CCOSKEG Discs in Simple Polygons

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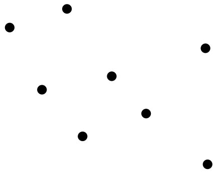
35th Canadian Conference on Computational Geometry

Problem



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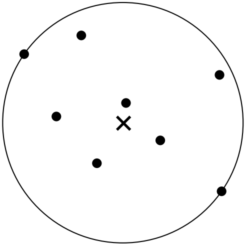
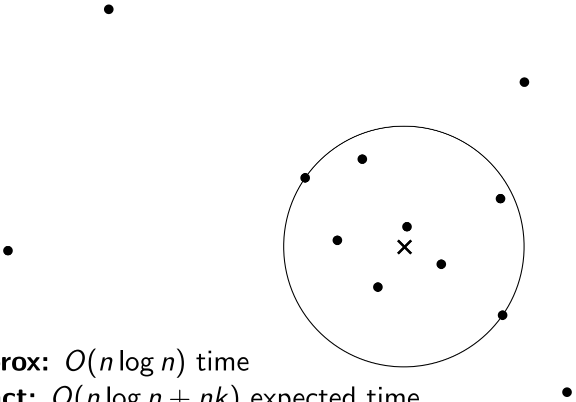


exact: $O(n)$ time
(Megiddo, '83), (Welzl, '91)

Problem

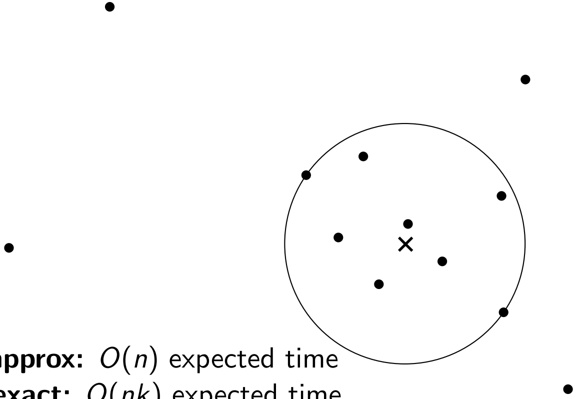


2-**approx:** $O(n \log n)$ time

exact: $O(n \log n + nk)$ expected time

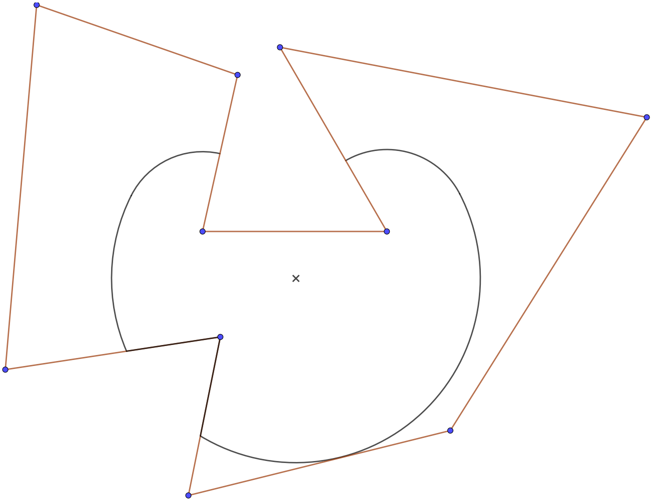
(Matoušek, '95)

Problem



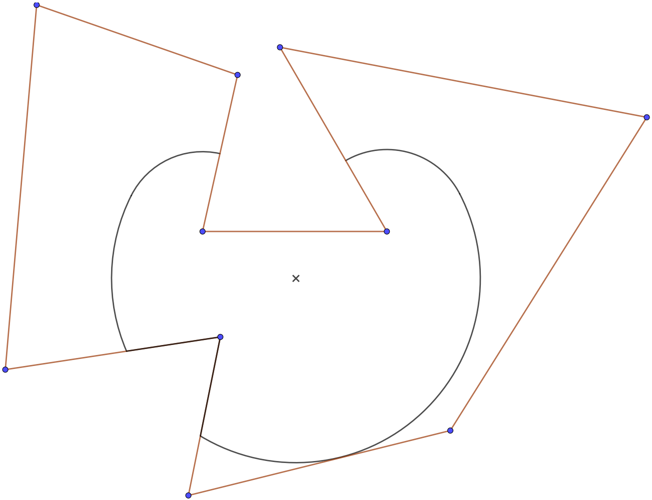
2-**approx:** $O(n)$ expected time
exact: $O(nk)$ expected time
(Har-Peled & Mazumdar, '05)

