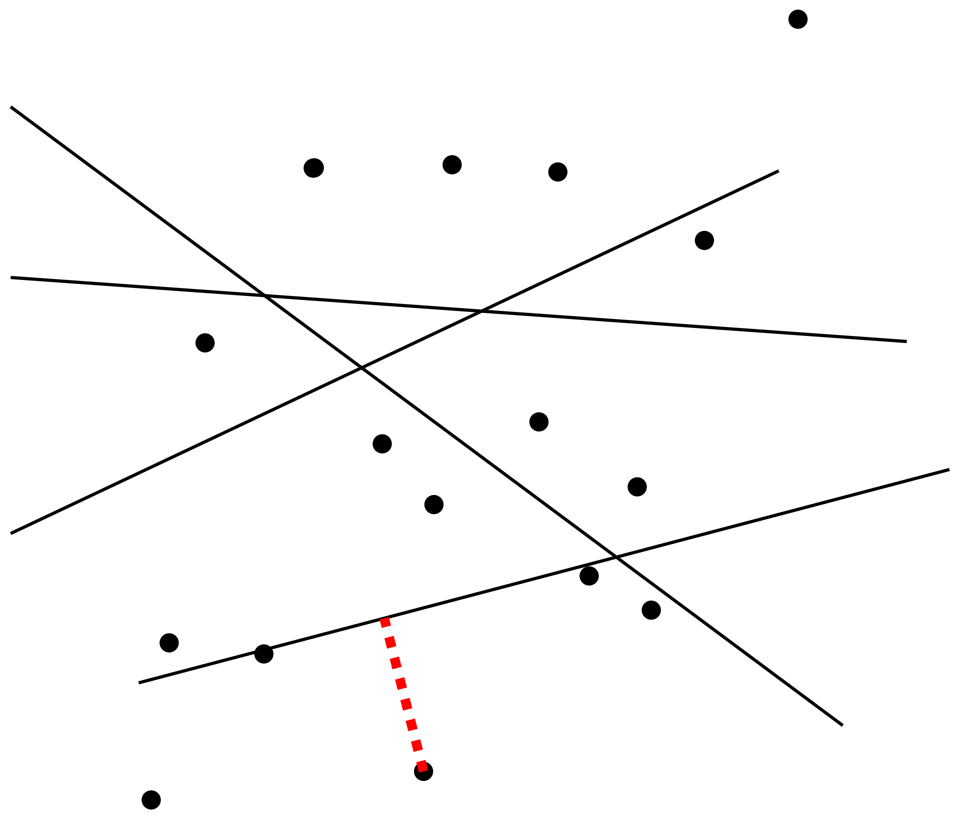




k-line-center problem



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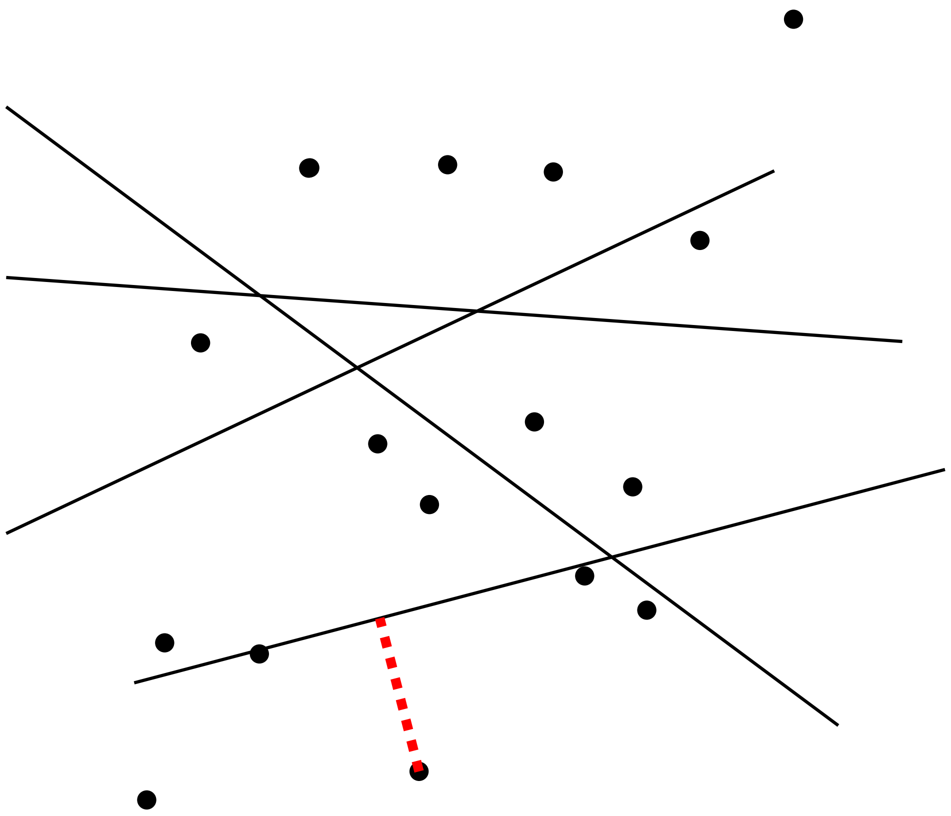
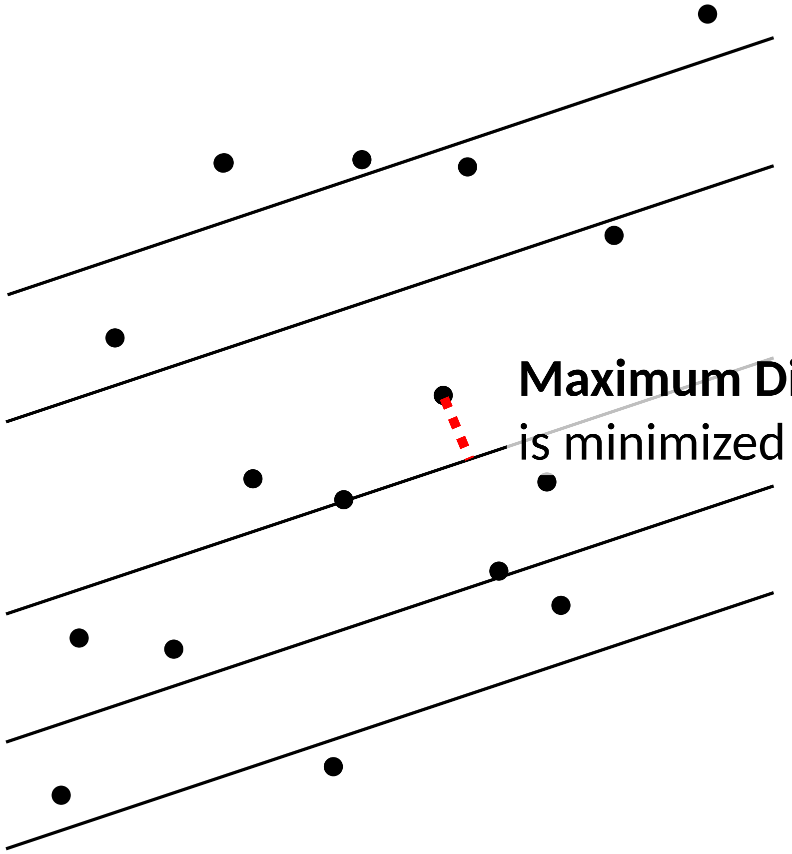


k-**parallel**-line-center problem



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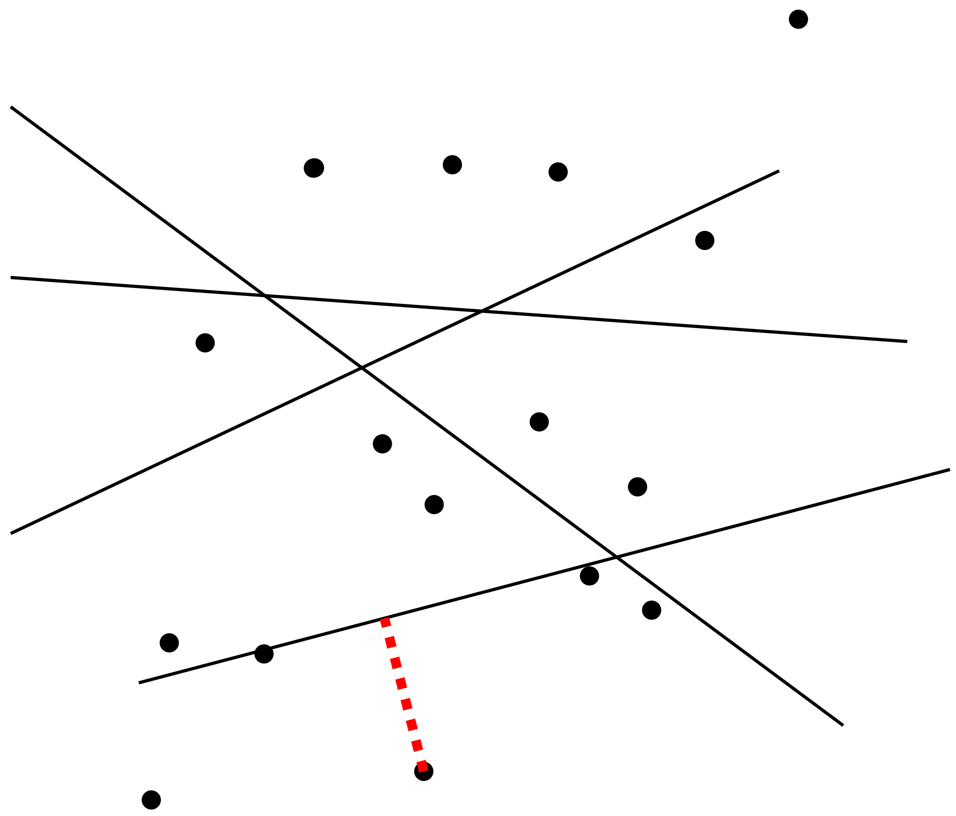

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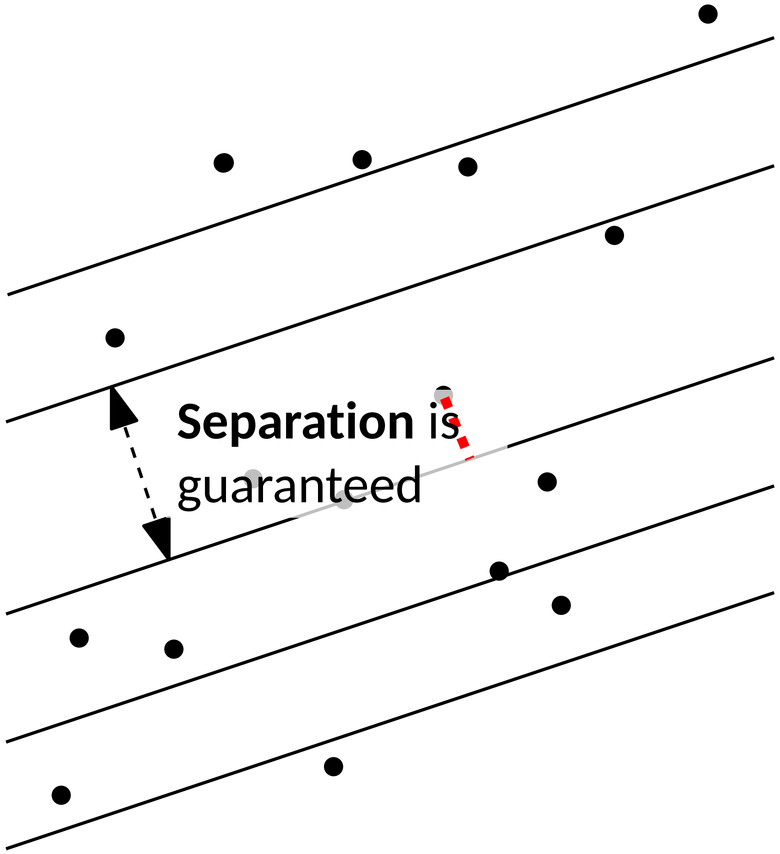

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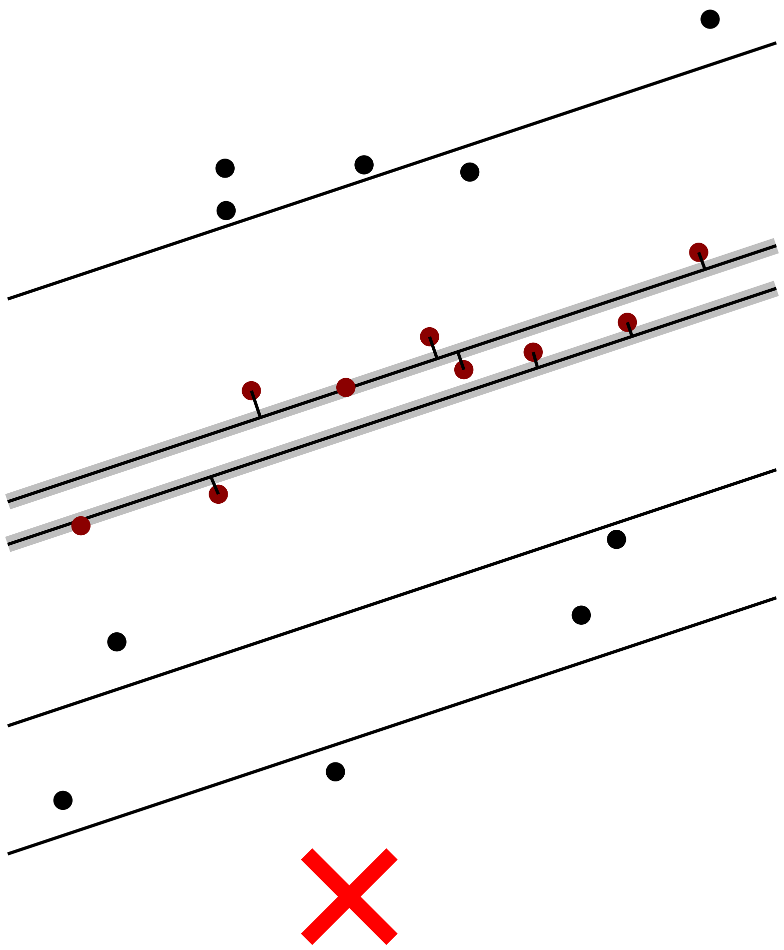

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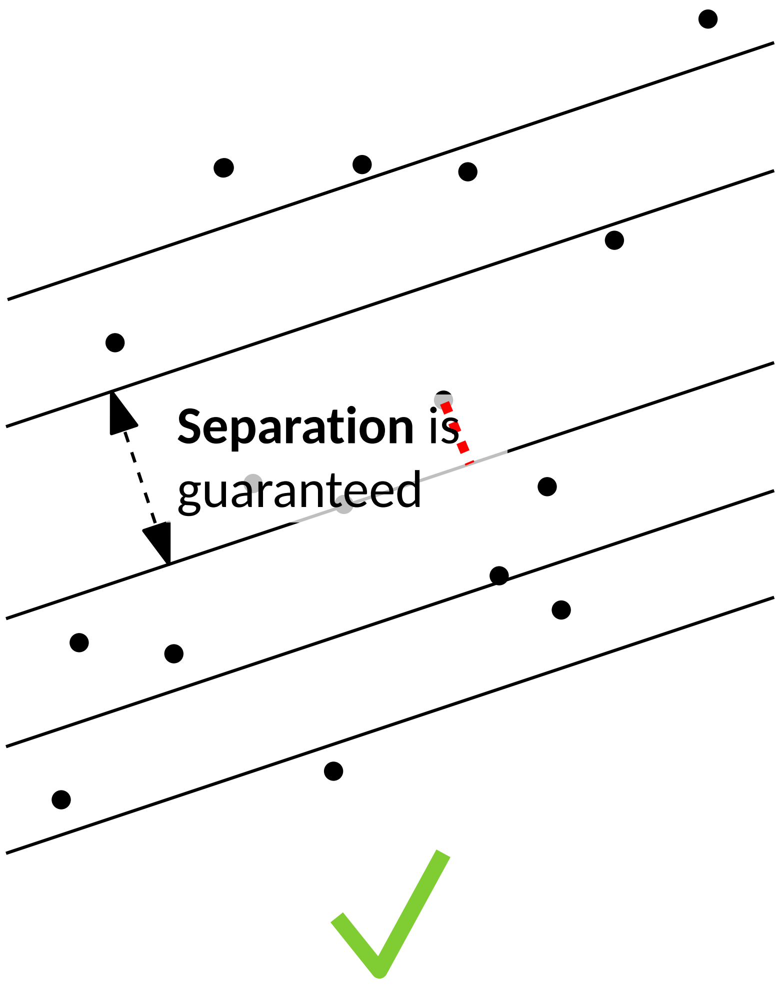
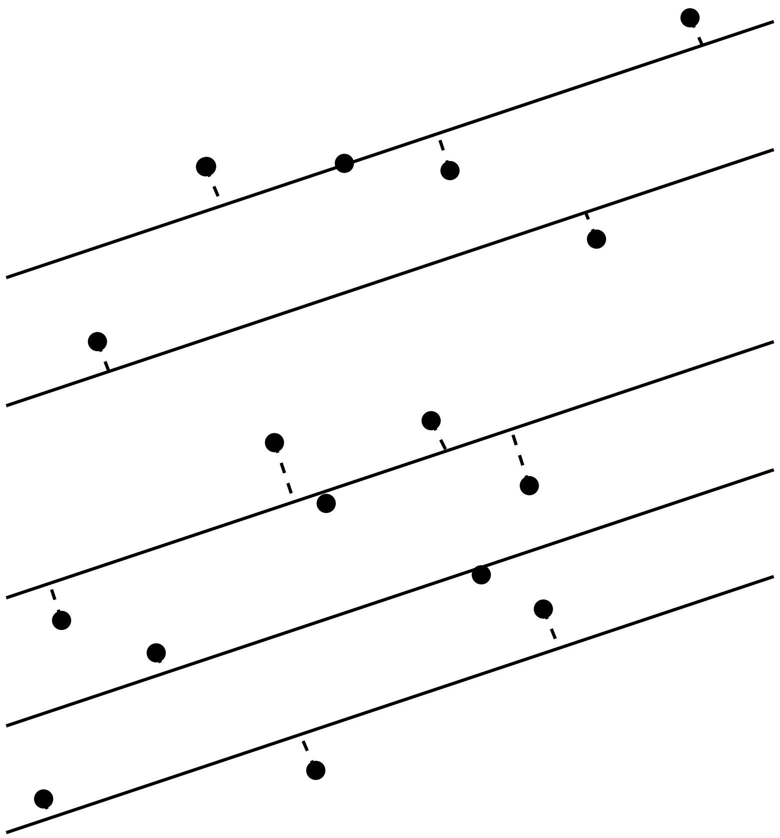
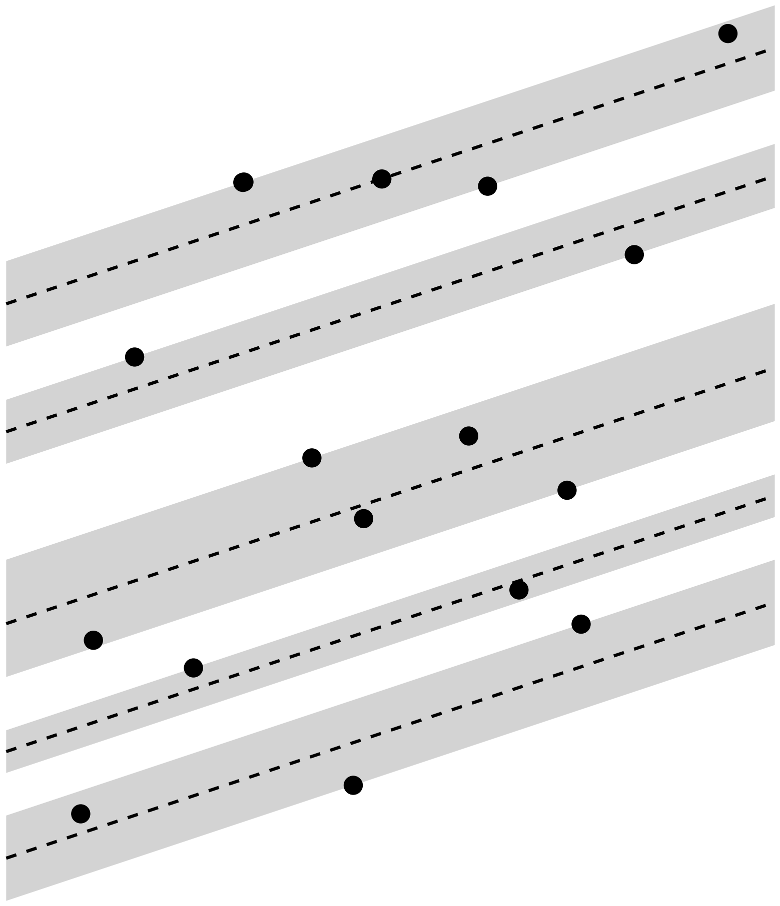