Problem



Problem

2-approx: $O(n \log^2 n \log r + m)$ expected time
(Bose, D'Angelo, Durocher, '23)



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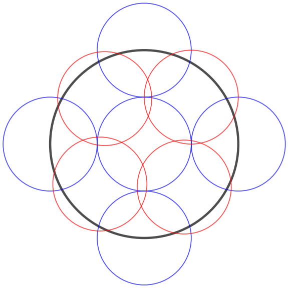
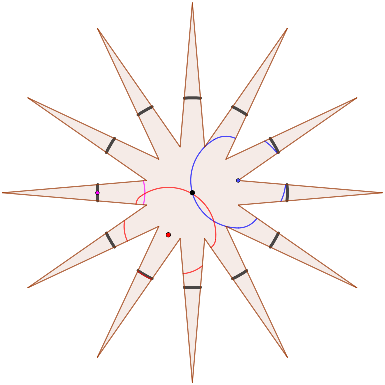


Euclidean



Geodesic

Problem

Chord-Constrained Smallest k-Enclosing Geodesic Disc

Given:

- a simple polygon P defined by a sequence of $\boldsymbol{m}$ vertices in $\mathbb{R}^2$, $\boldsymbol{r}$ of which are reflex vertices
- a set S of $\boldsymbol{n}$ points of $\mathbb{R}^2$ contained in P
- integer $k \leq n$
- a chord $\ell \subset P$

Find a geodesic disc of minimum radius $\boldsymbol{\rho}^*$ centred on ℓ and contained in P that contains $\boldsymbol{k}$ points of S.

Problem

KEG disc k-**enclosing geodesic disc**
geodesic disc centred and contained in P that contains k points of S

Problem

KEG disc — k-**enclosing geodesic disc**
geodesic disc centred and contained in P that contains k points of S

SKEG disc — **smallest** k-**enclosing geodesic disc**
KEG disc with smallest radius

Problem

KEG disc **k-enclosing geodesic disc**
geodesic disc centred and contained in P that contains k points of S

SKEG disc **smallest k-enclosing geodesic disc**
KEG disc with smallest radius

CCOSKEG disc **chord-constrained SKEG disc**
a SKEG disc whose centre is constrained to lie on an input chord of P

CCOSKEG

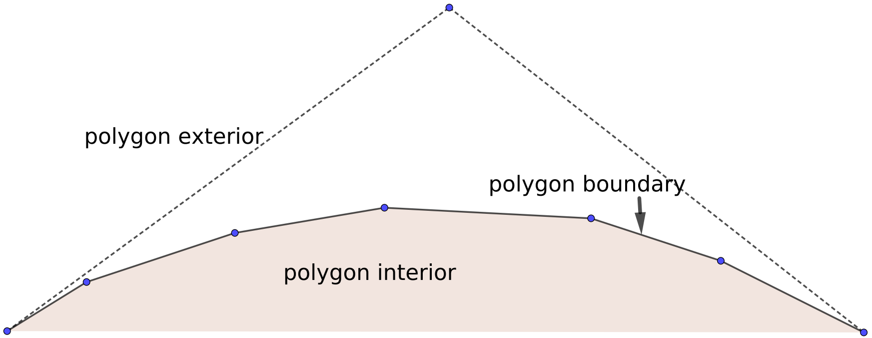
Theorem

We compute a CCOSKEG disc in $O(n \log^2 n + m)$ time with high probability ($\geq 1 - e^{-\log^b n}$ for some constant $b > 0$) using $O(n \log r + m)$ space.