For a given point set P,

k-parallel-line-center of P

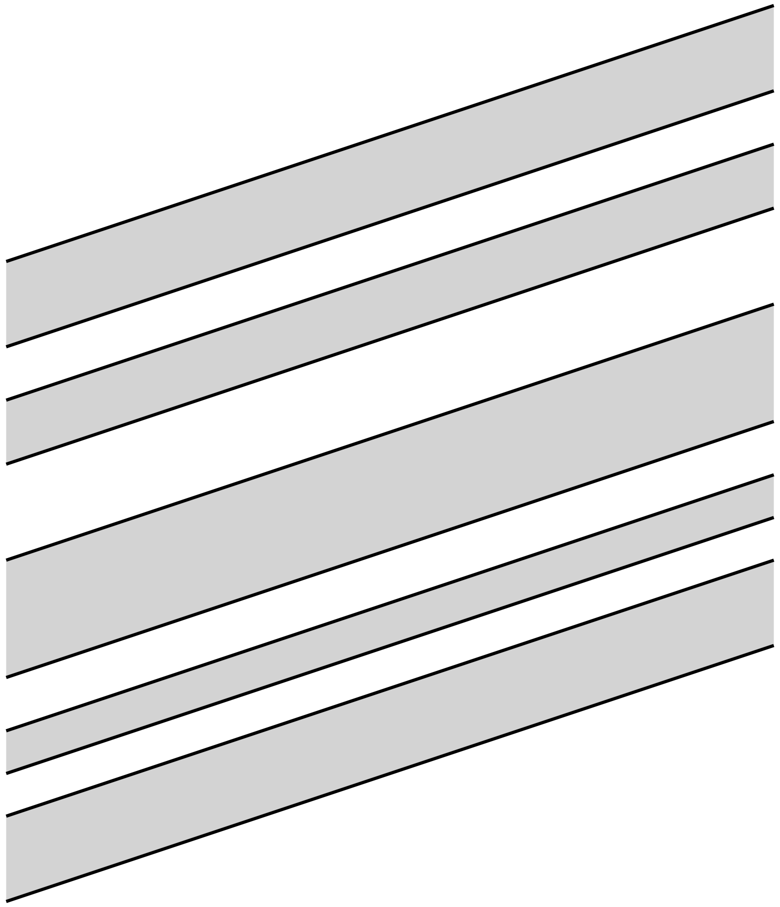


k-**slab cover** of P



Given a set of *k* slabs,

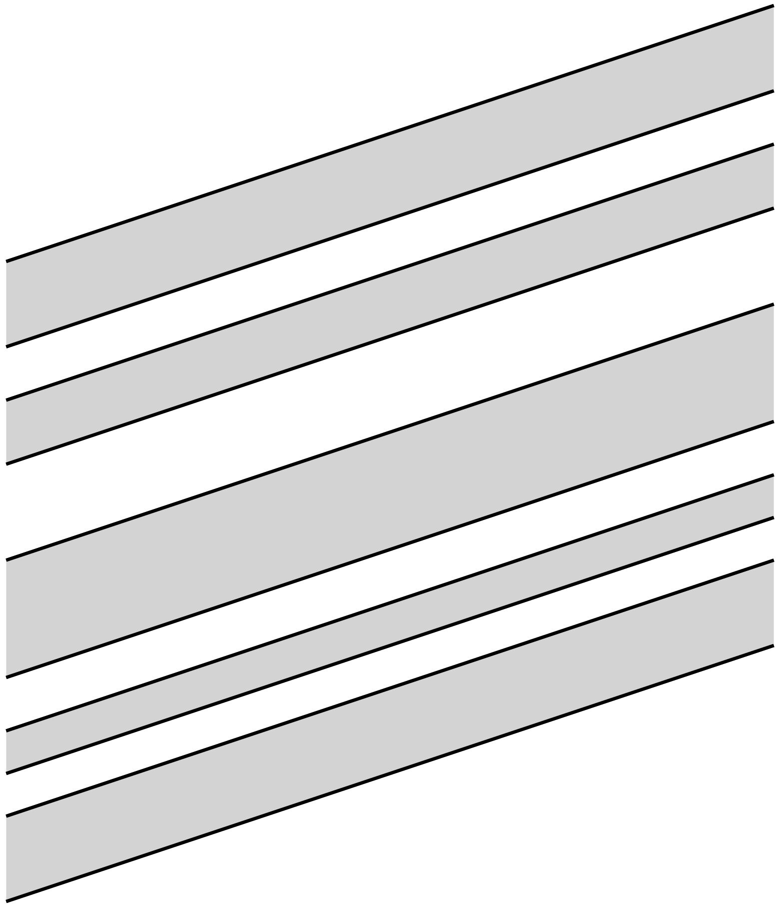
***k*-slab**



Given a set of k slabs,

Given a set P of points,

***k*-slab**



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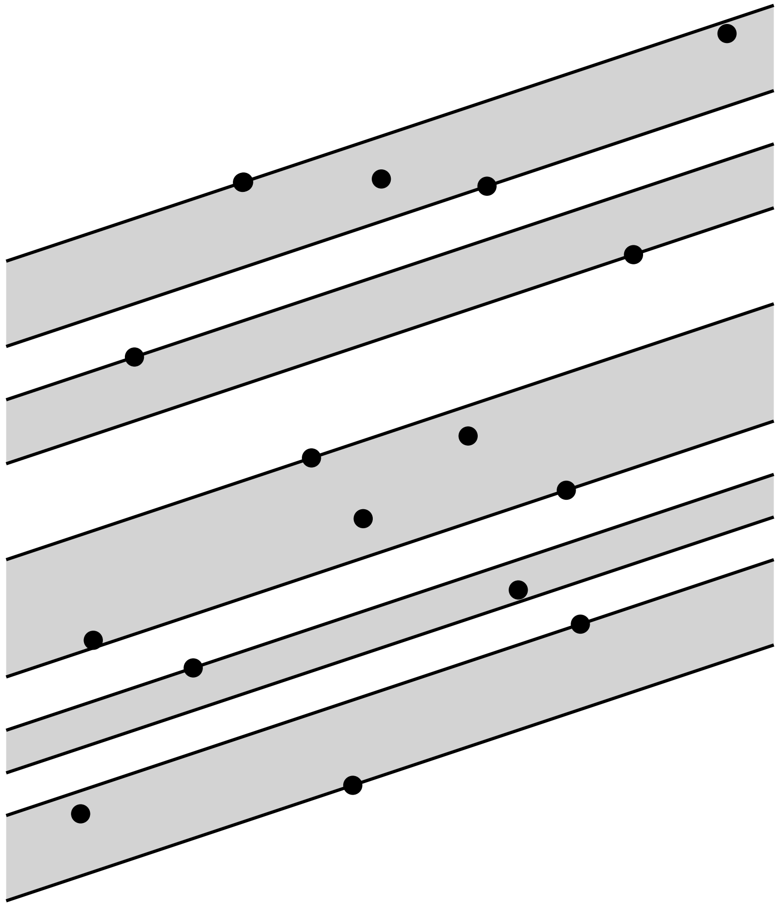
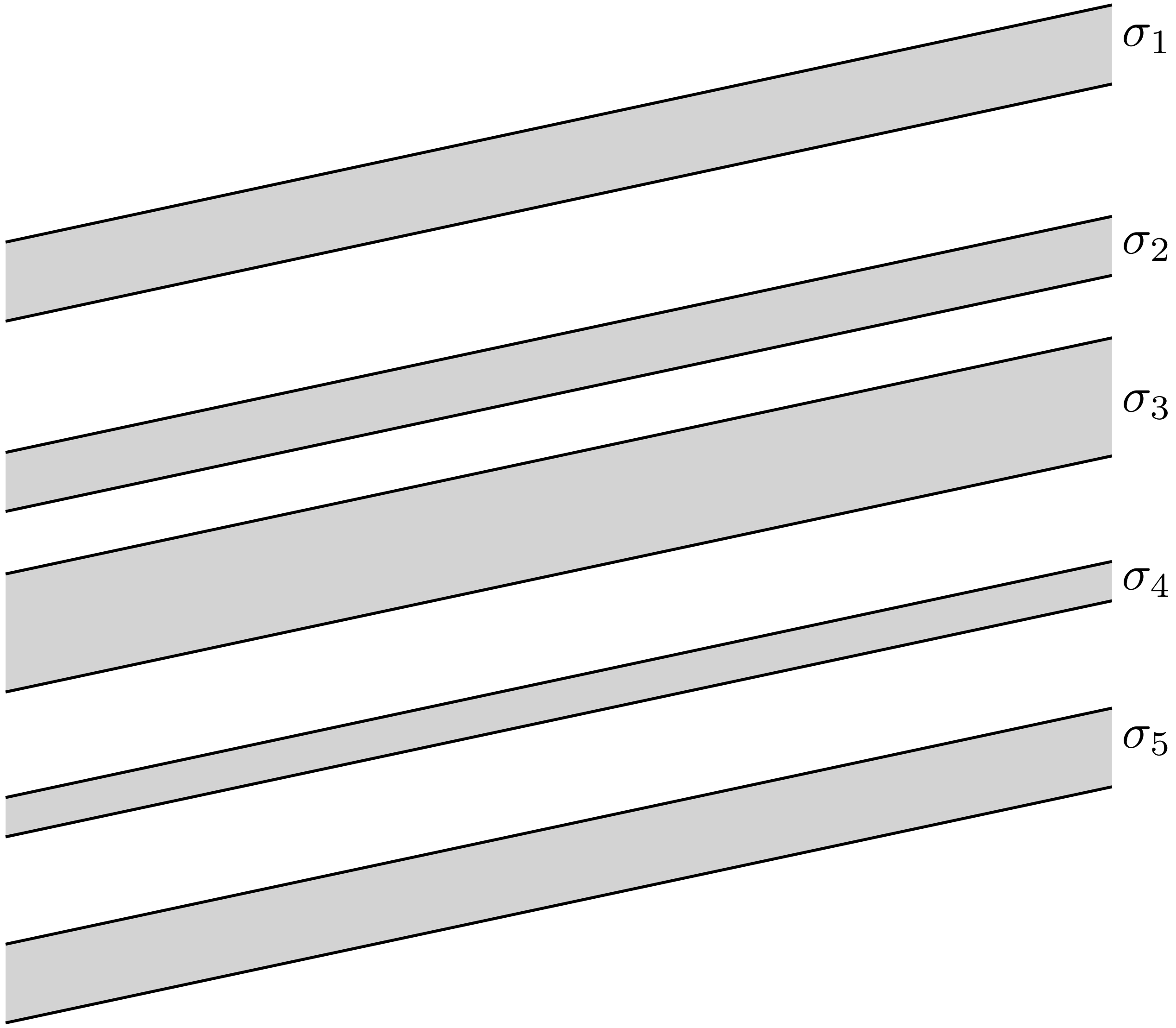


k-slab $\mathsf{S} = (\sigma_1, \ldots, \sigma_k)$ when $k = 5$



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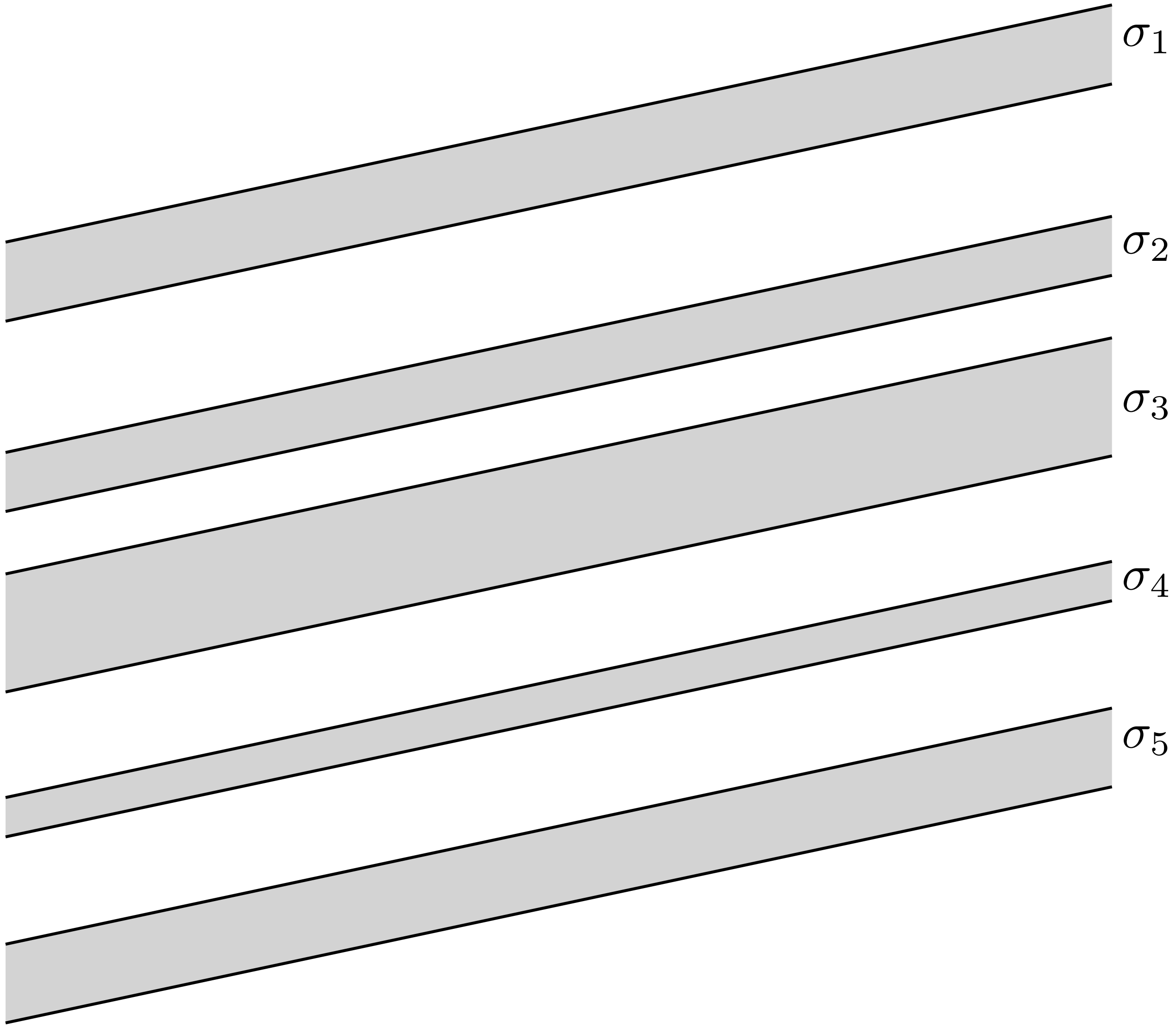

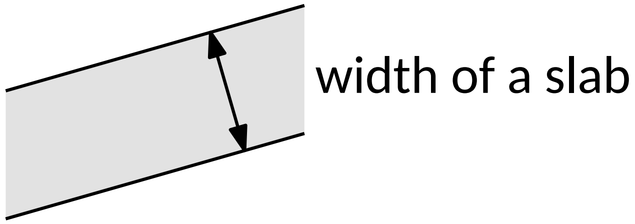

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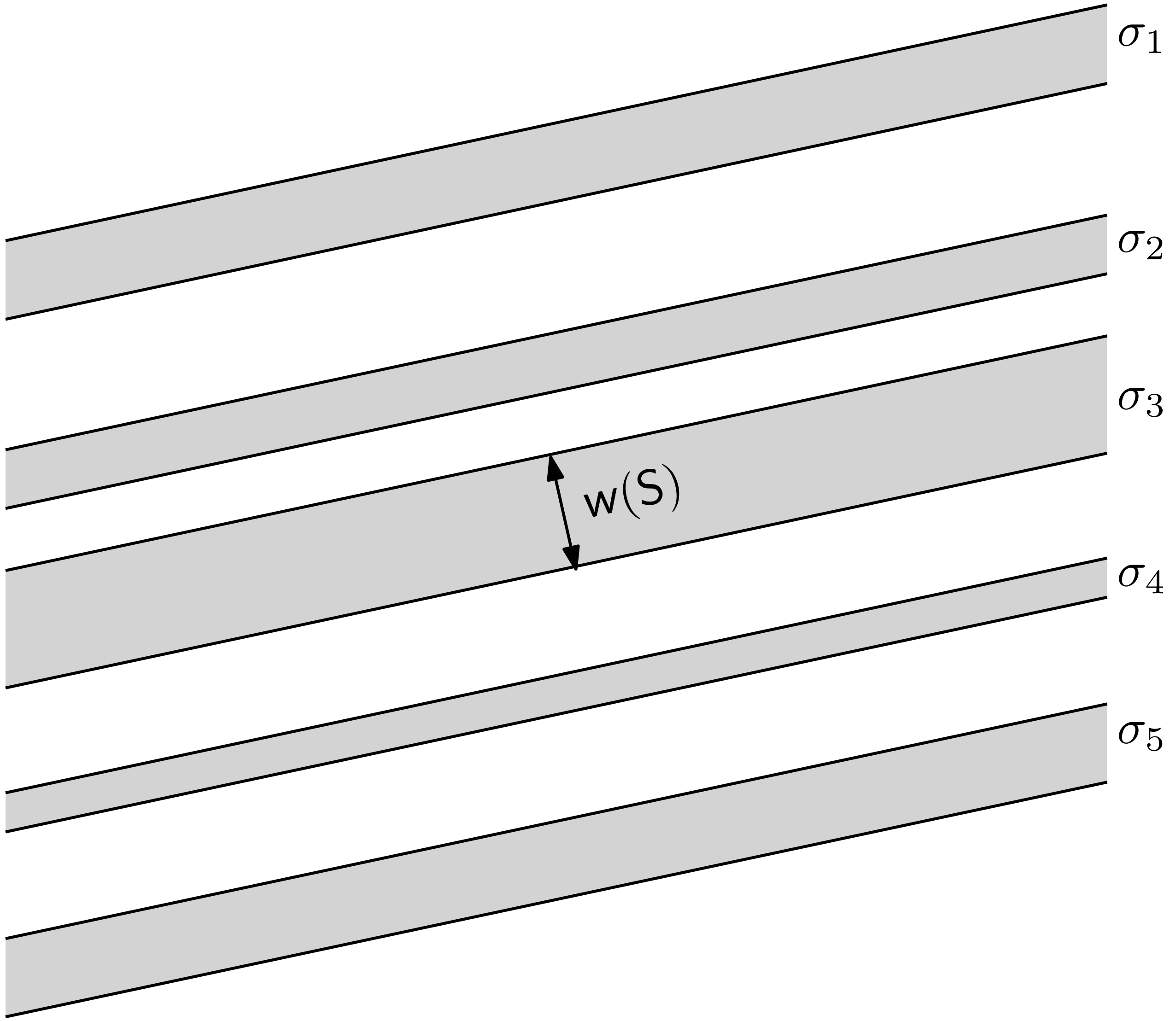

The width of S

- $w(S) := \max\{w(\sigma_1), w(\sigma_2), \dots\}$

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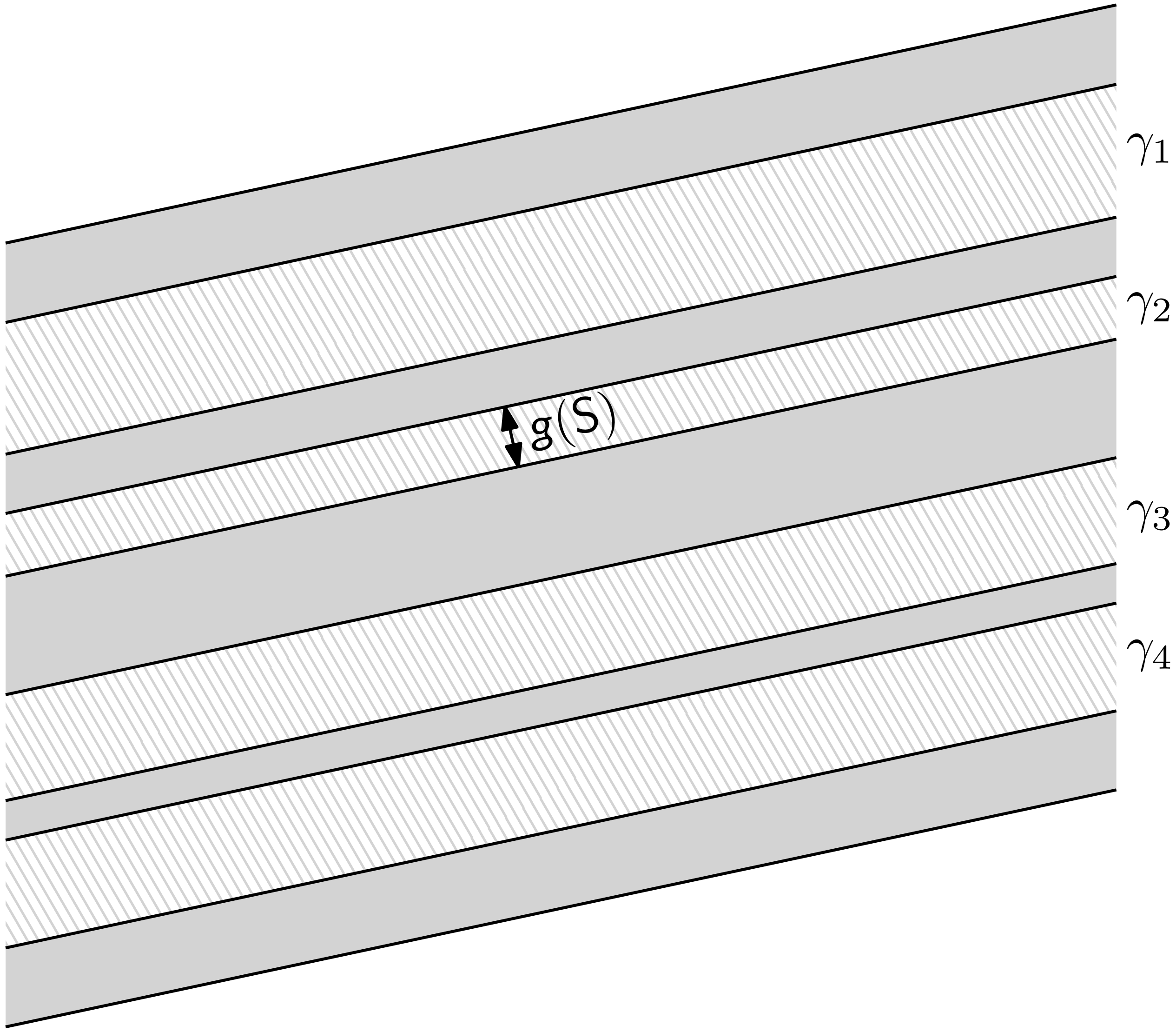

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The gap-width of S

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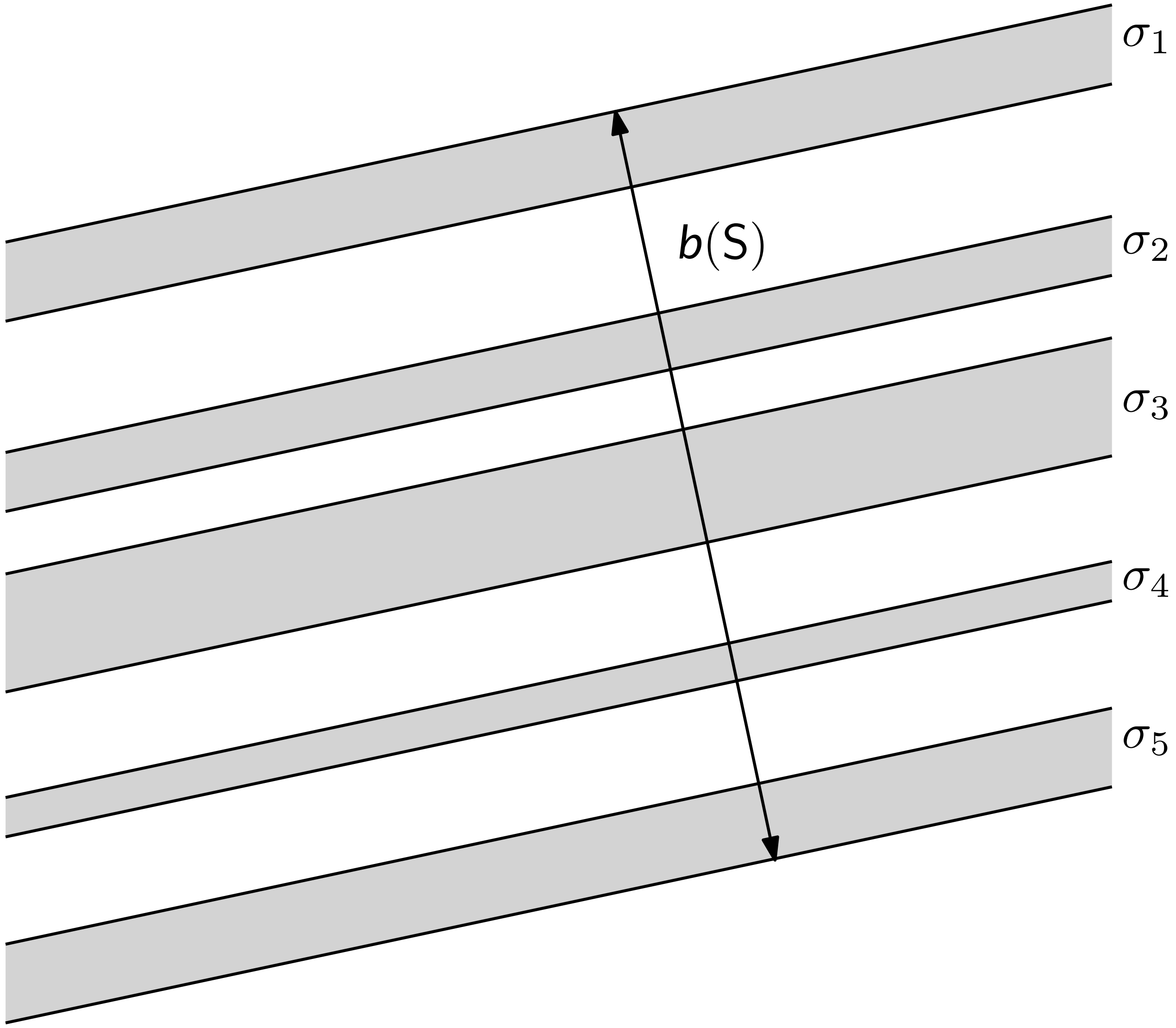

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The breadth of S

- $b(S)$

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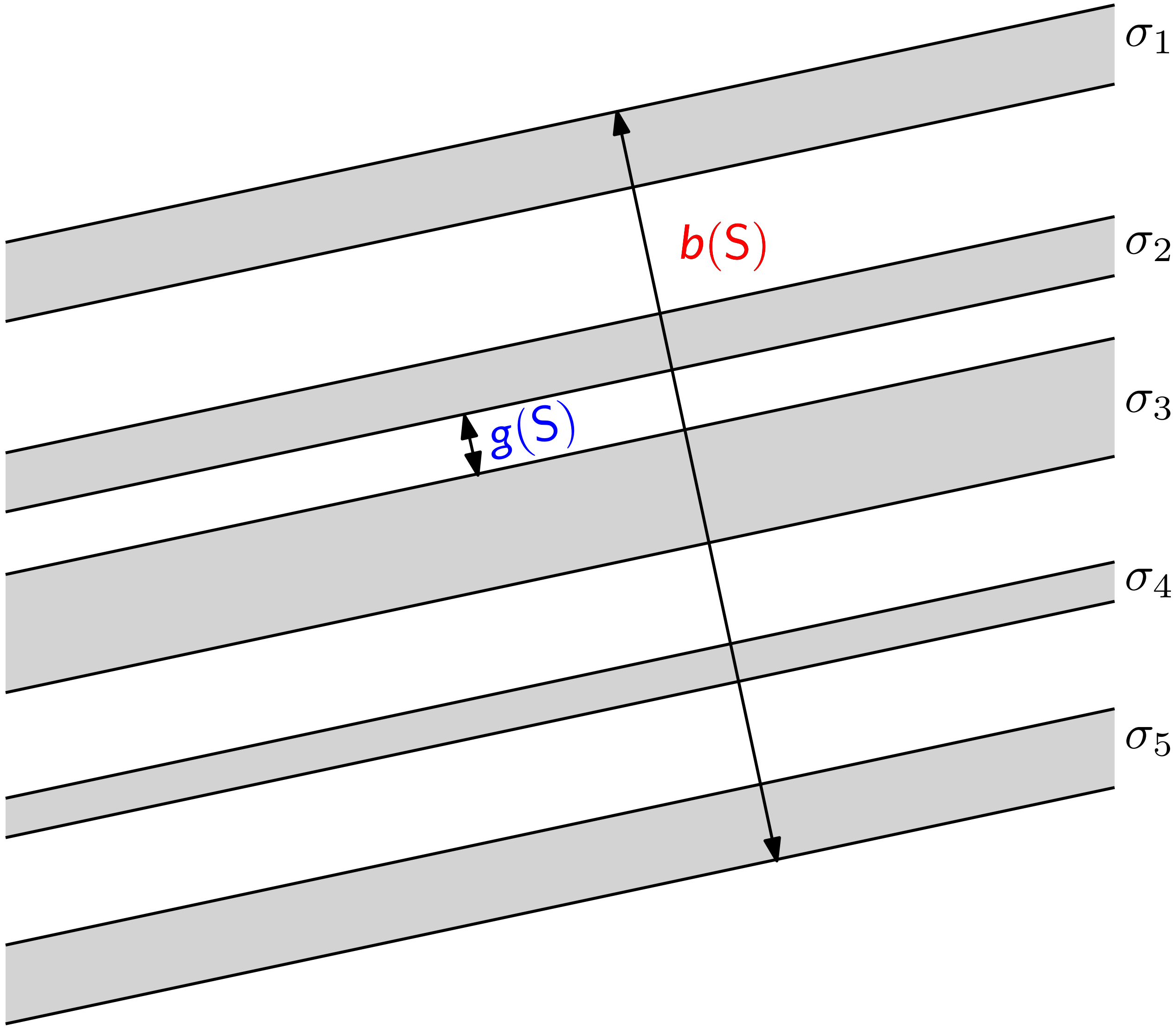

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The breadth of S

- $b(S)$

The gap-ratio of S

- $\rho(S) := g(S)/b(S)$

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For given $k \geqslant 2$, a set P of n points, and a real $\rho \in (0, 1]$,

find a minimum-width k-slab cover of P whose gap-ratio is at least ρ .

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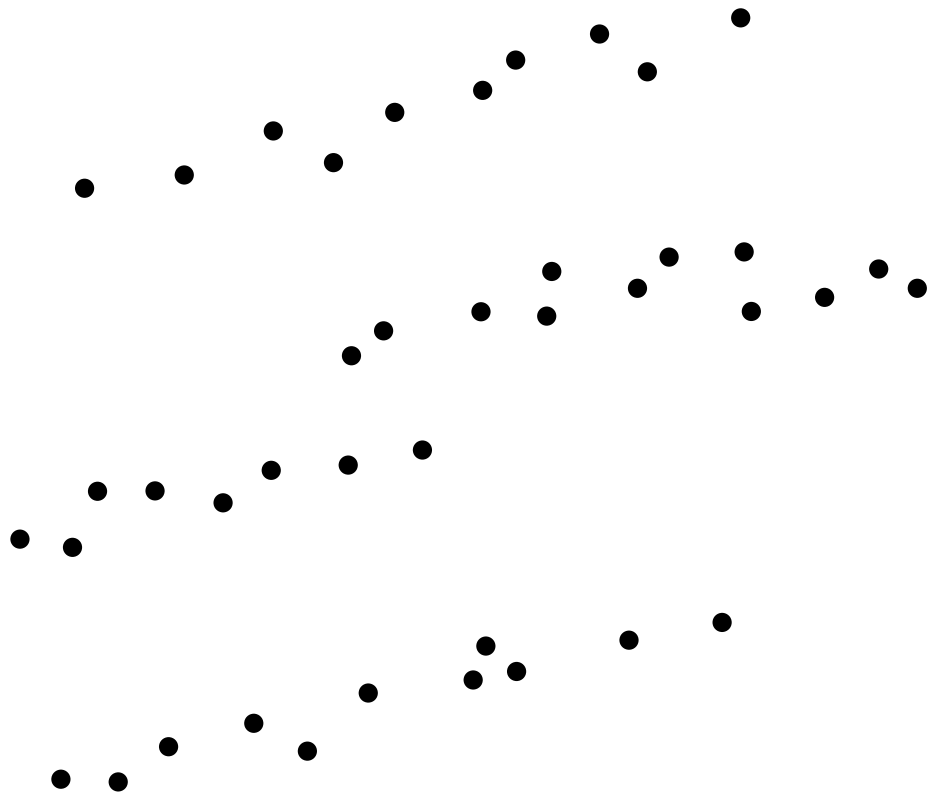

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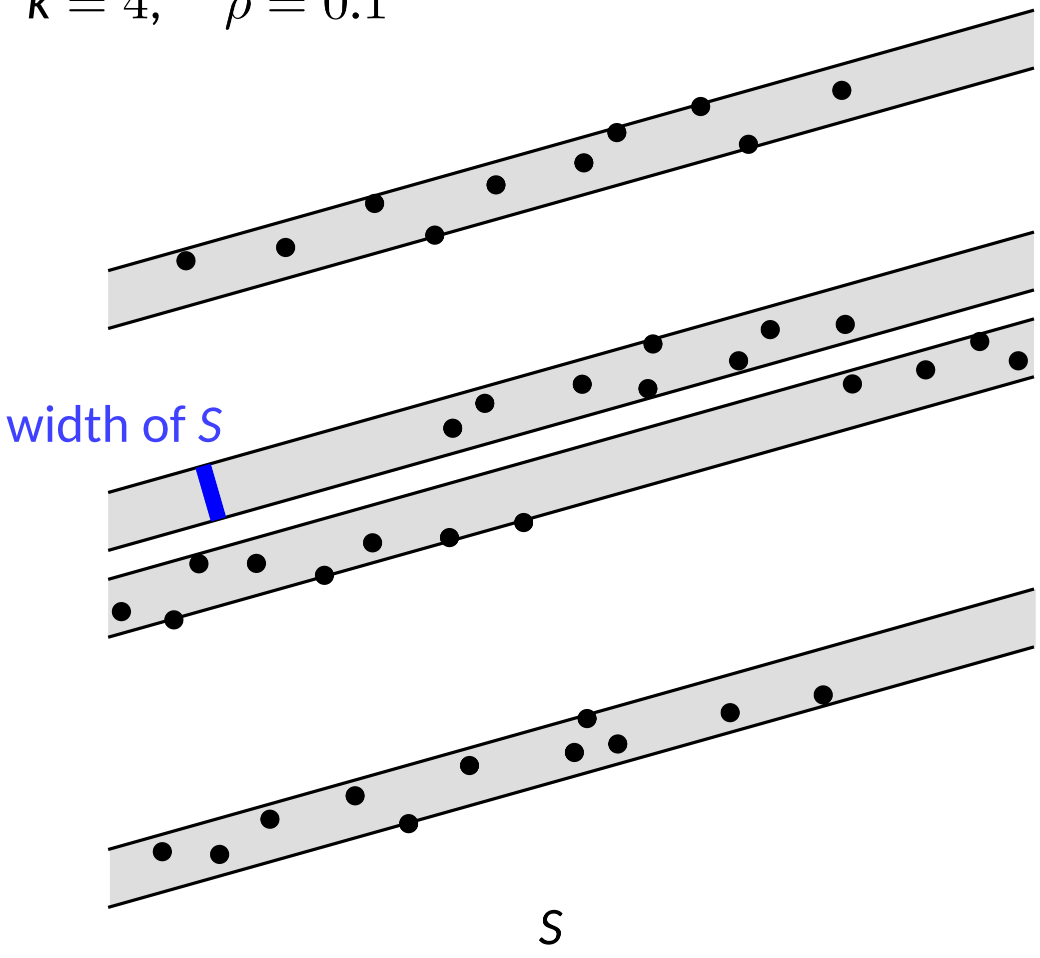

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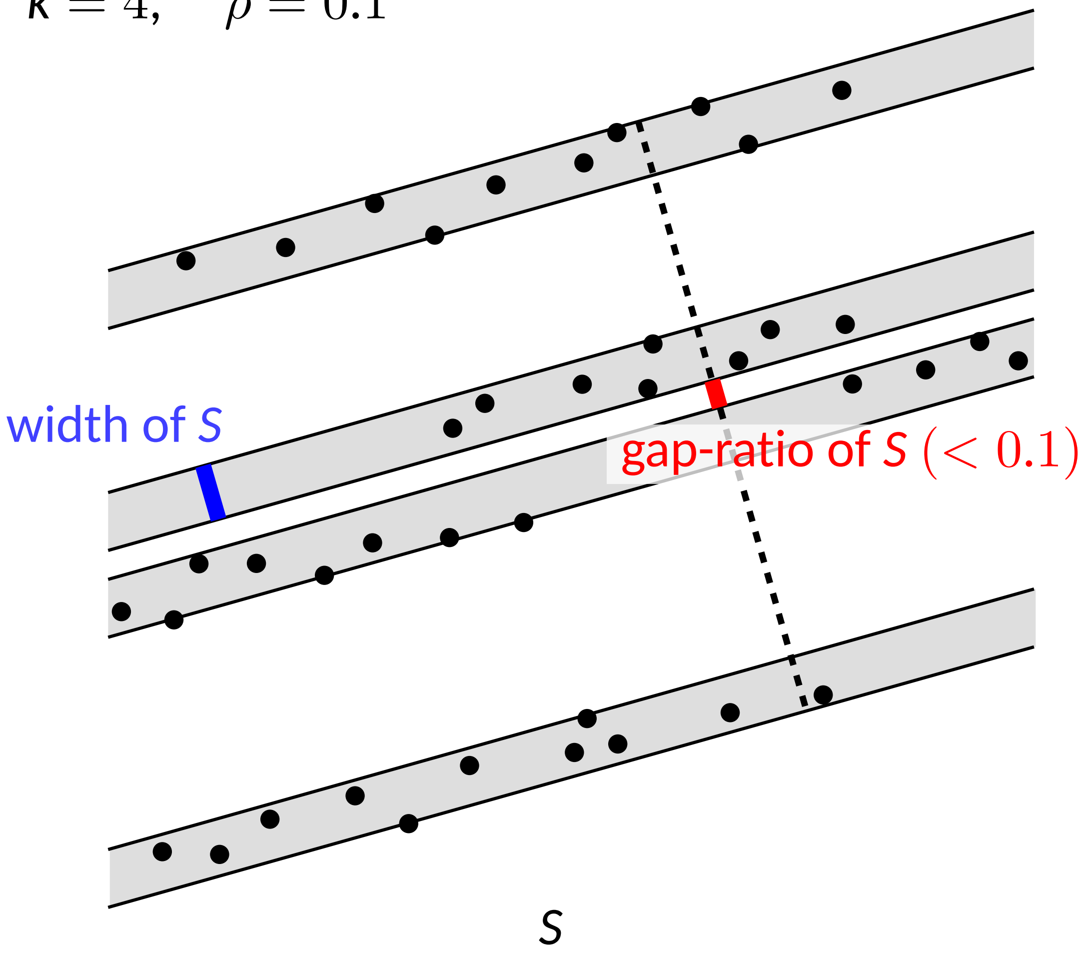

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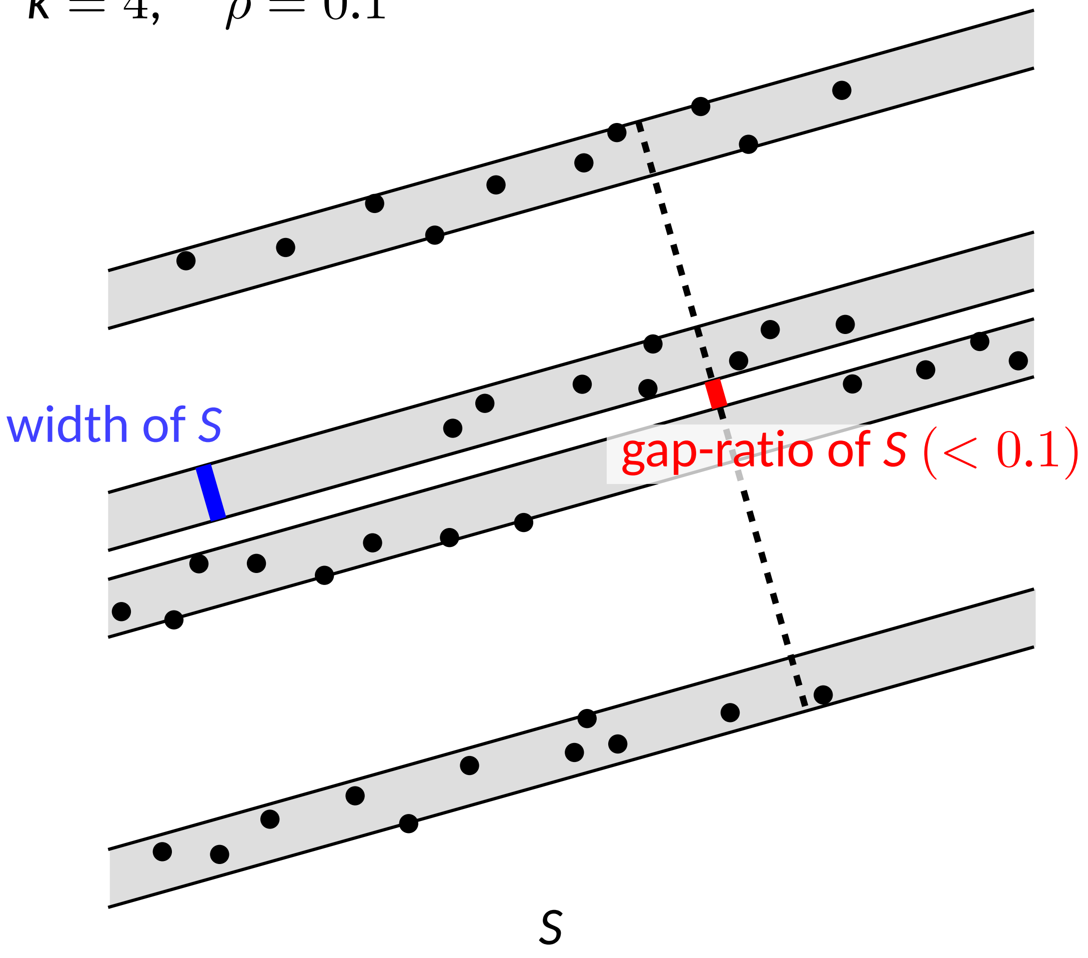

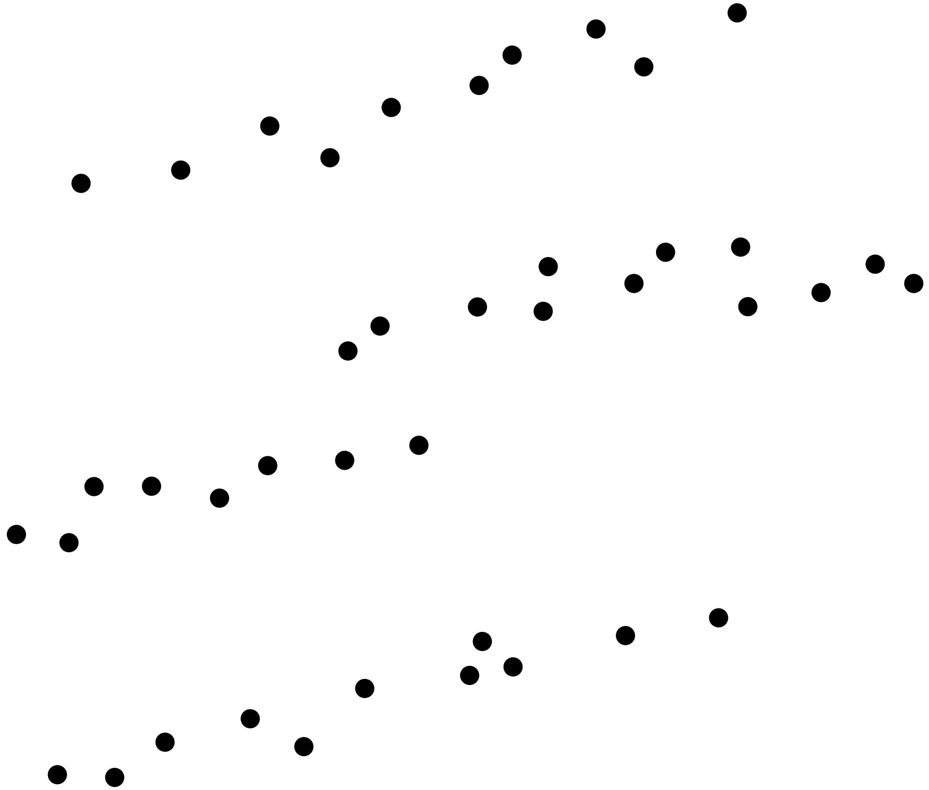

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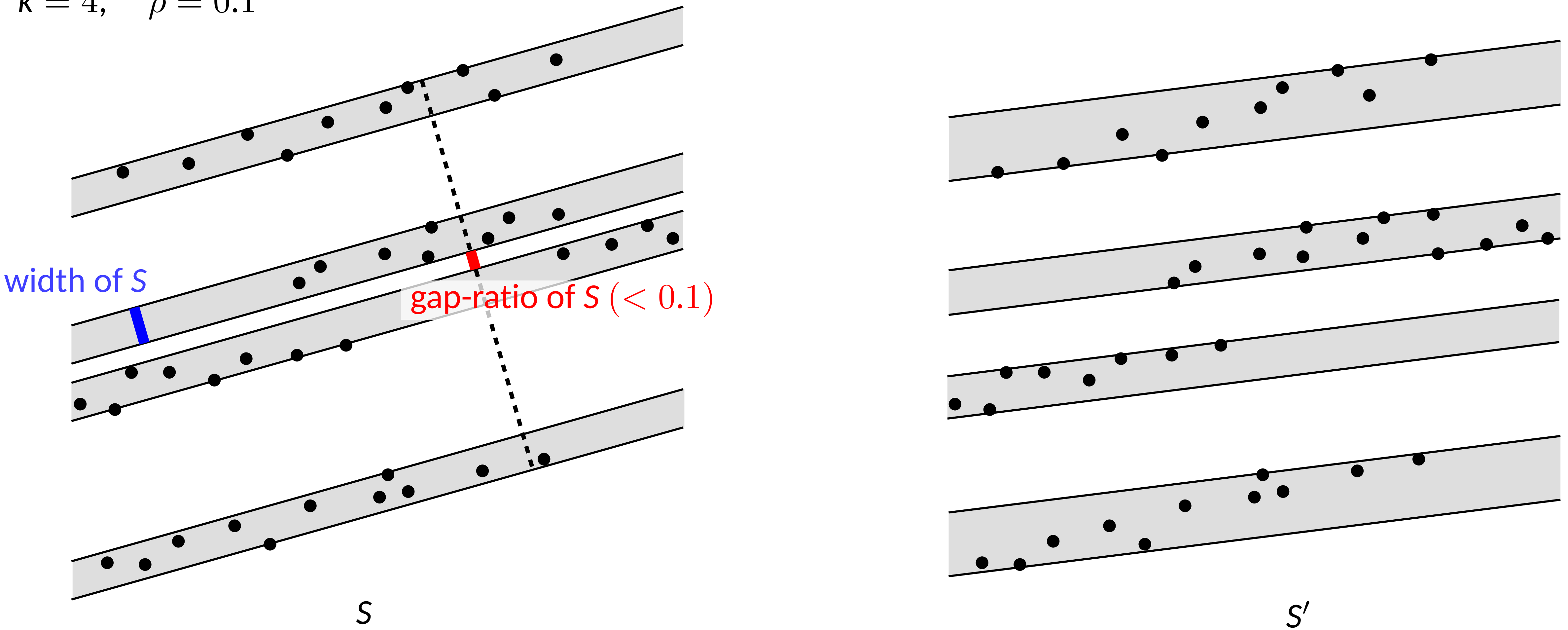

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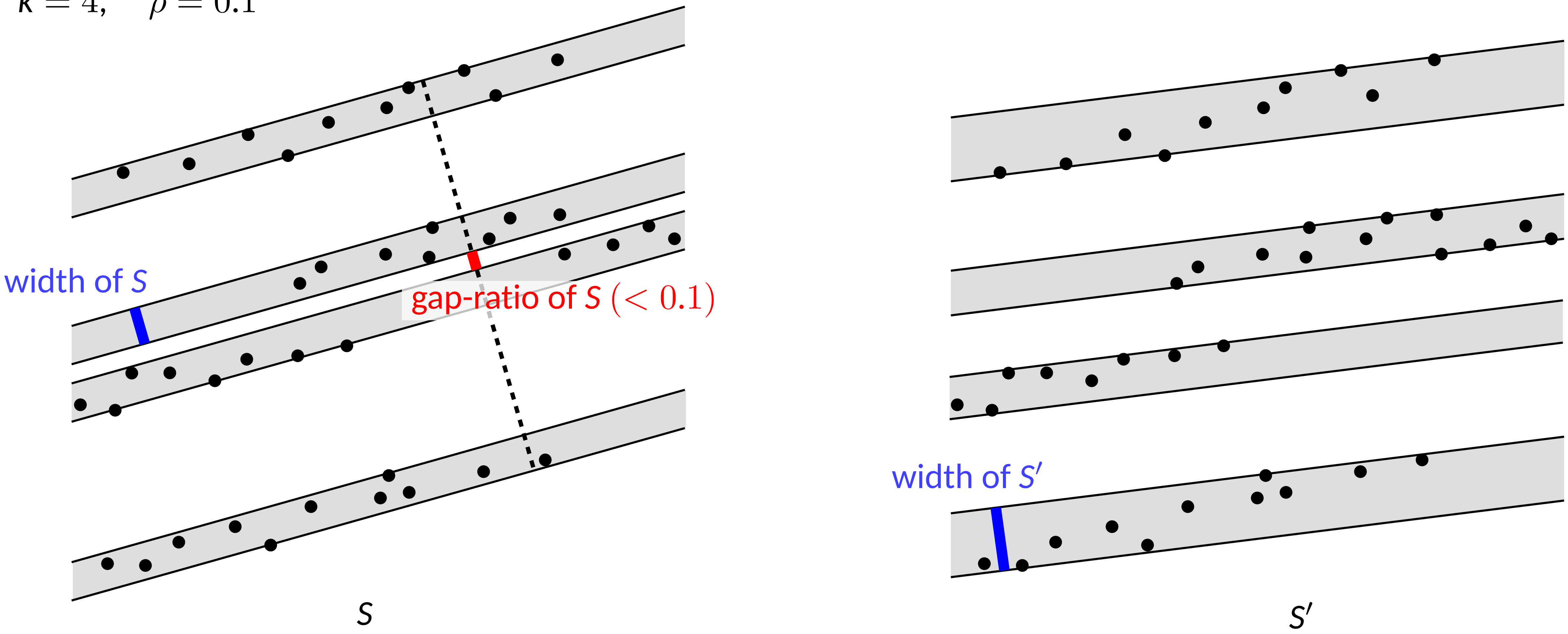

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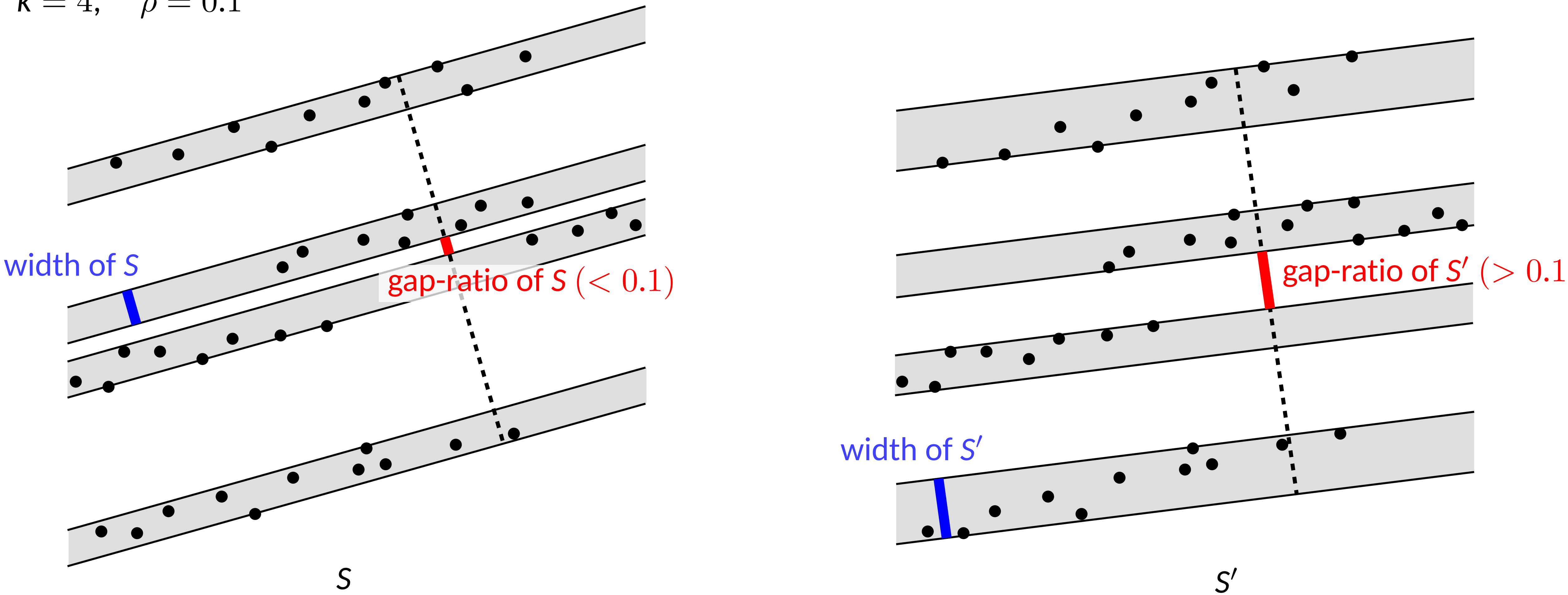

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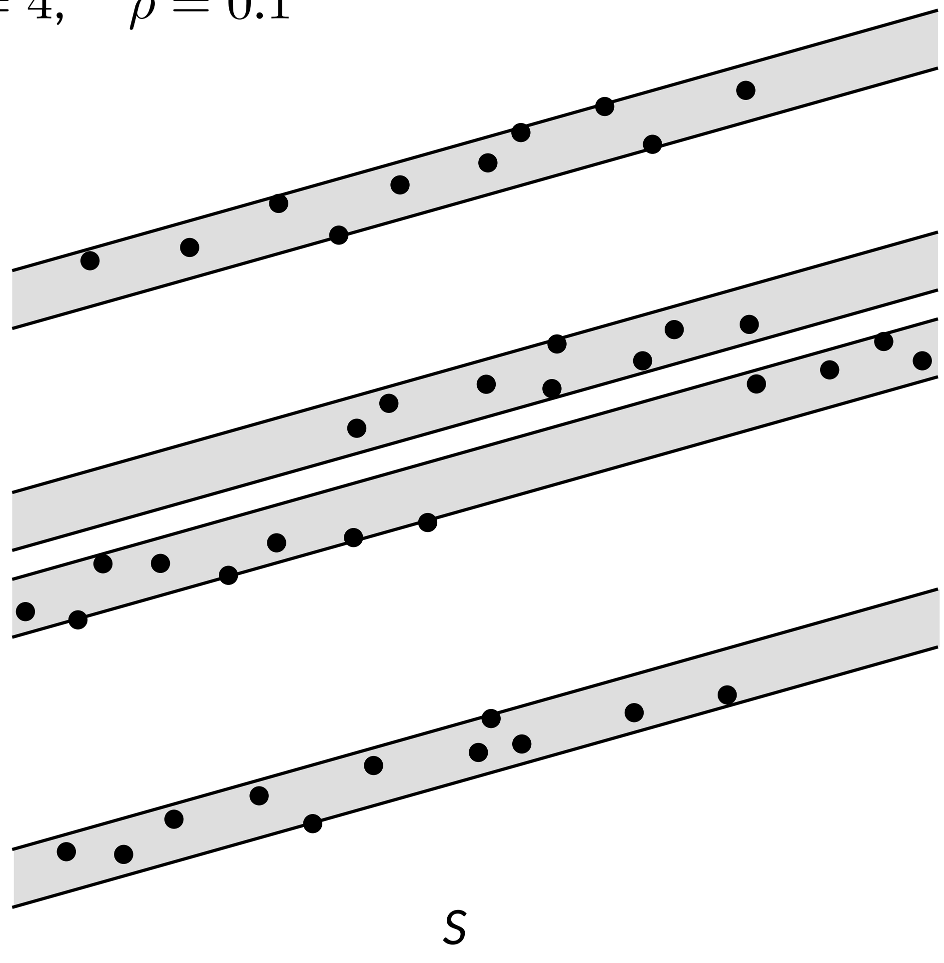

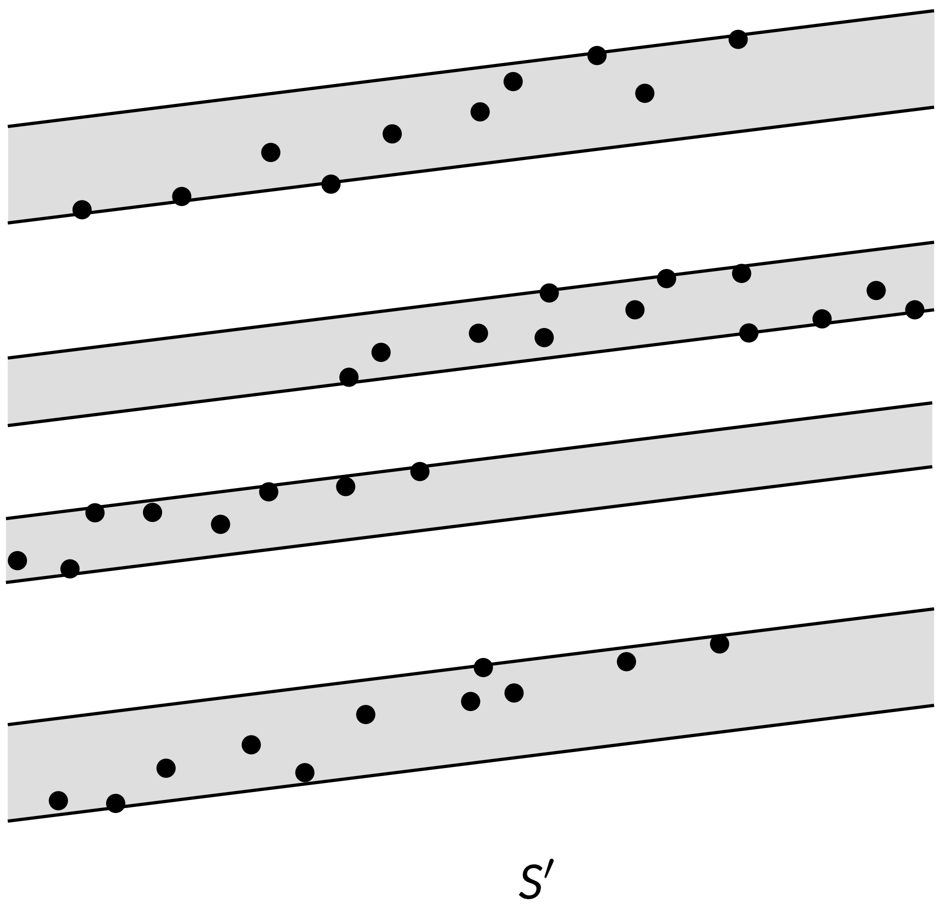

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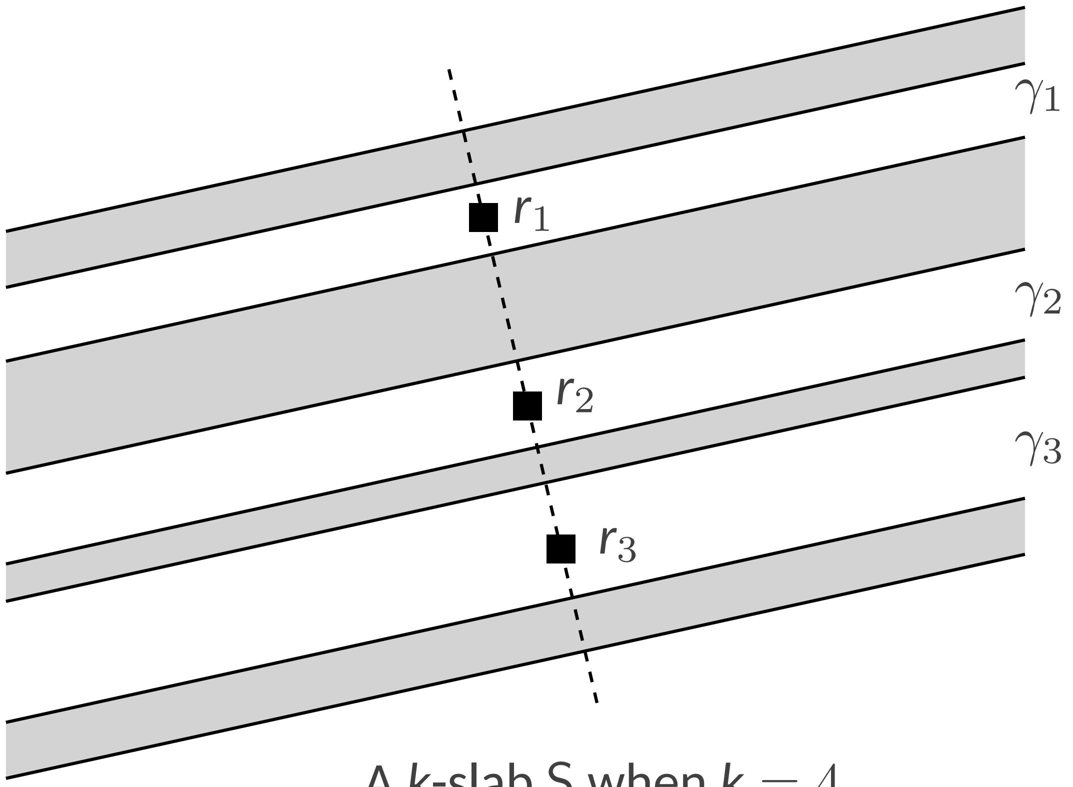
A **separator** of a k-slab S:
a sequence of $k - 1$ points on a common line each of which lies in its distinct gap.



A k-slab S when $k = 4$

R $= (r_1, r_2, r_3)$
It holds that $r_i \in \gamma_i$ for each $i = 1, \ldots, k - 1$.

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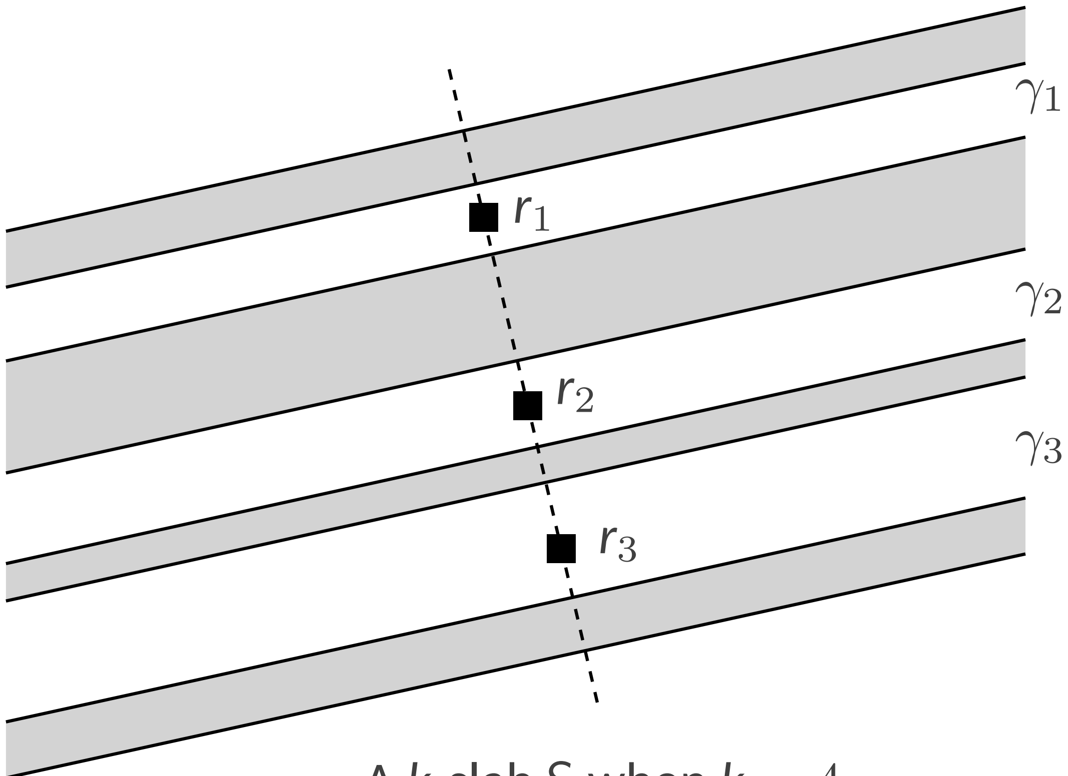

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A k-slab S **respects** the separator R .

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Step 1. An algorithm to compute a minimum-width k-slab cover of P which **respects** a given separator R in $O(kn \log n)$ time and $O(n)$ space.

Step 2. An algorithm to compute $O(\rho^{-k})$ candidate separators.

Step 3. Compute a minimum-width k-slab cover of P by testing $O(\rho^{-k})$ candidate separators in $O(\rho^{-k} kn \log n)$ time and $O(n)$ space.

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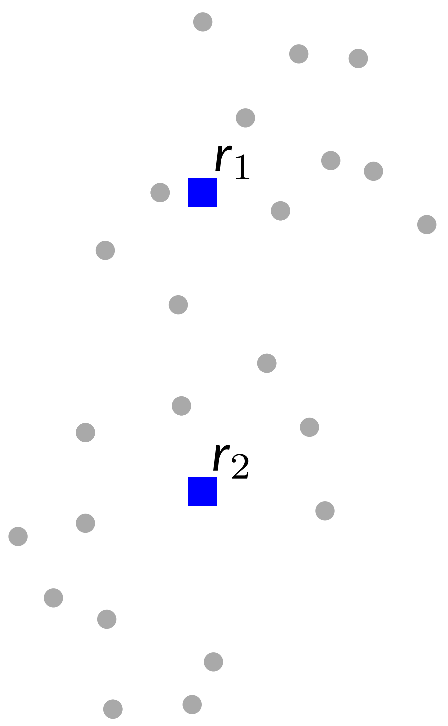

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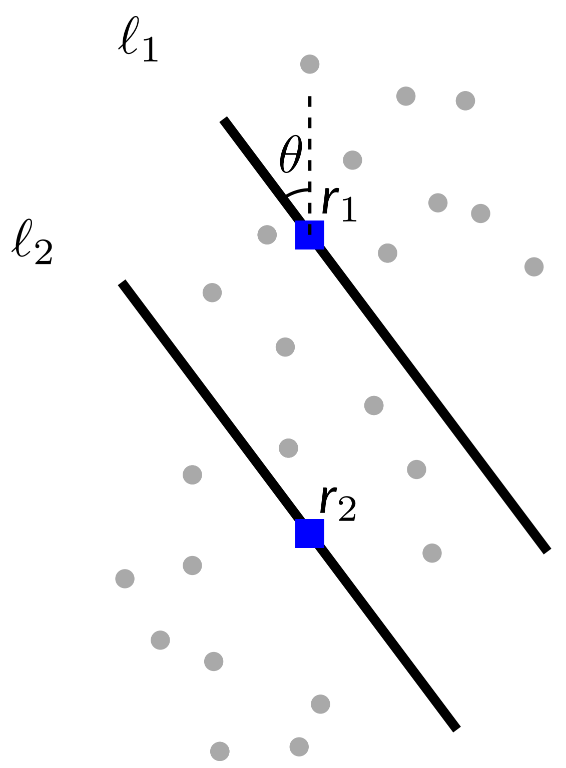

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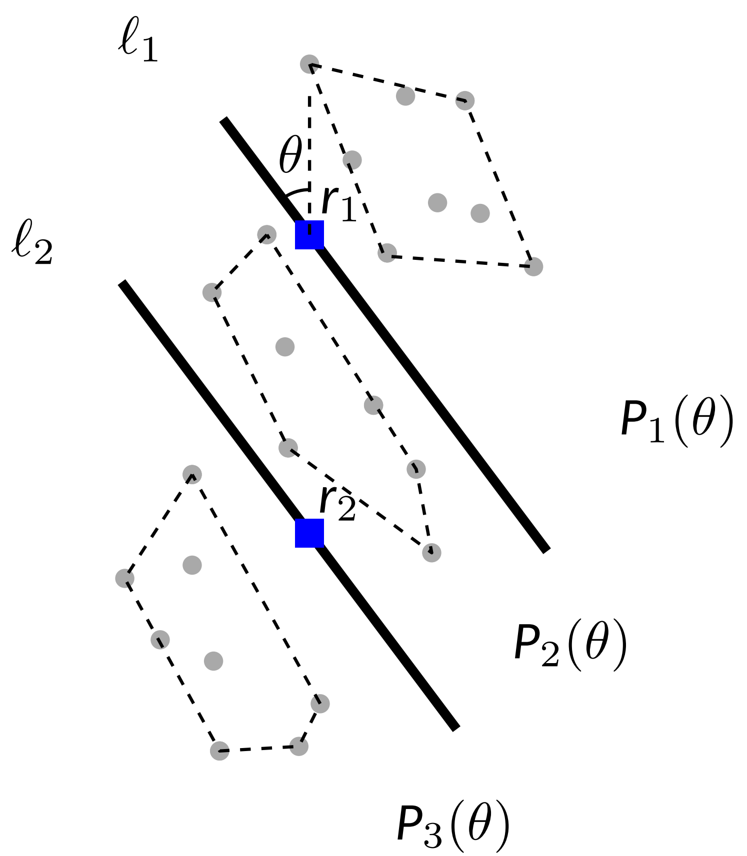

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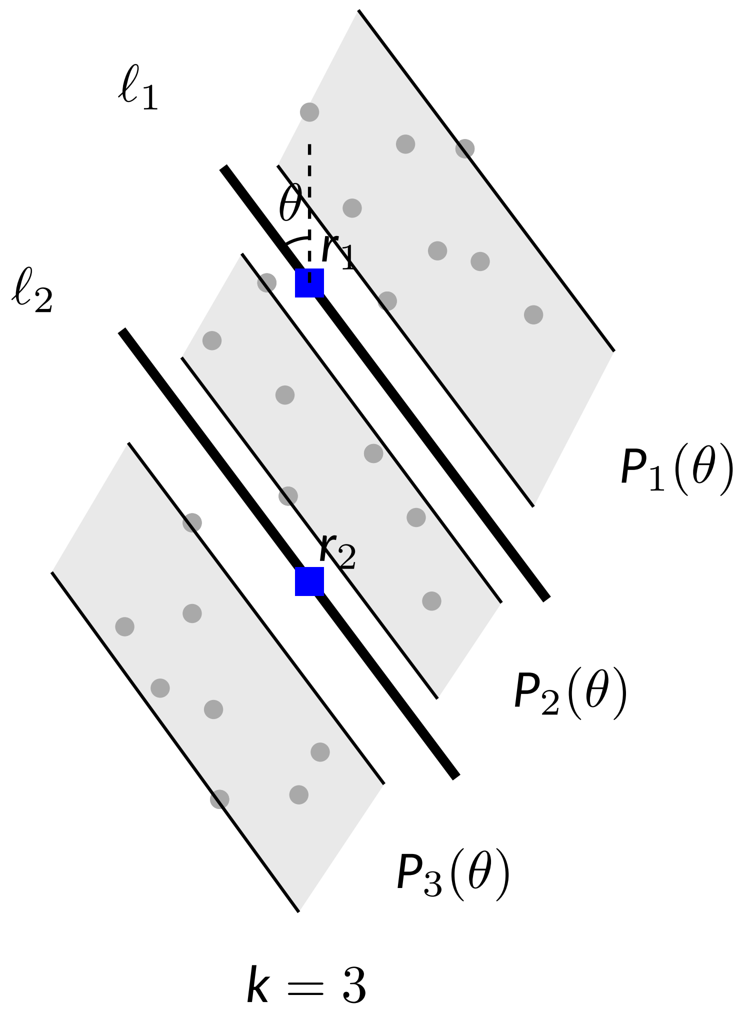

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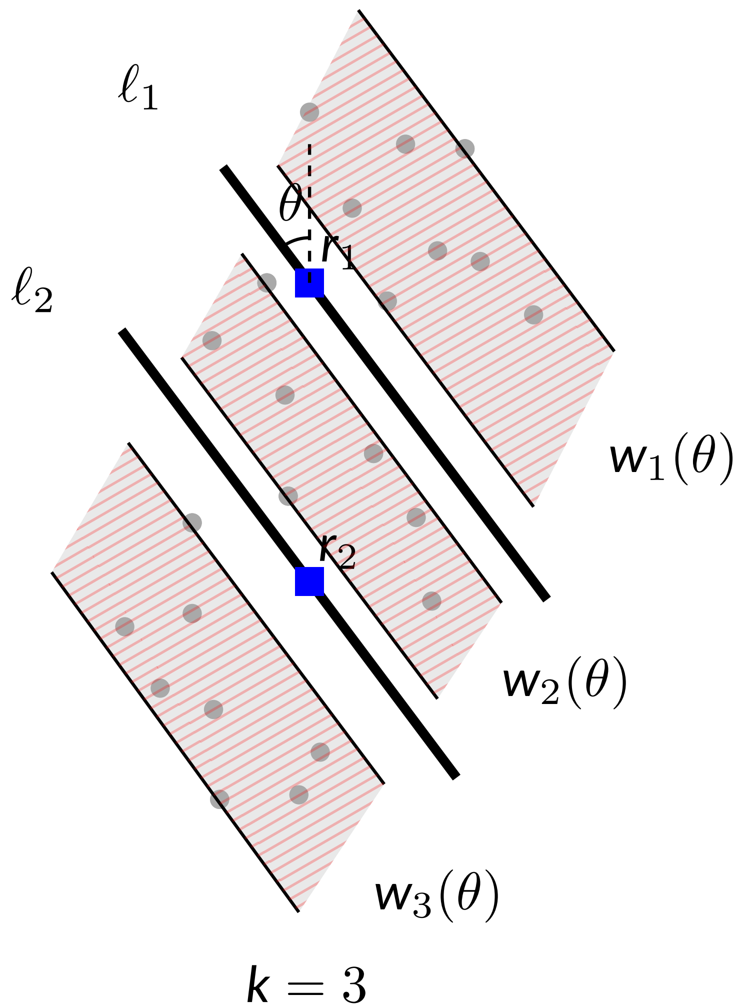

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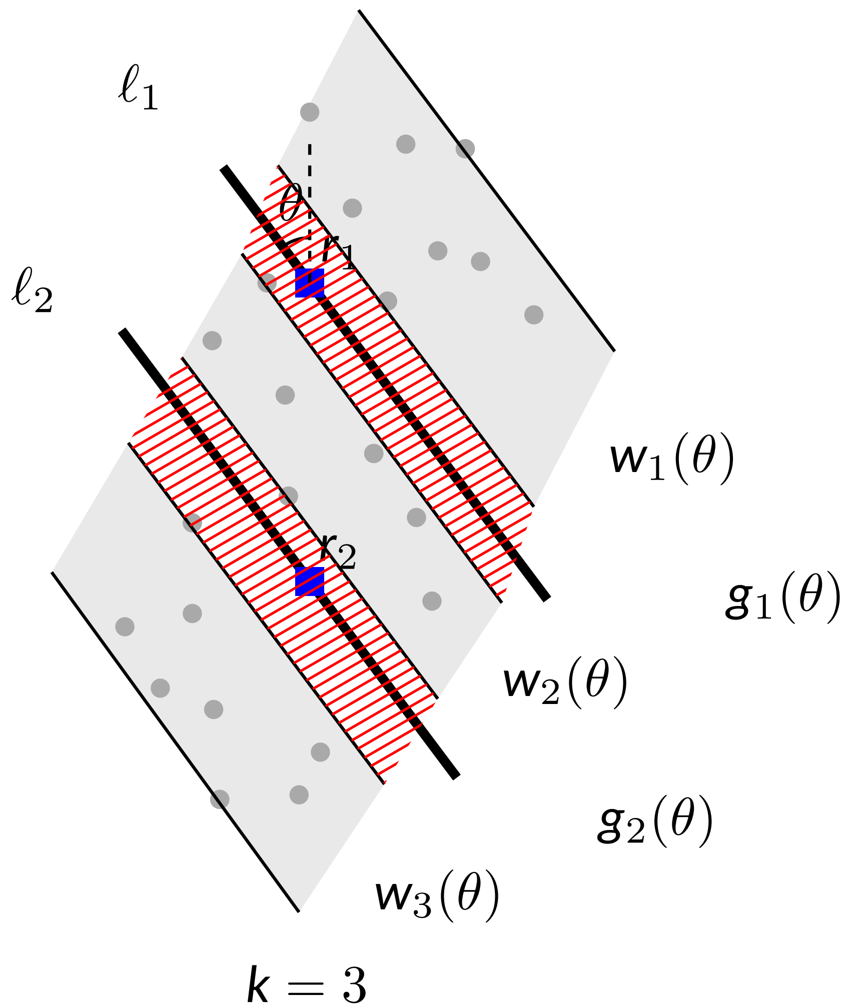

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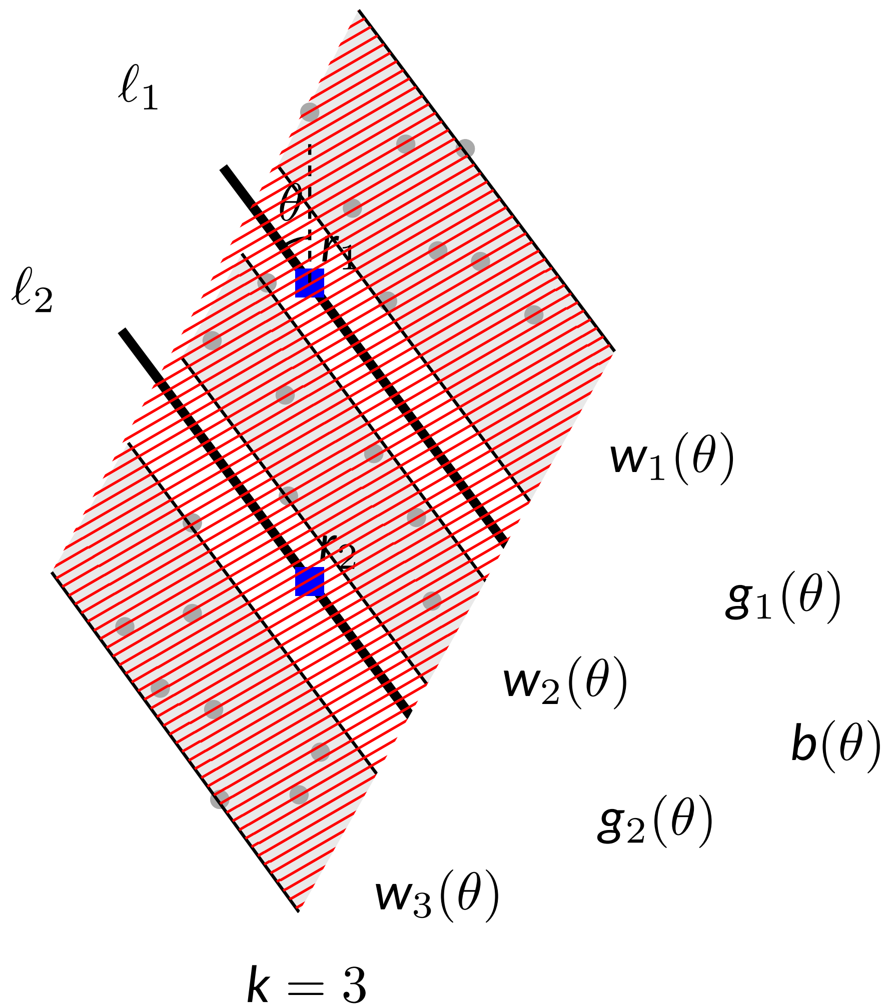

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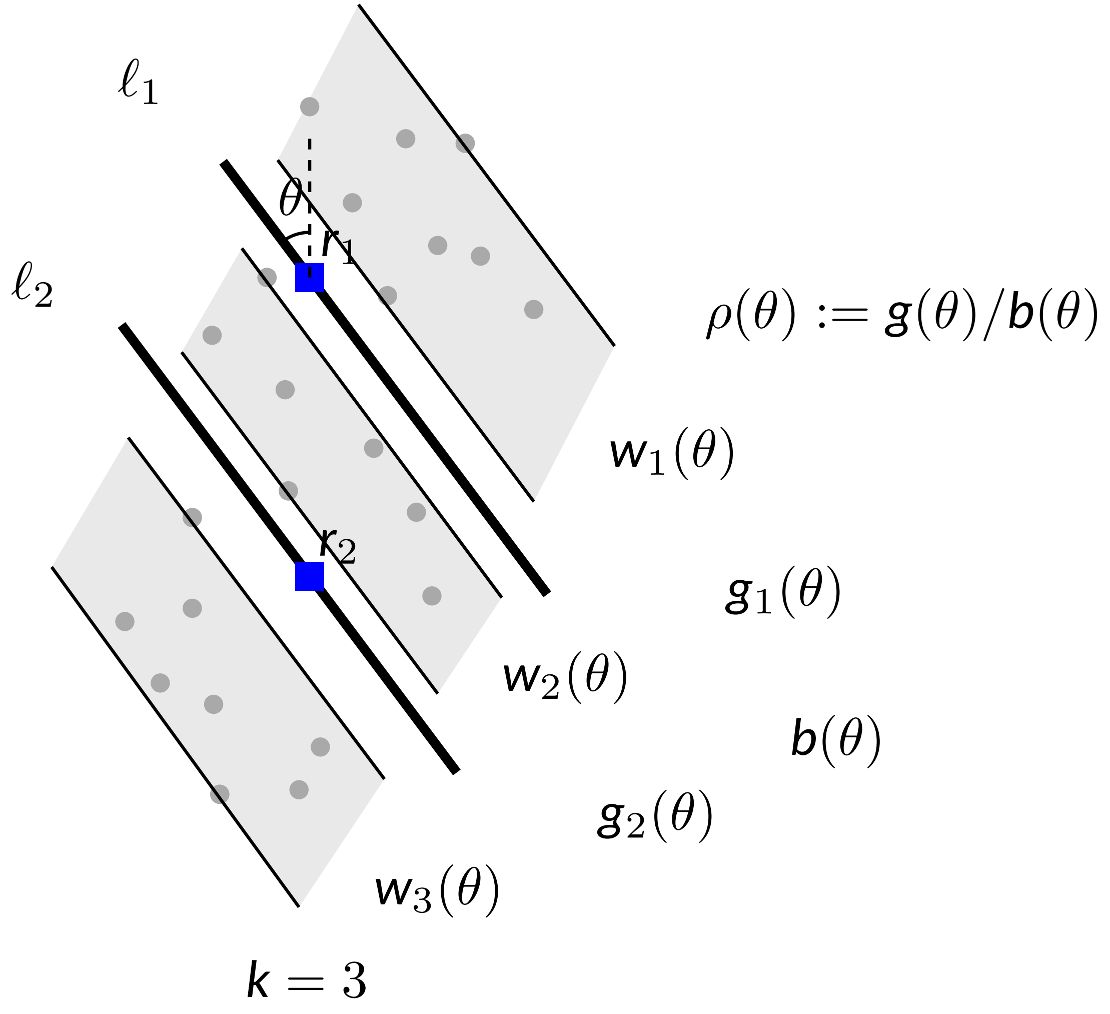

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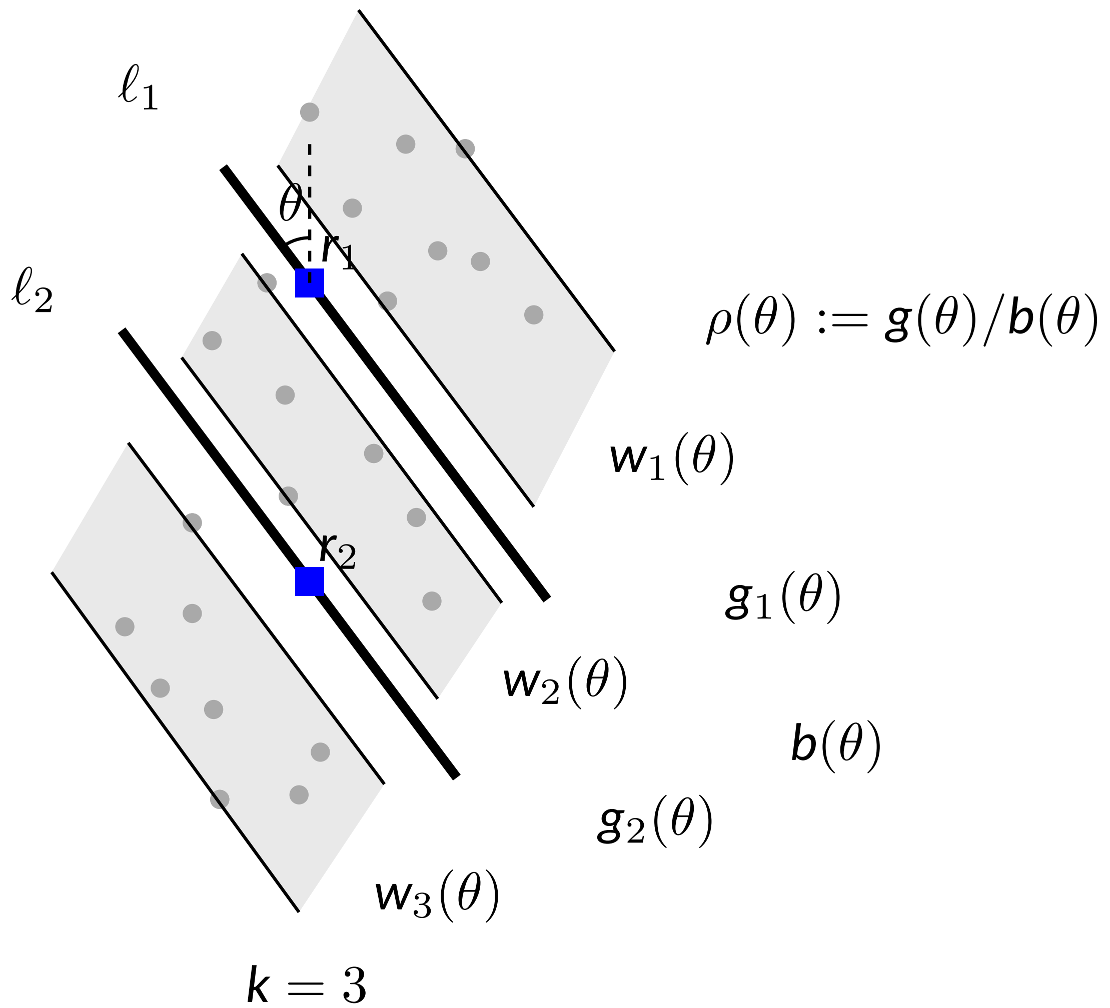

Each of the function

- $w_1, \ldots, w_k$,
- $g_1, \ldots, g_{k-1}$
- b

: piecewise sinusoidal with $O(n)$ breakpoints

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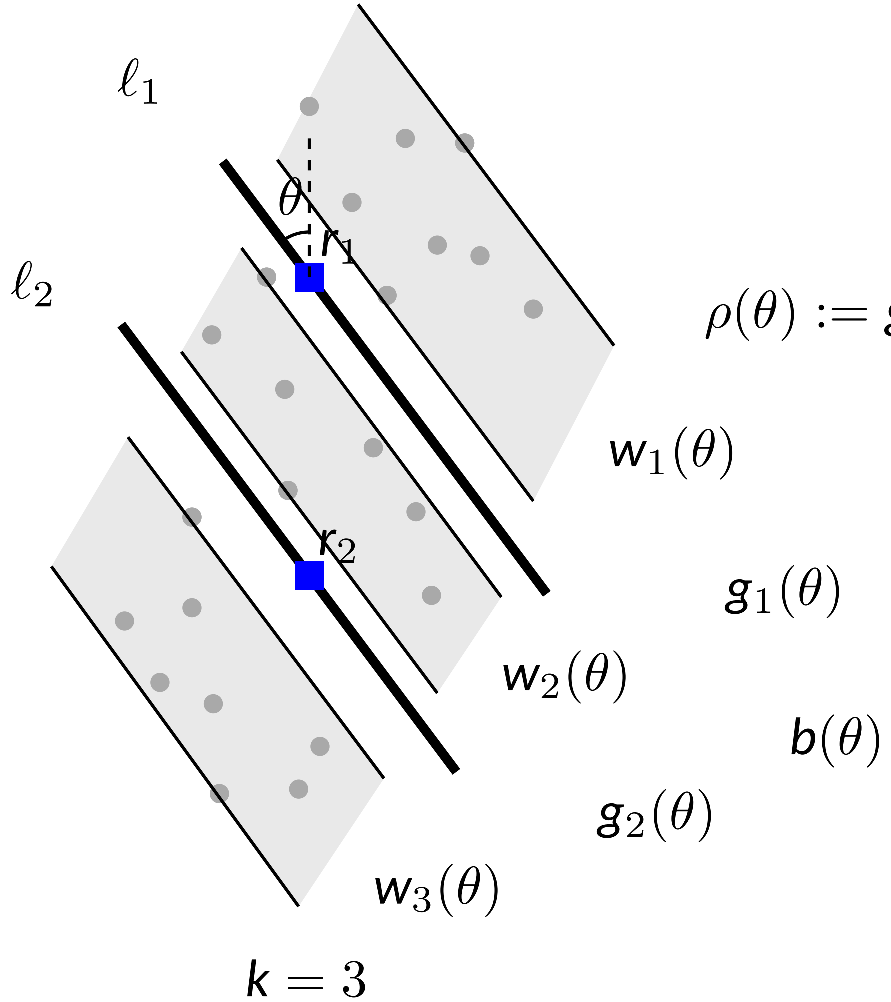

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$\rightarrow$ We can compute **the width, gap-width, breadth**, and **the gap ratio** for a fixed orientation.

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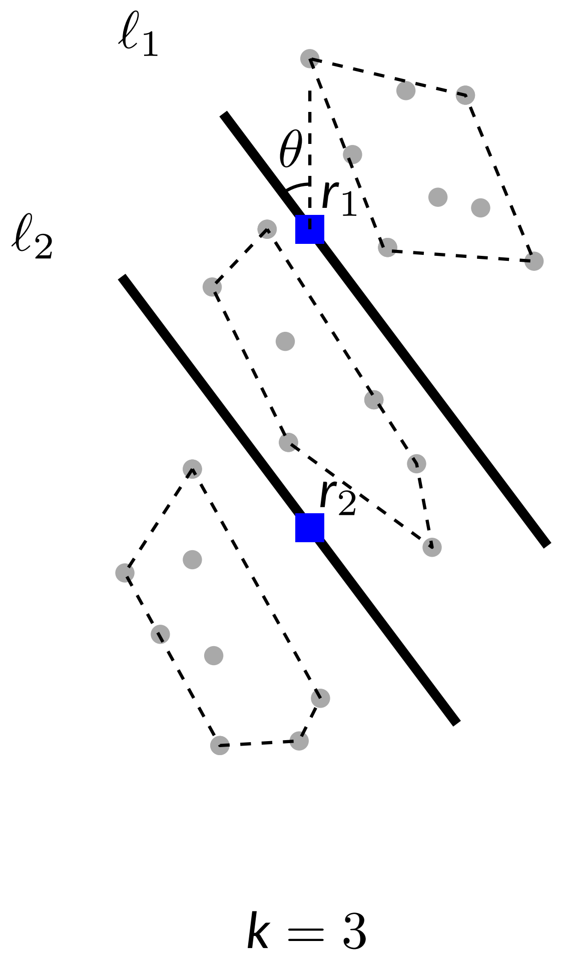

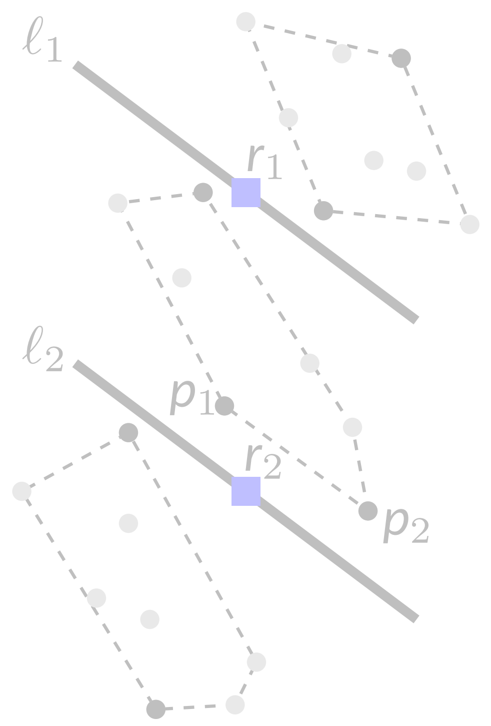

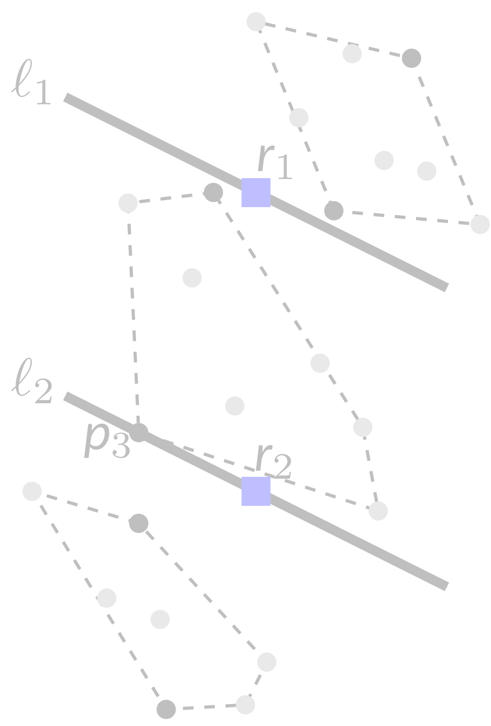

2. A min-width k-slab cover for a Given Separator

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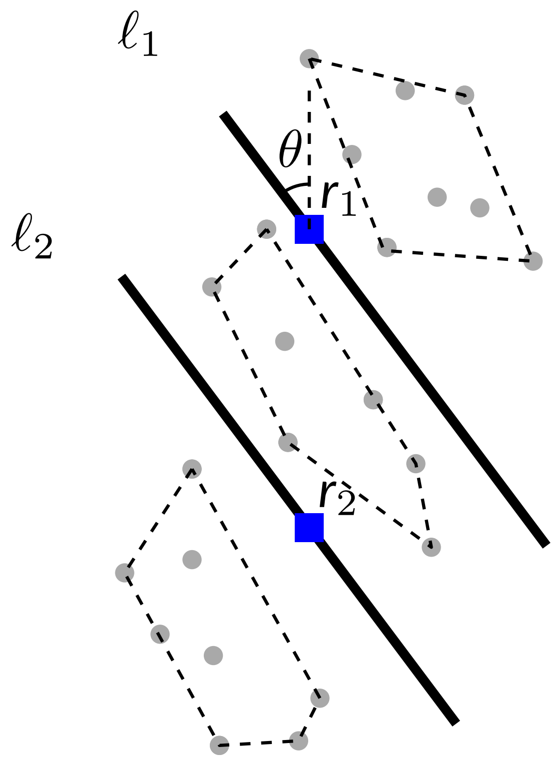
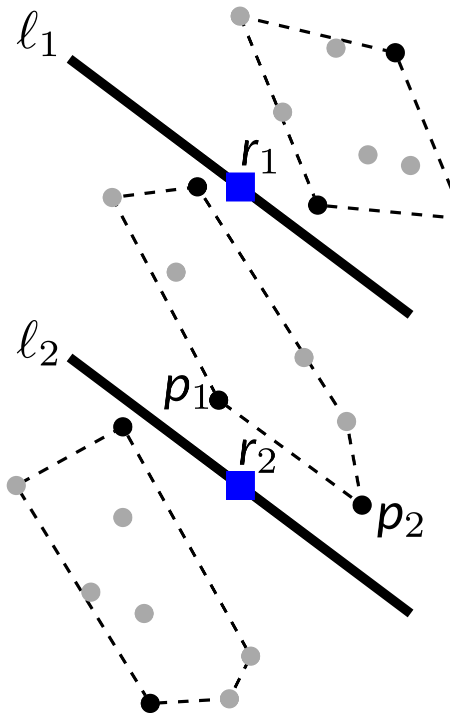
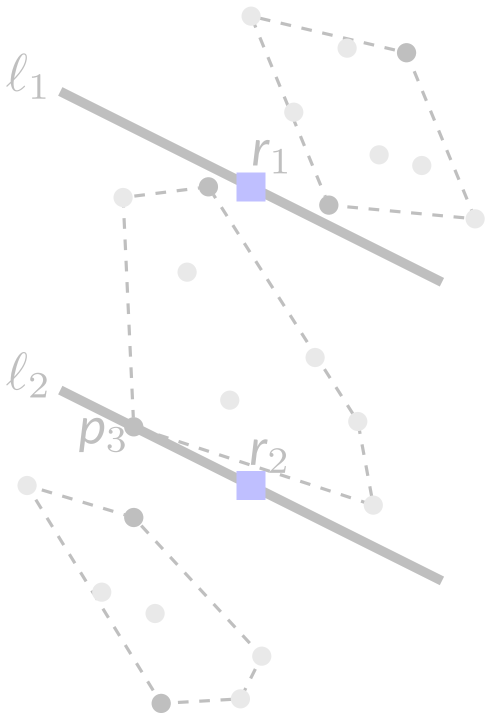


$k = 3$





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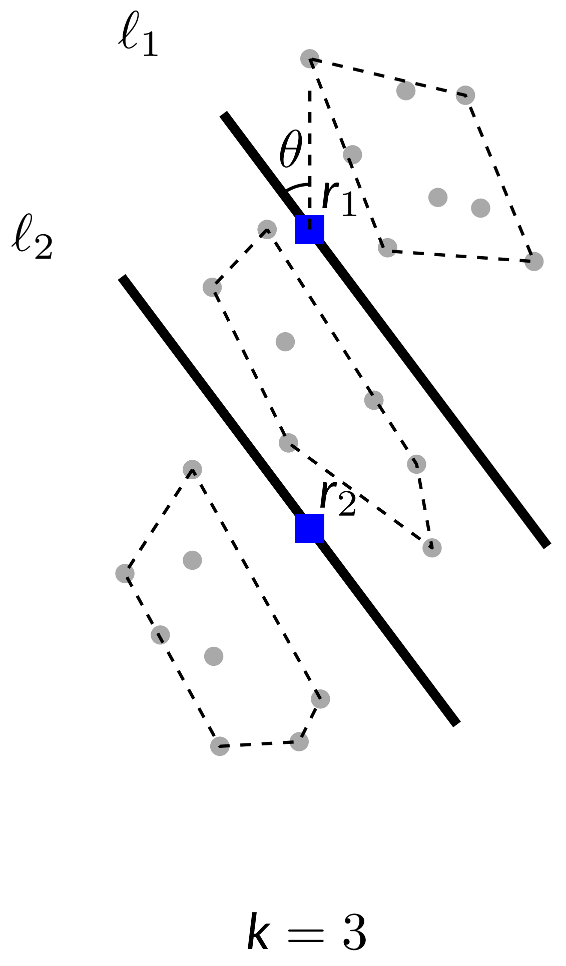

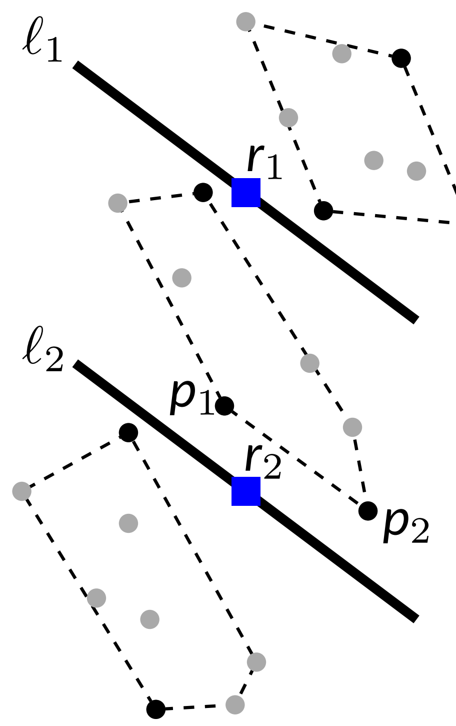

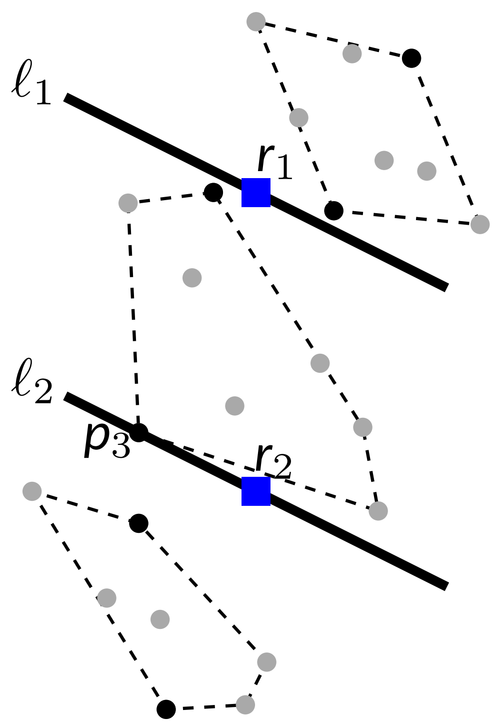

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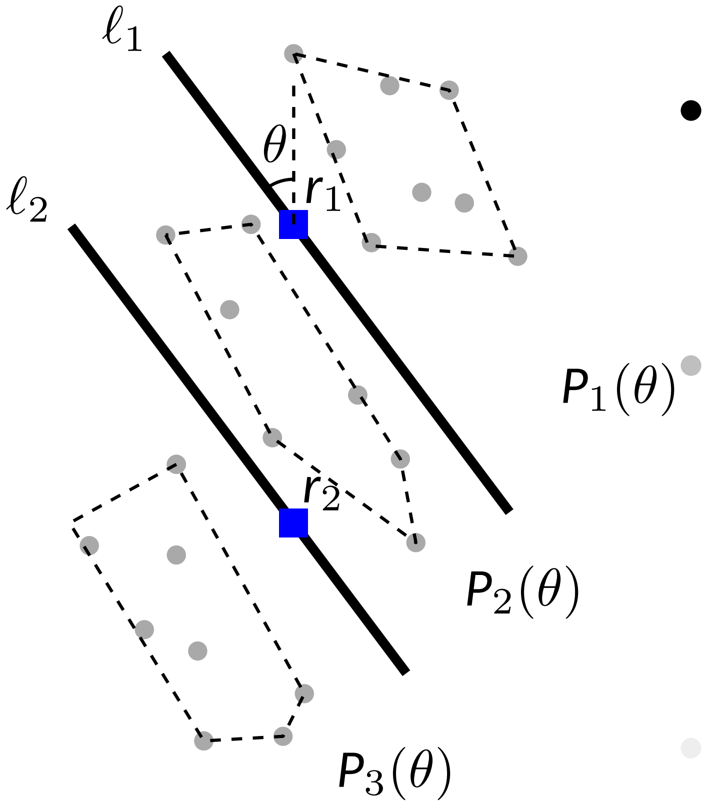

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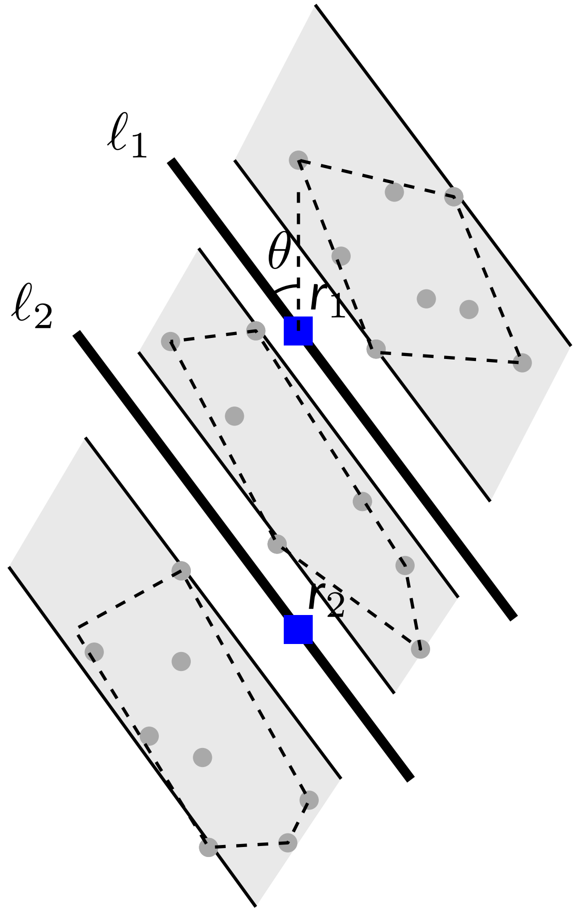

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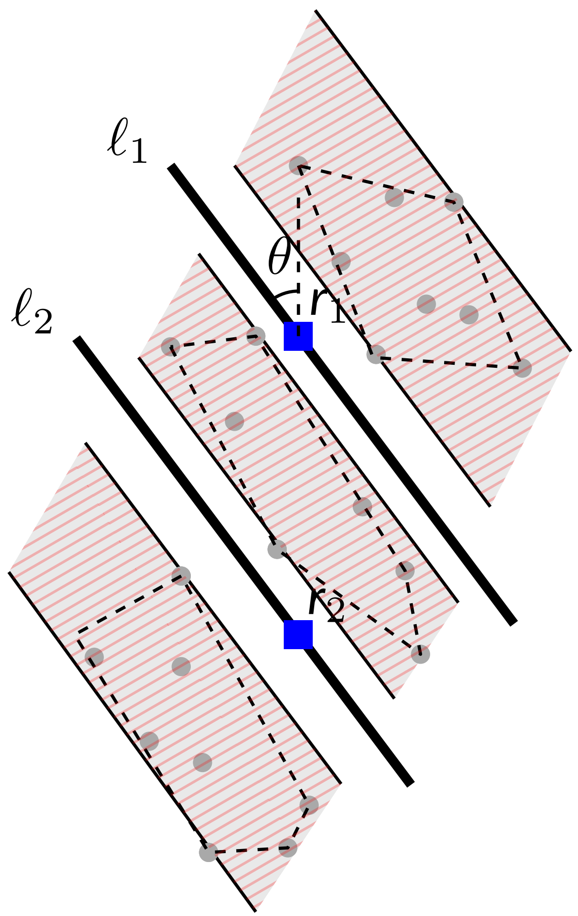

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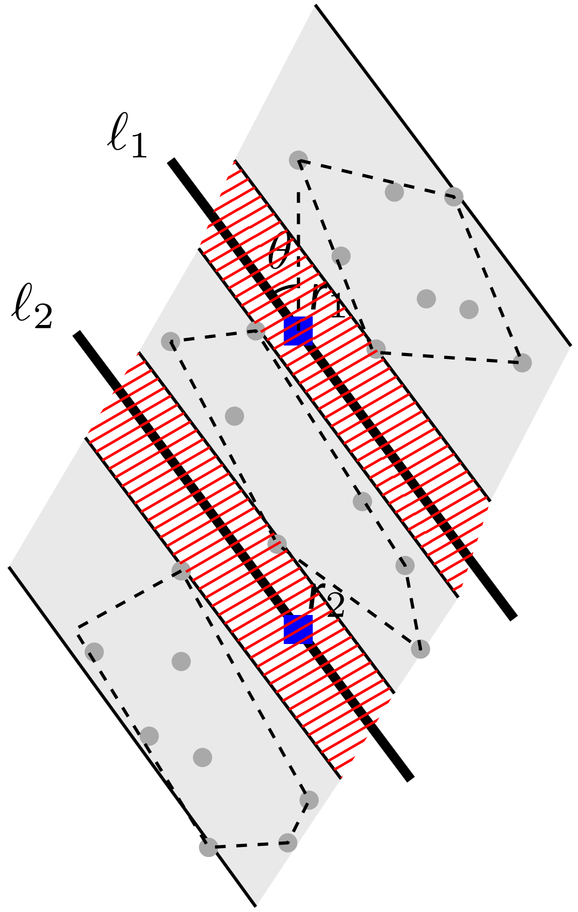

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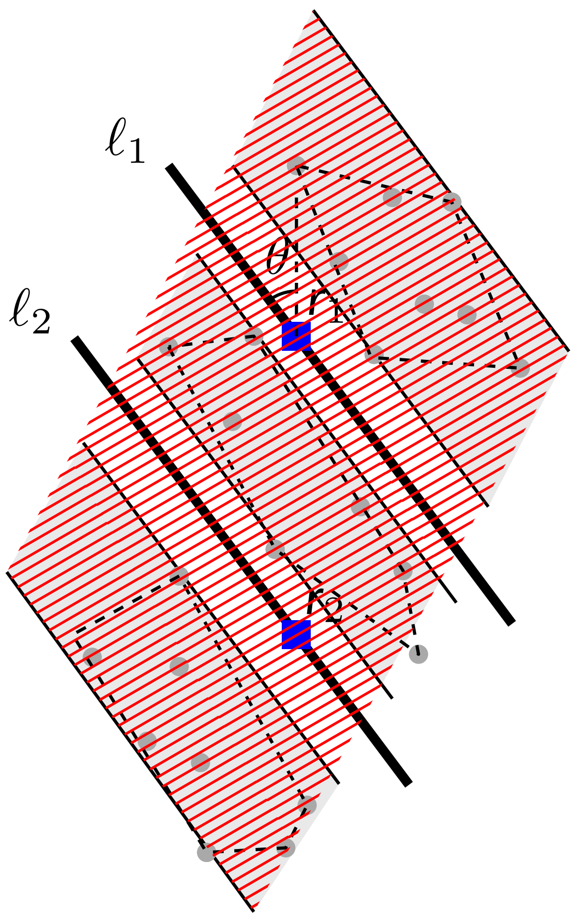

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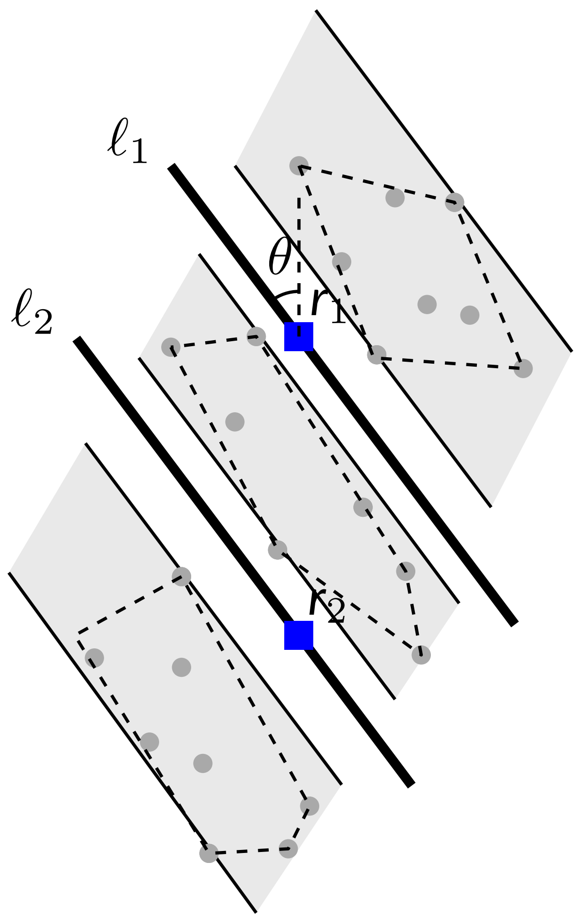

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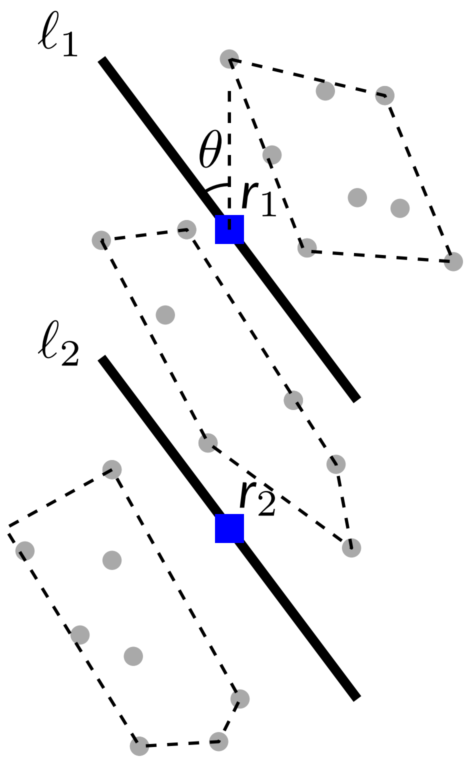
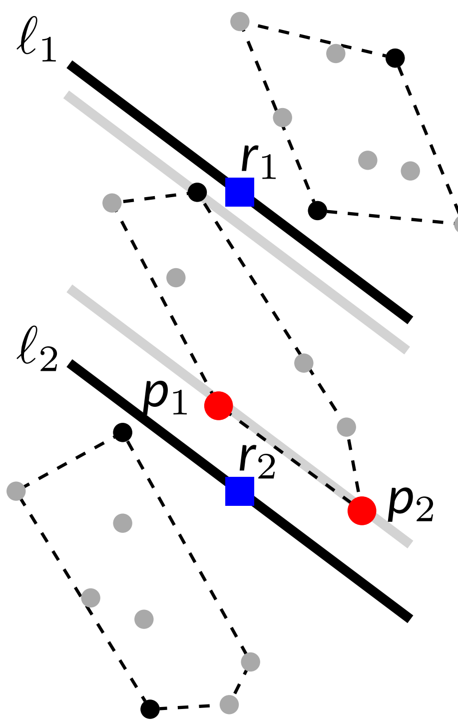
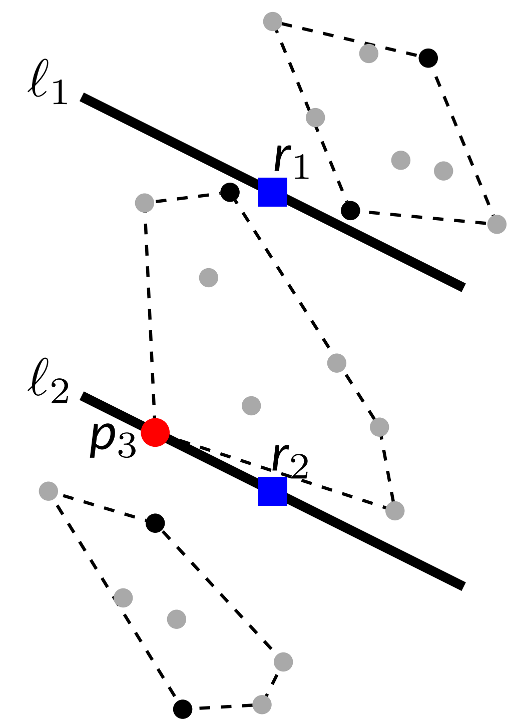




1) Slab event

: when two or more points of P are contained in a boundary line of slab of $P_i(\theta)$



2) Cross event

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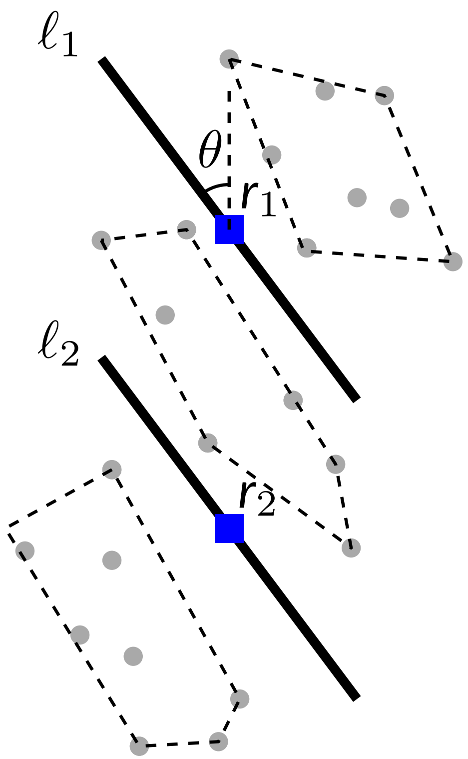

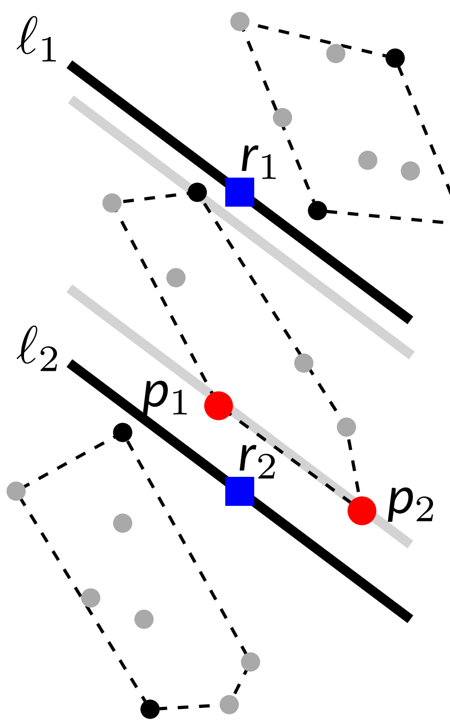

- Update W_i
- Update $\mathrm{G}_{i-1}, \mathrm{G}_i$
- Update B, if needed

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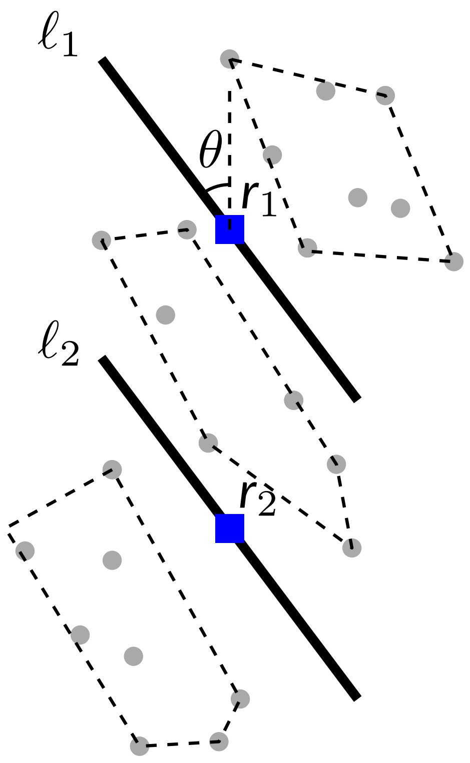

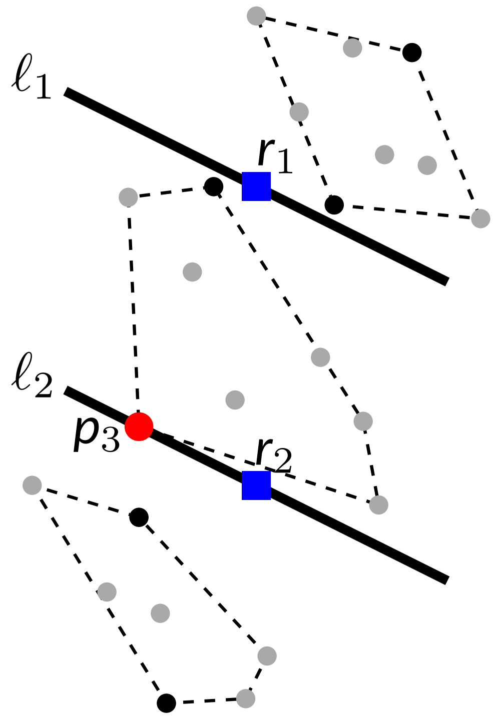

- Update CH_i, C_{i+1}
- Update W_i, W_{i+1}
- Update G_i

2) Cross event

: when a point in P lies on $\ell_i(\theta)$

2. A min-width k-slab cover for a Given Separator

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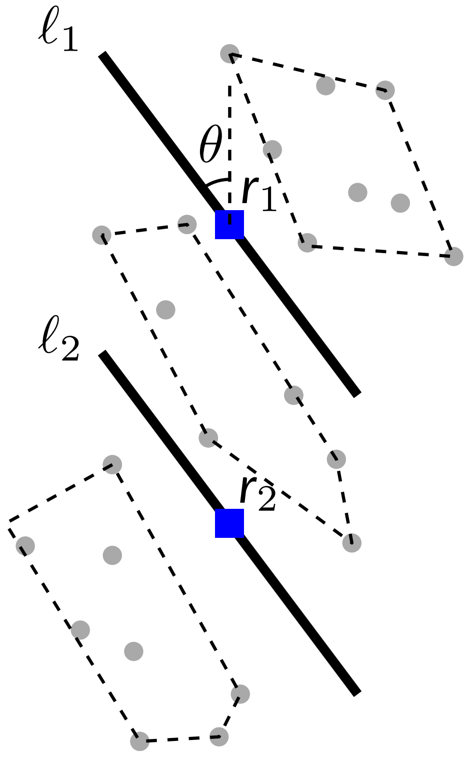

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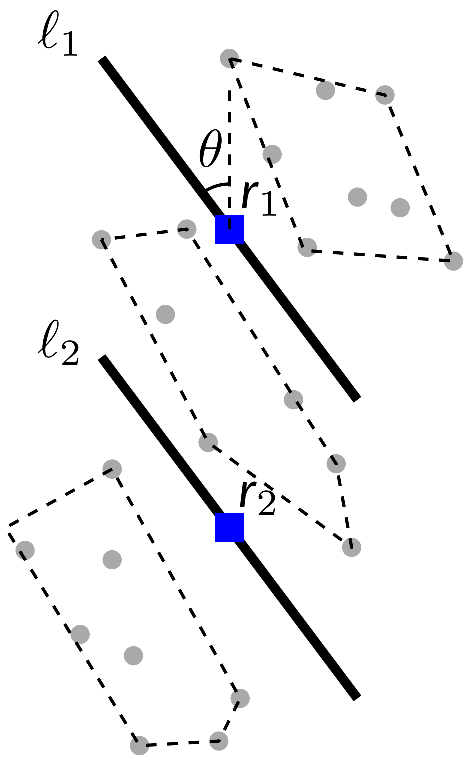

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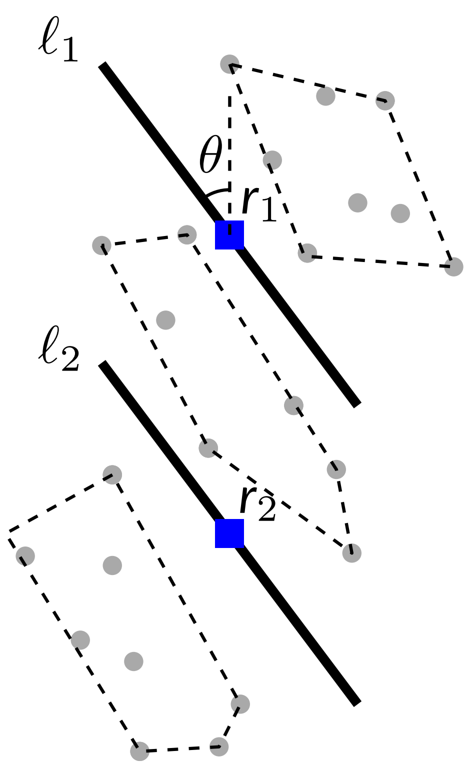

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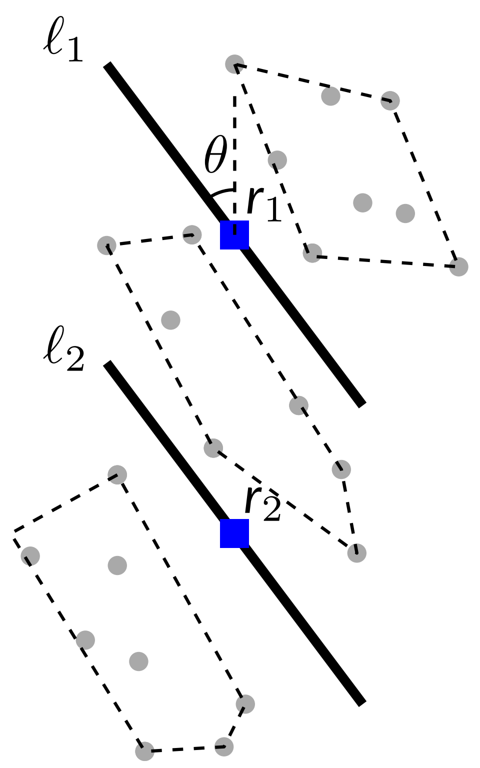

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: Compute the exact orientation θ where the gap-ratio is at least ρ and the width is minimized.

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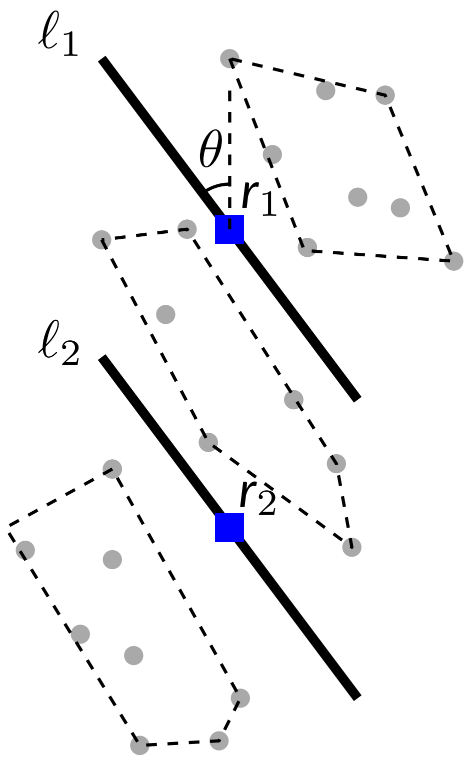

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Theorem. A minimum-width k-slab cover of P respecting R can be computed in $O(kn \log n)$ time and $O(n)$ space.

3. Computing Candidate Separators

Step 2. Given a set *P* of *n* points, compute candidates for an optimal separator.

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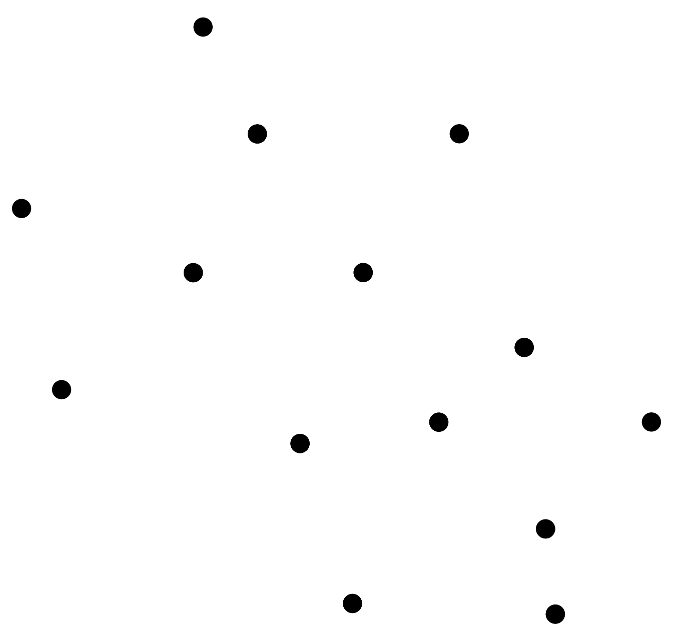
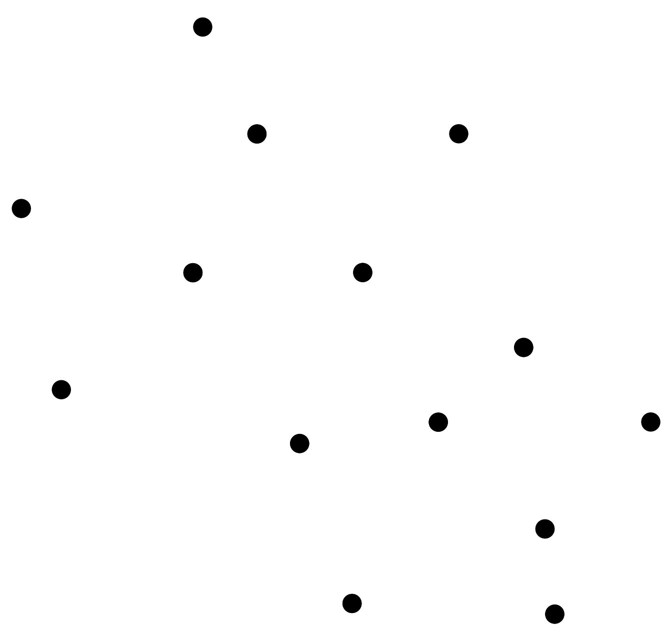
The **directional width** of *P* in orientation θ, $d_\theta(P)$



An ϵ**-coreset** for the directional width of *P*



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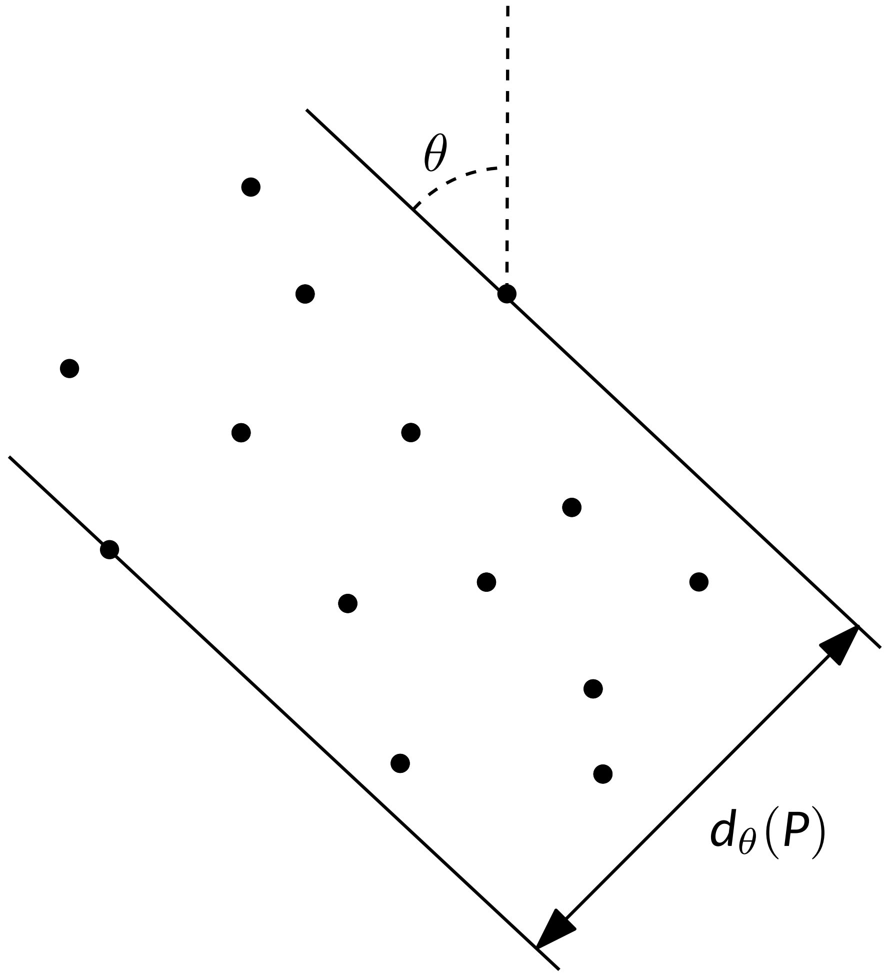

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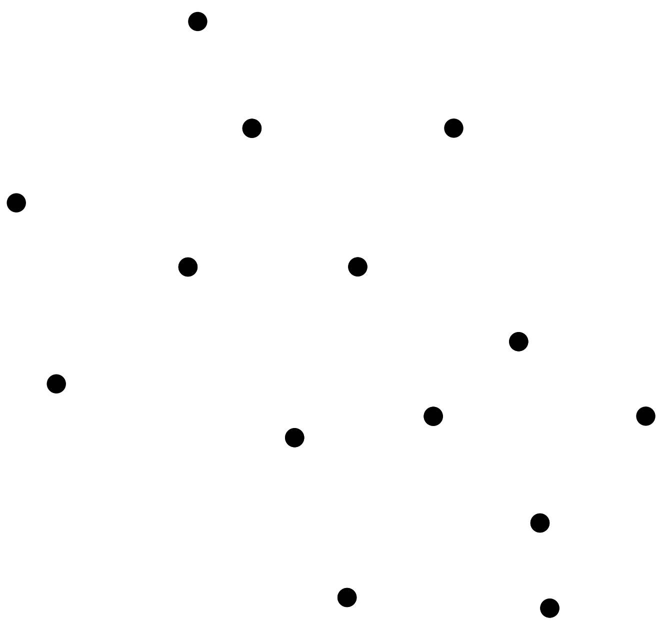

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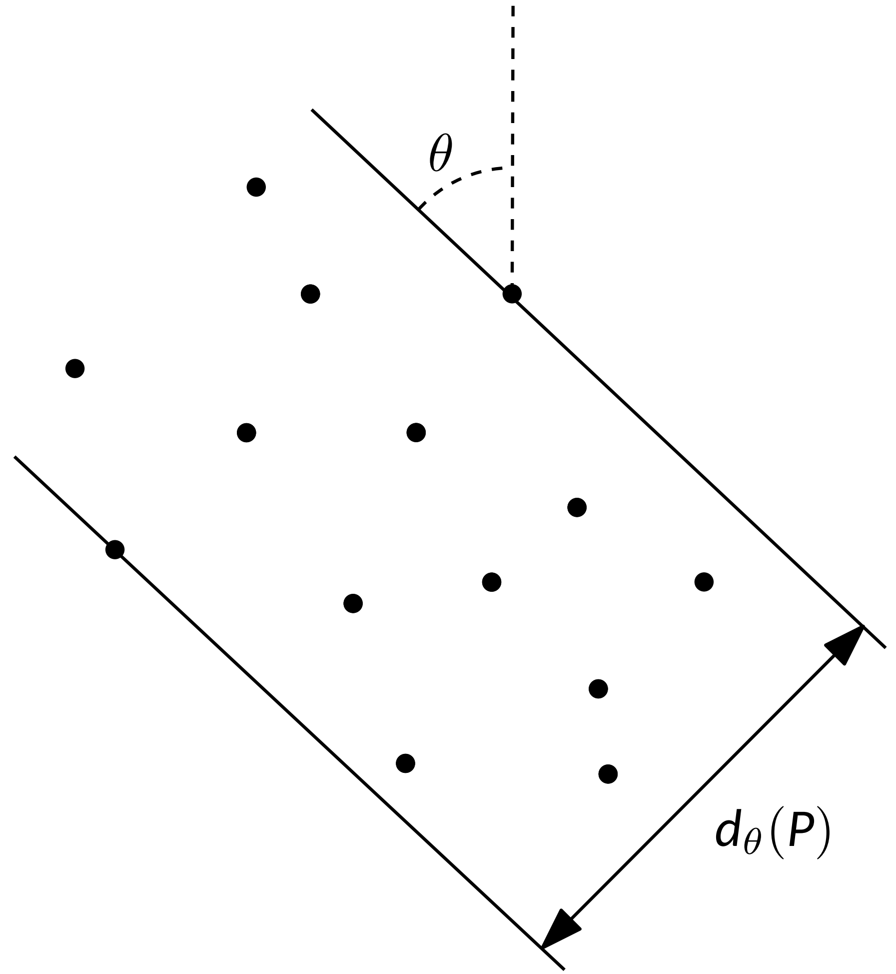

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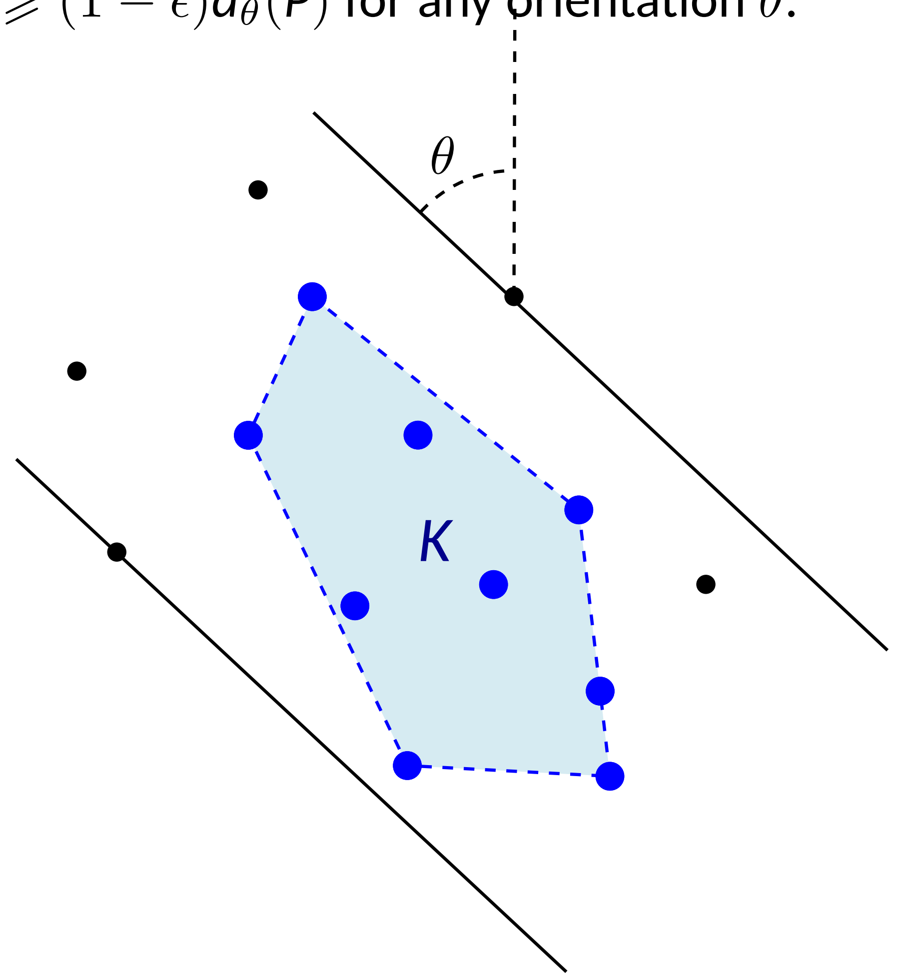
$d_\theta(K) \geqslant (1-\epsilon)d_\theta(P)$ for any orientation θ.



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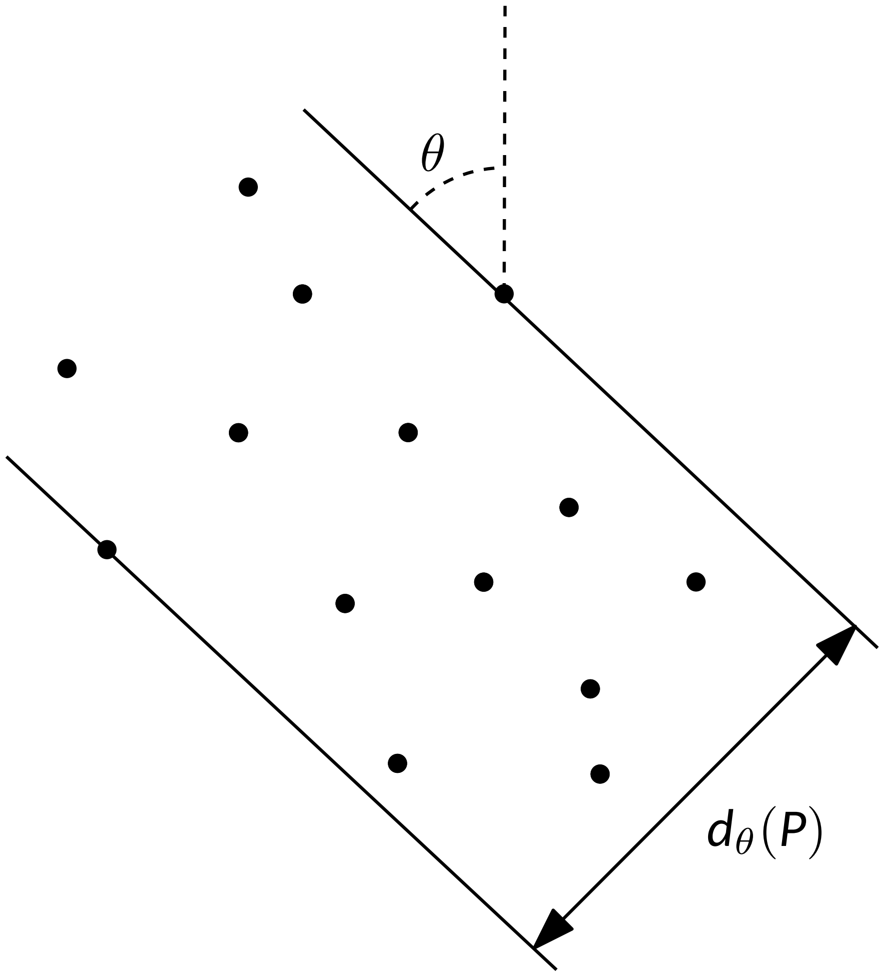

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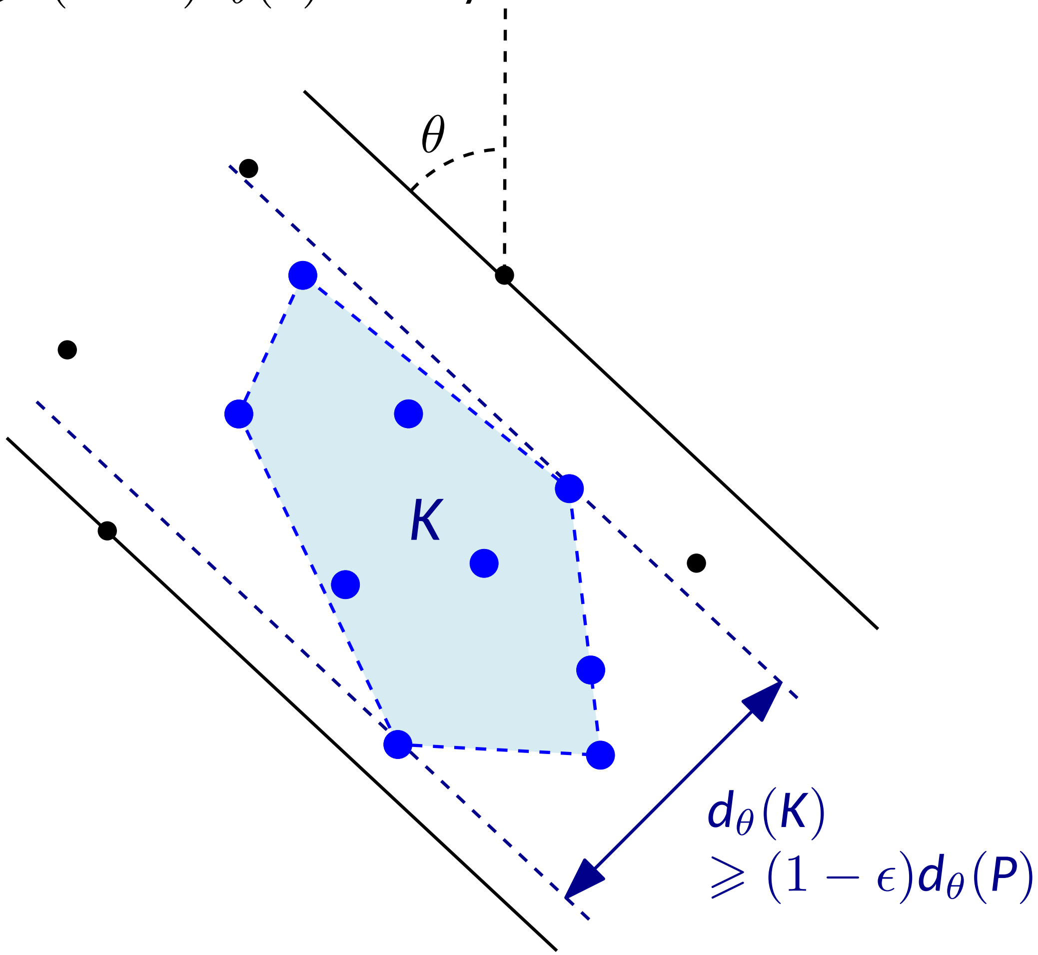

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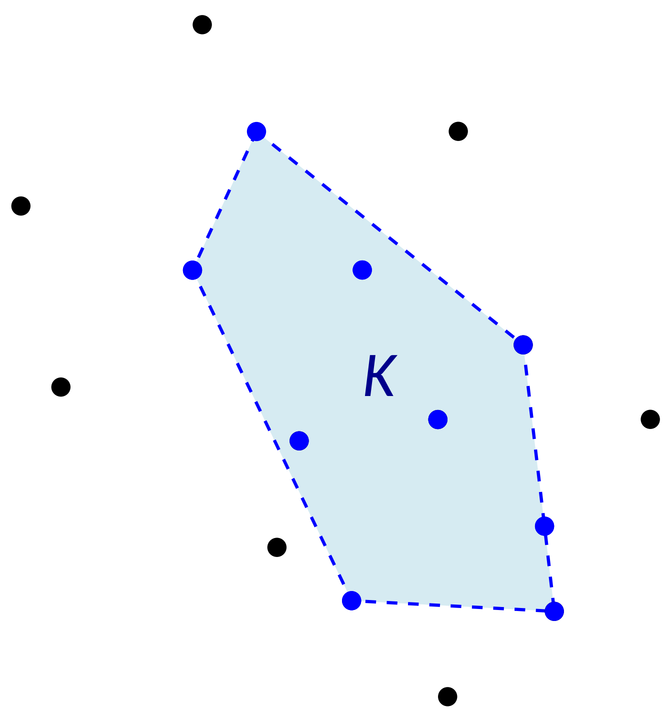
Let $\mathbf{K} \subset P$ be a $(\rho/2)$-coreset for directional width of P.



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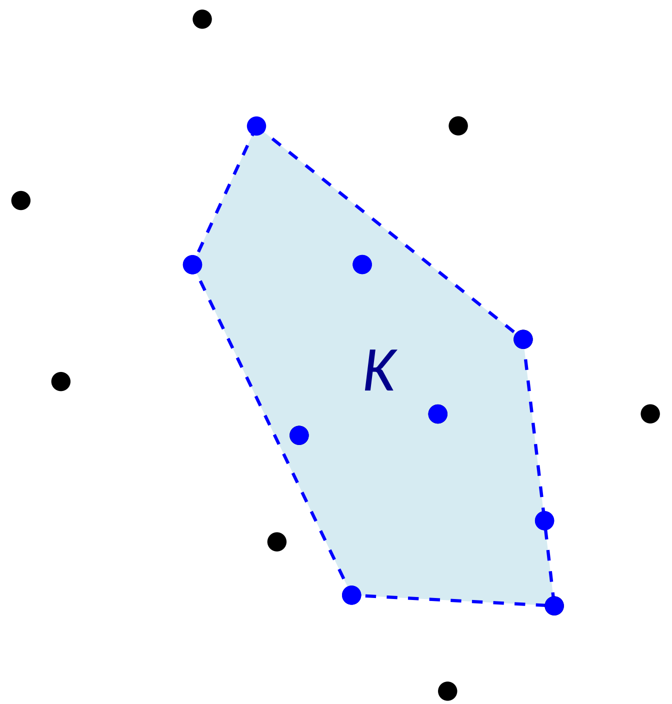

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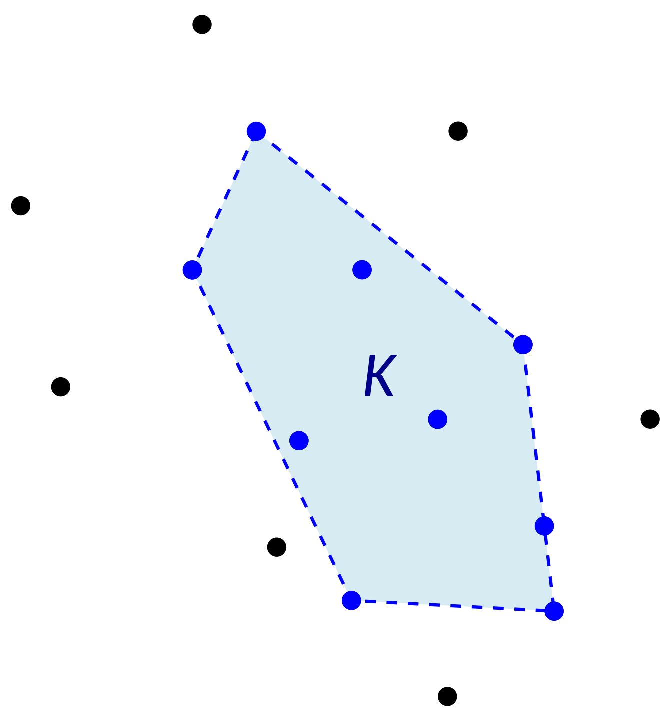

Lemma. For any k-slab cover $S = (\sigma_1, \dots, \sigma_k)$ of P, it holds that $K \cap \sigma_1 \neq \emptyset$ and $K \cap \sigma_k \neq \emptyset$.

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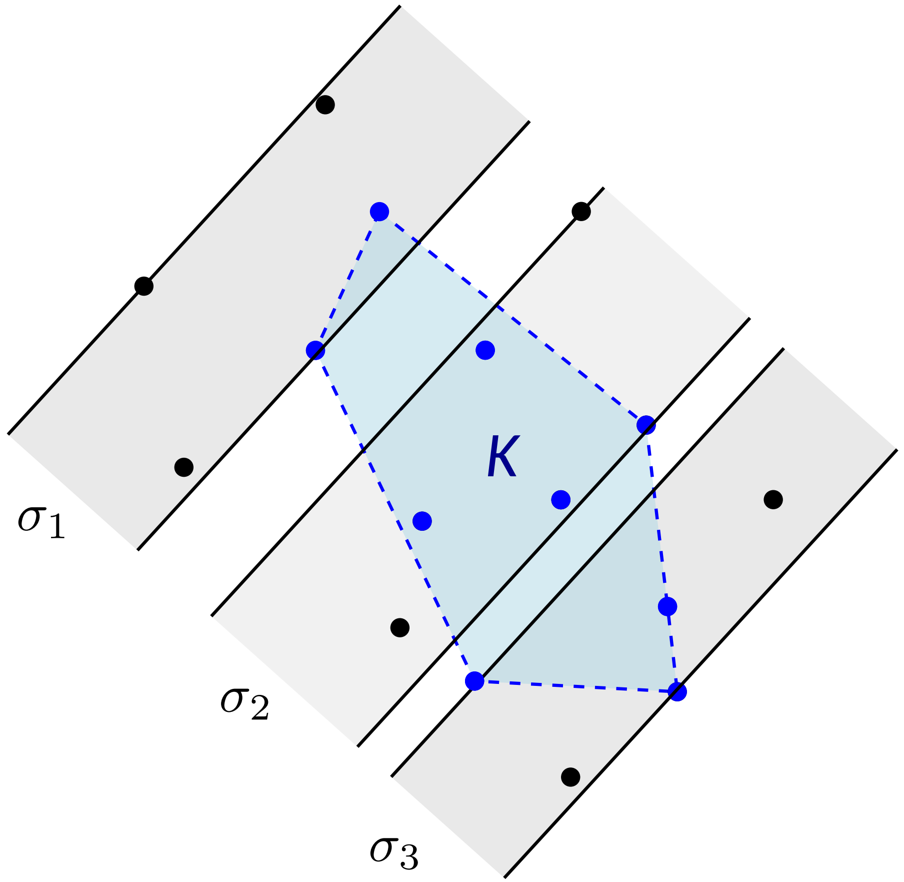

Lemma. For any k-slab cover $S = (\sigma_1, \ldots, \sigma_k)$ of P, it holds that $K \cap \sigma_1 \neq \emptyset$ and $K \cap \sigma_k \neq \emptyset$.

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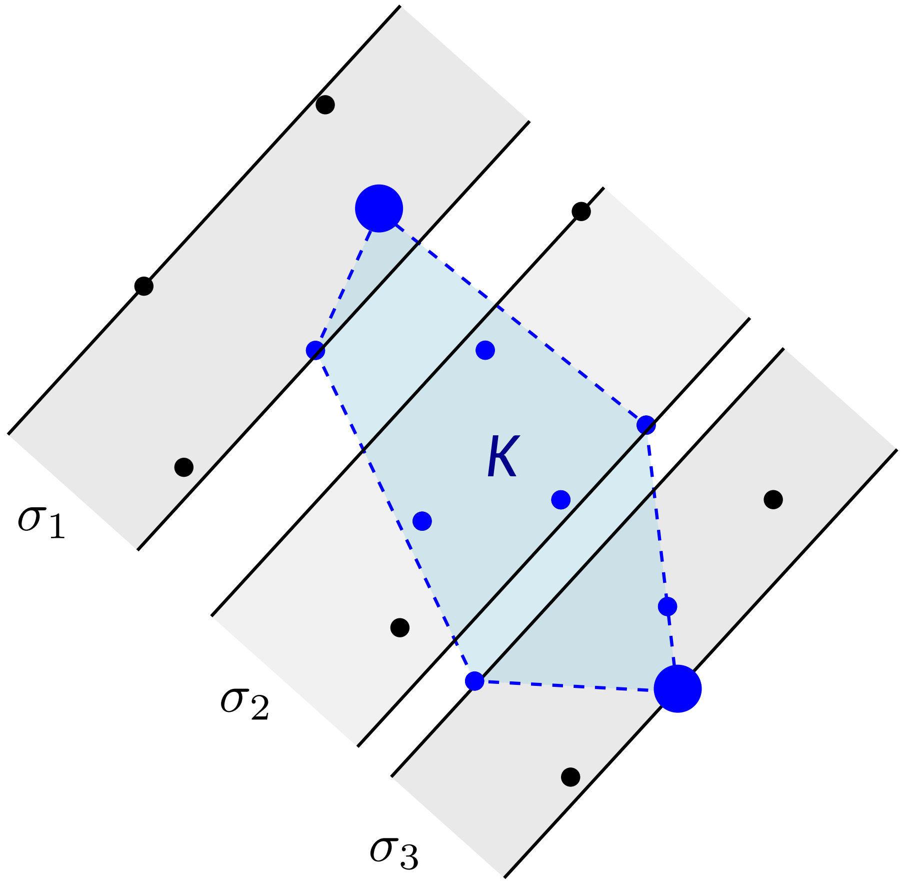

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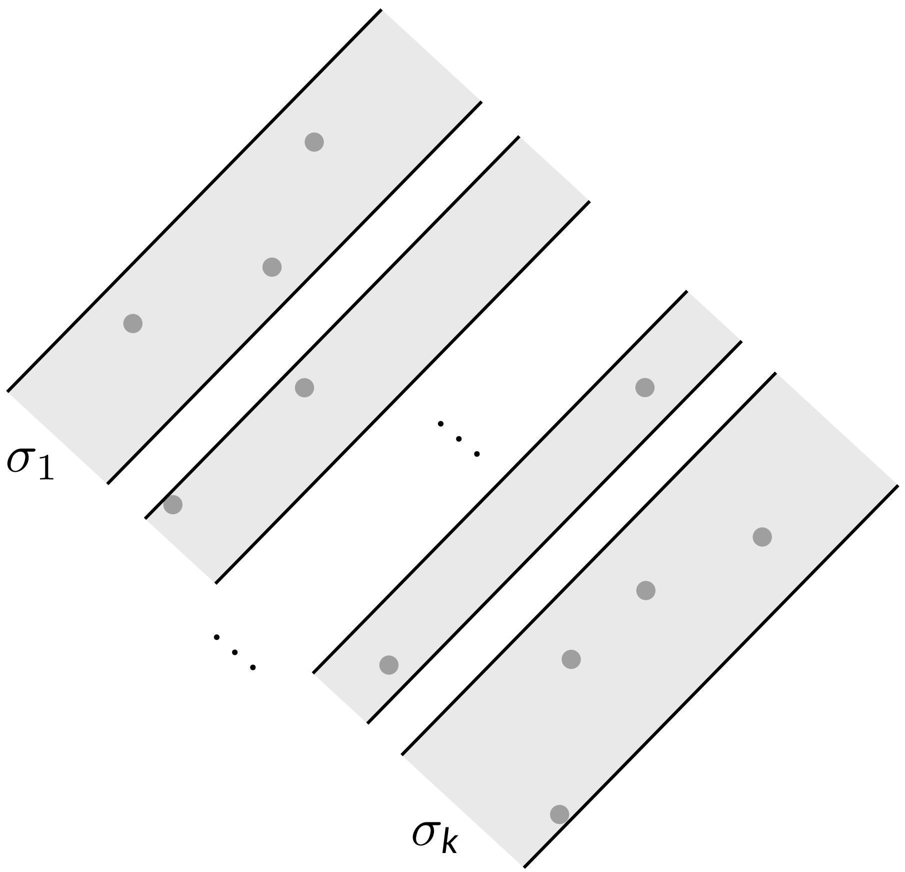

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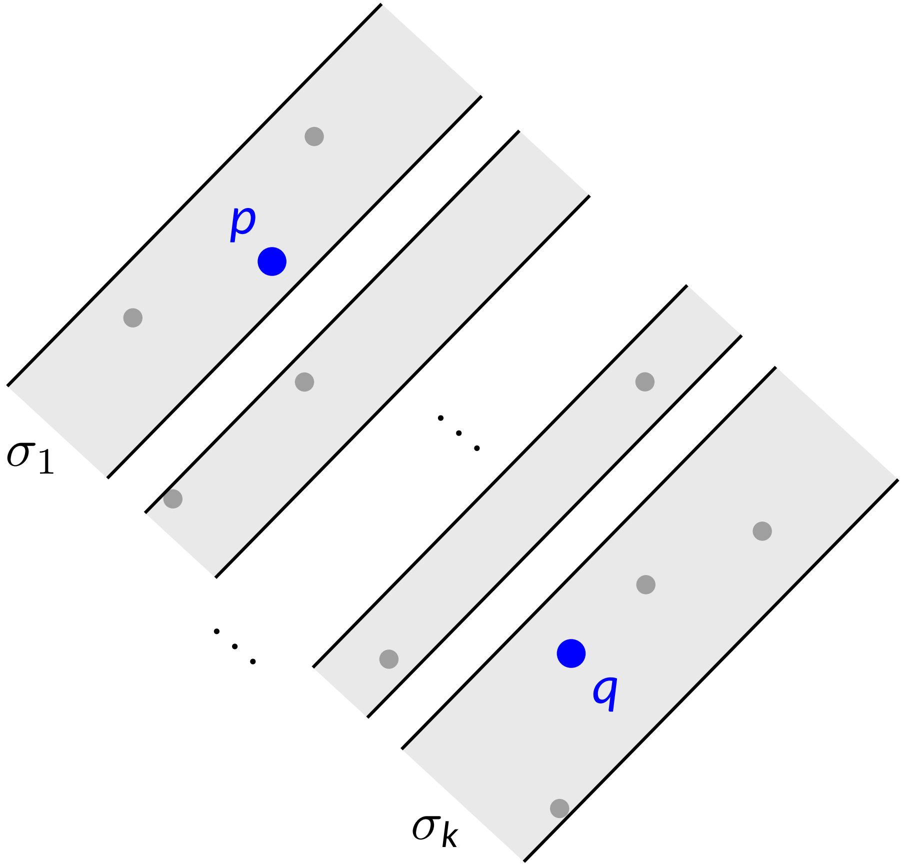

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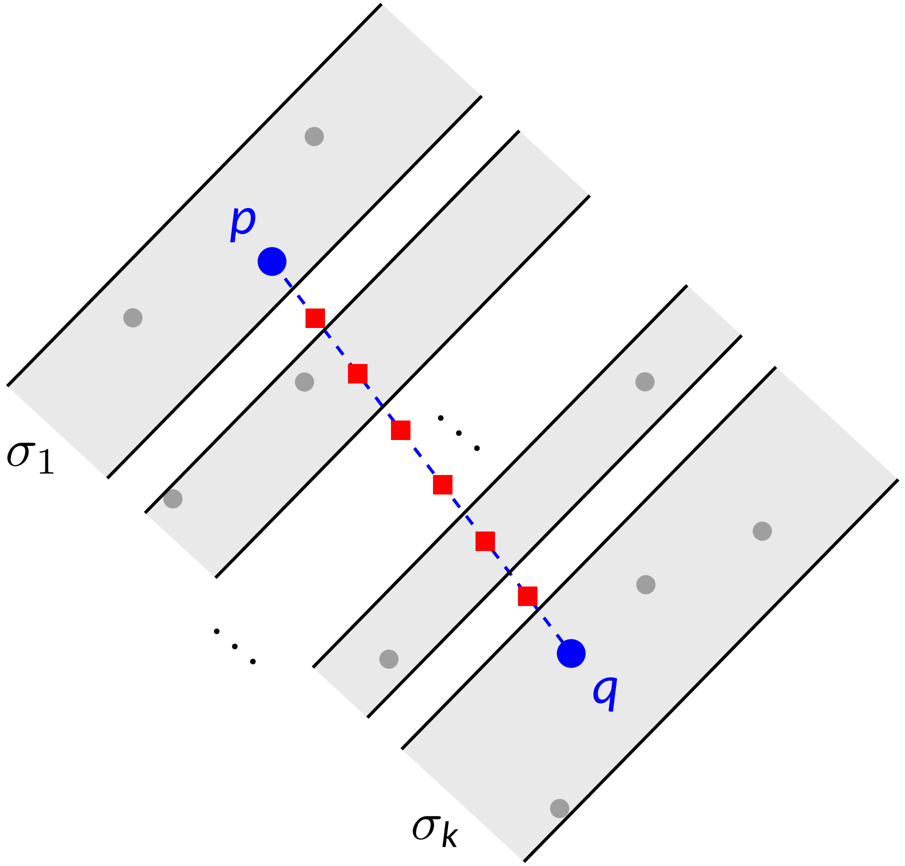

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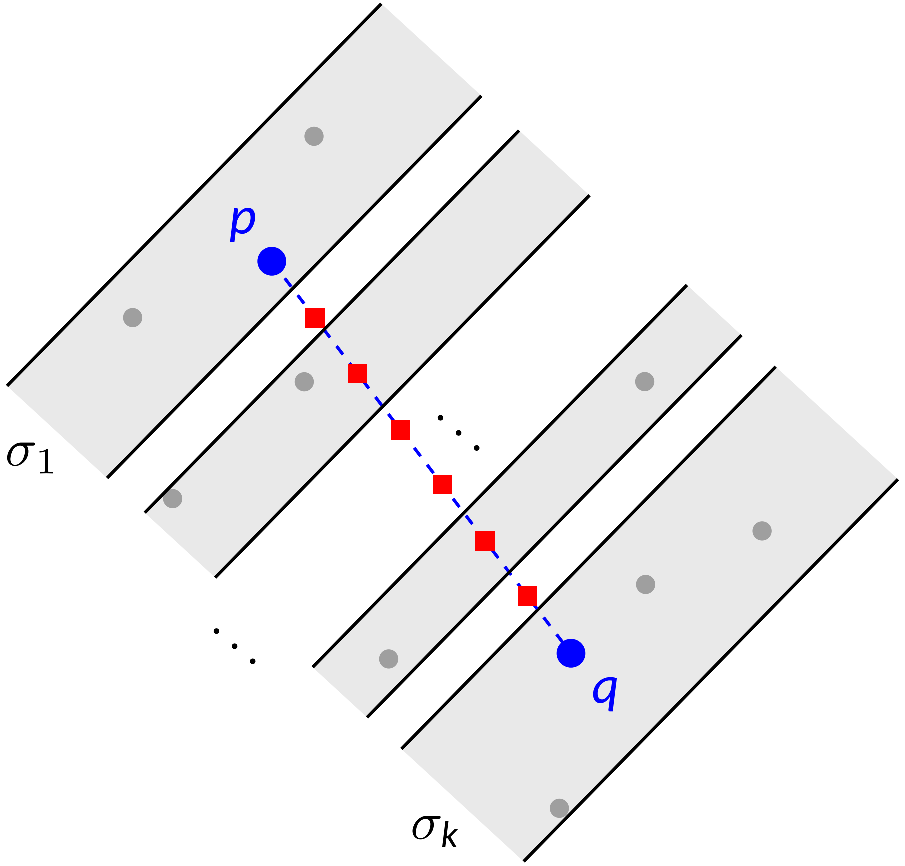

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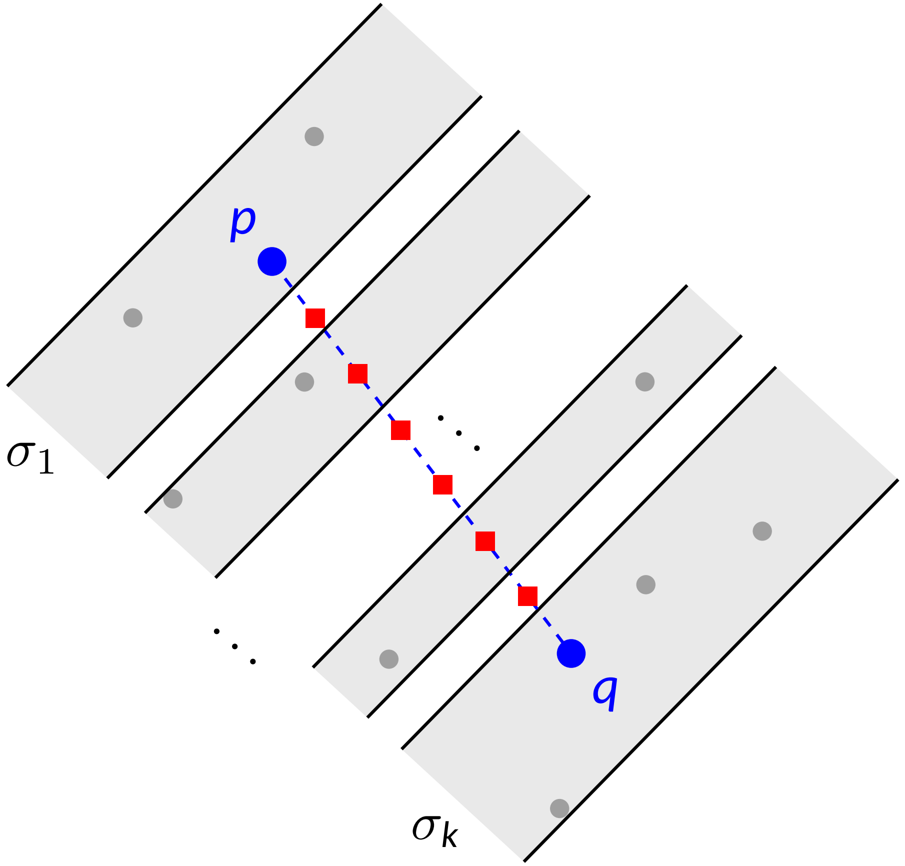

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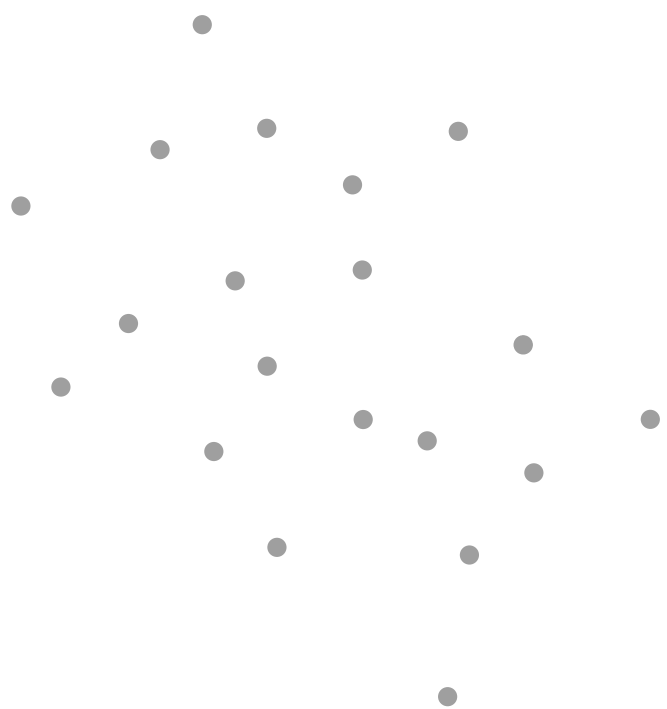

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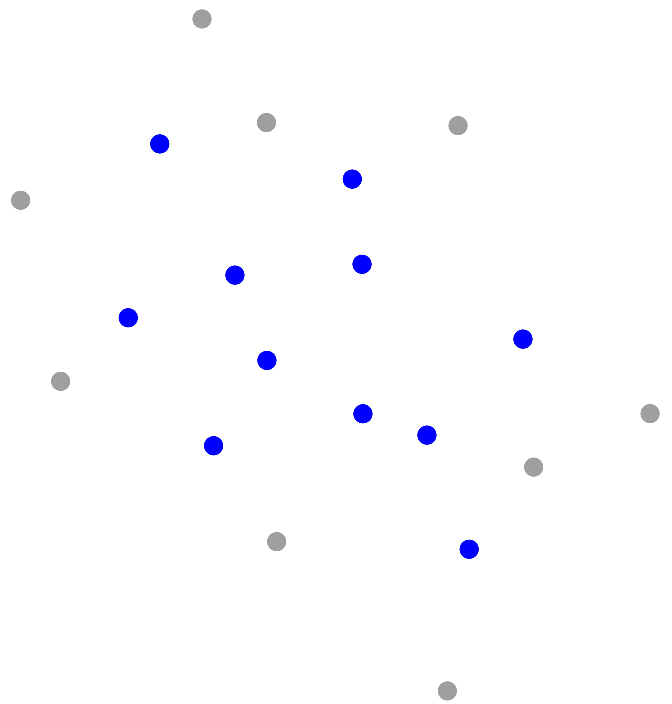

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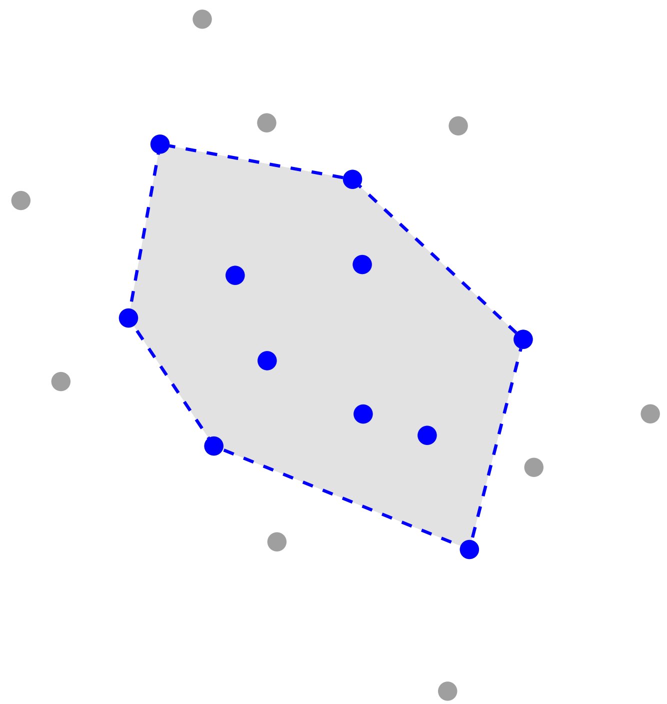
Corollary. There is an antipodal pair (p, q) of K such that $p \in \sigma_1$ and $q \in \sigma_k$.



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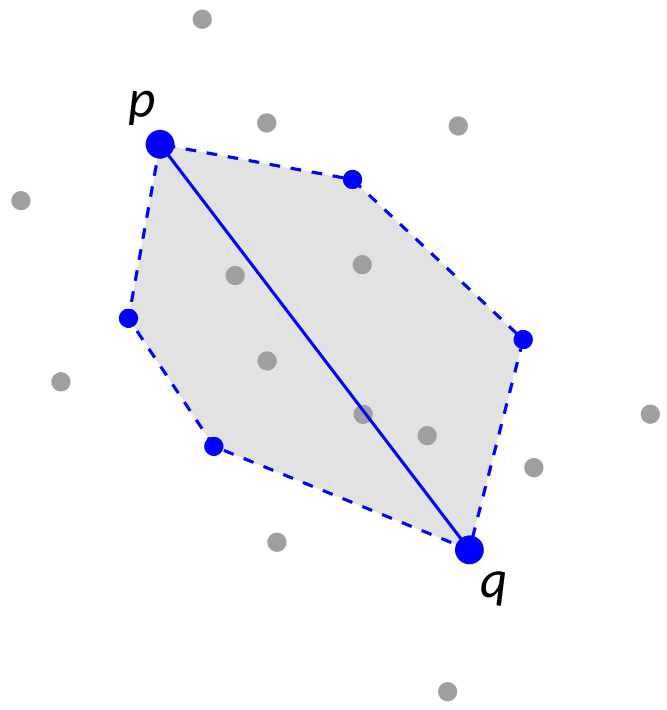

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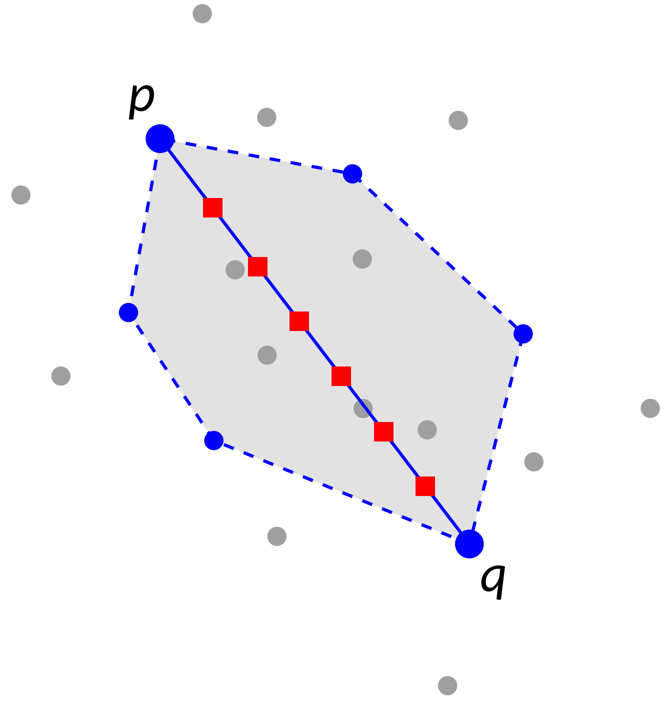

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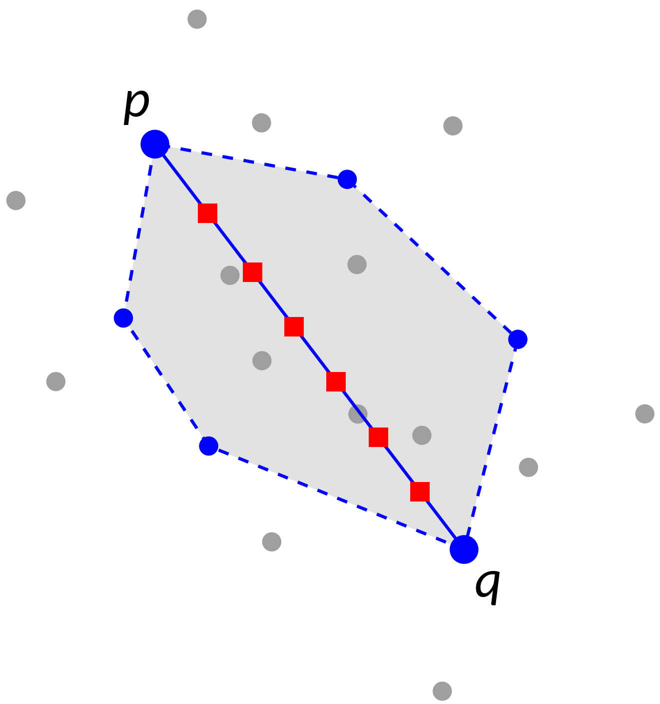
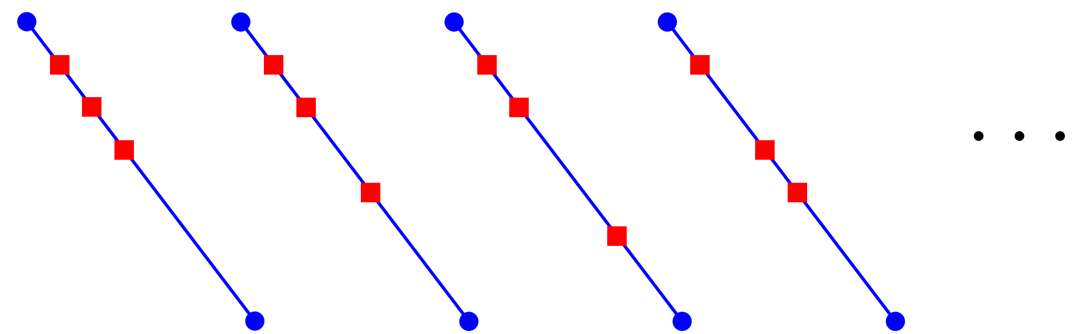


When $k = 4$



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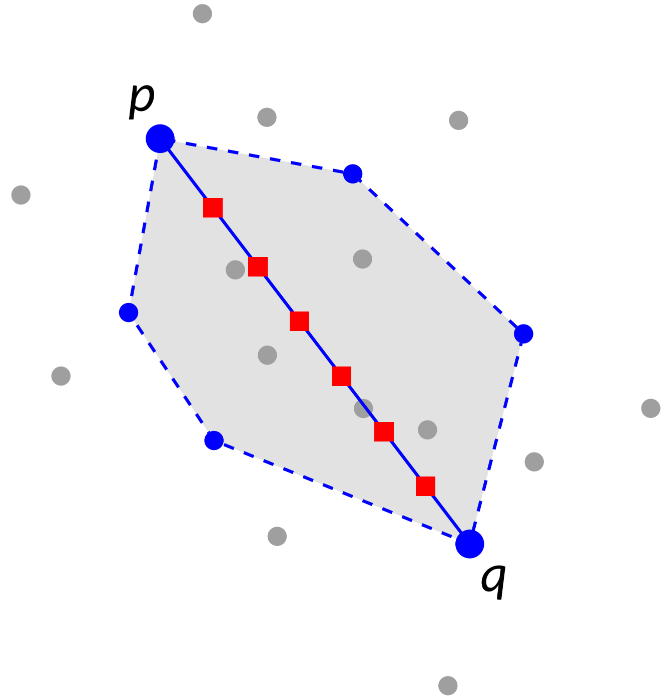

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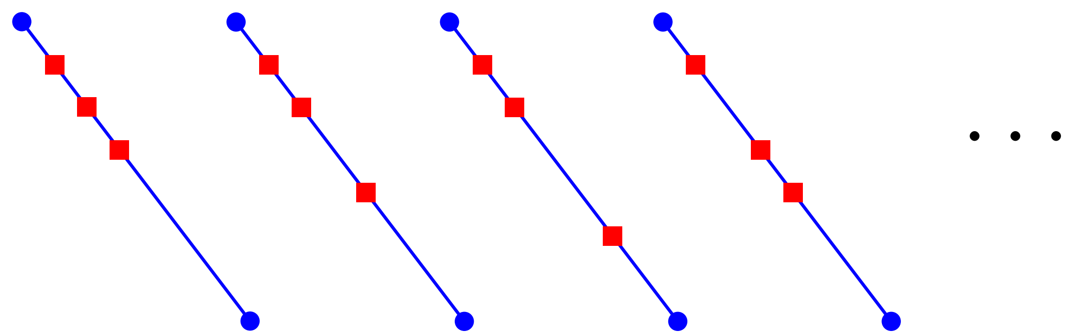

$|K| = O(1/\rho)$, # of combinations per antipodal pair = $O(1/\rho^{k-1})$

$\rightarrow$ # of candidate separators is $O(1/\rho^k)$

4. Algorithm

1) Given a separator R, we can compute a minimum width k-slab cover of P respecting R in $O(kn \log n)$ time and $O(n)$ space.

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Theorem. A minimum-width k-slab cover of P whose gap-ratio is at least ρ can be computed in $O(\rho^{-k} \cdot kn \log n)$ time and $O(n)$ space, if exists.

Thank you!