Tools

Polygon Simplification

$O(m)$ time **(Aichholzer et al., '14)**
Size $O(r)$, preserves visibility, shortest paths

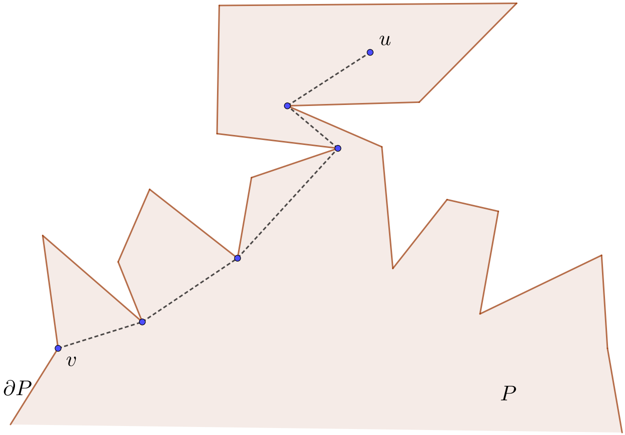


Tools

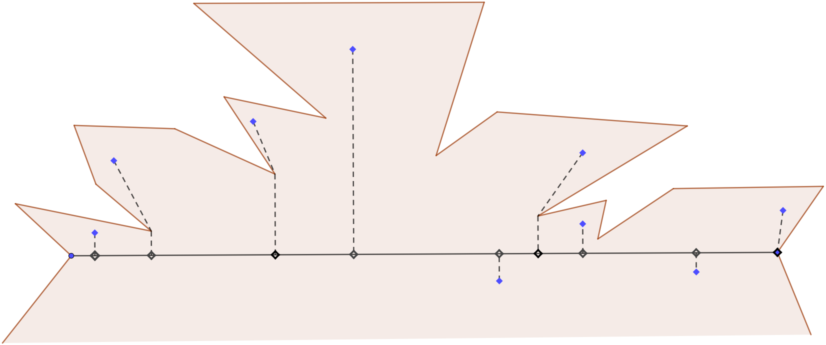
Shortest-Path Query Data Structure

$O(r)$ build time **(Guibas, Hershberger, '89)**

$O(\log r)$ query time

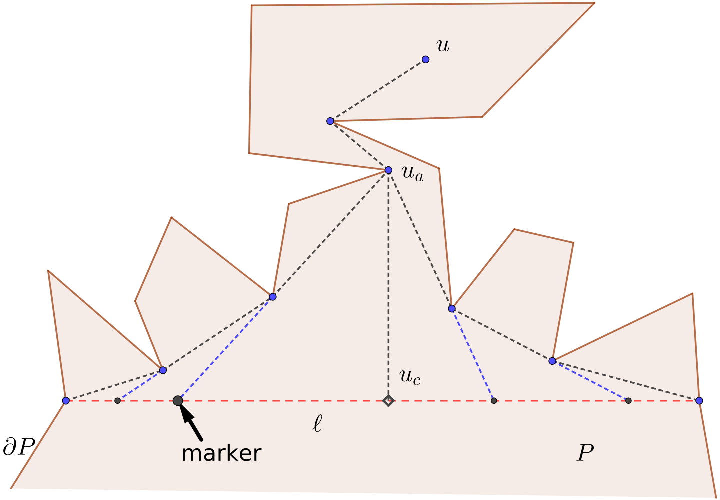


Projections



Searching Along ℓ

Binary Search



Parametric Search

(Megiddo, '79, '83)

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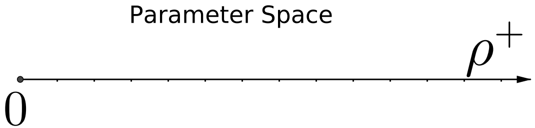
Search through parameter space looking for a "solution"

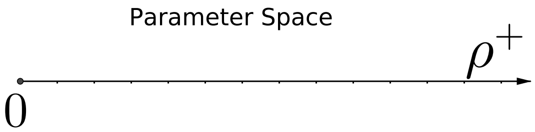


Parametric Search

(Megiddo, '79, '83)

Search through parameter space looking for a "solution"

- decision algorithm to test candidates

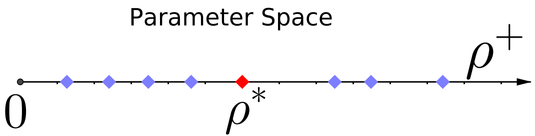


Parametric Search

(Megiddo, '79, '83)

Search through parameter space looking for a "solution"

- decision algorithm to test candidates
- candidates need "monotonicity" property

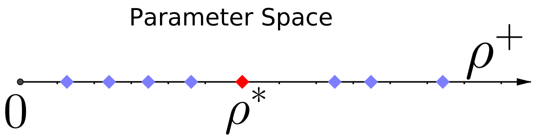


Parametric Search

(Megiddo, '79, '83)

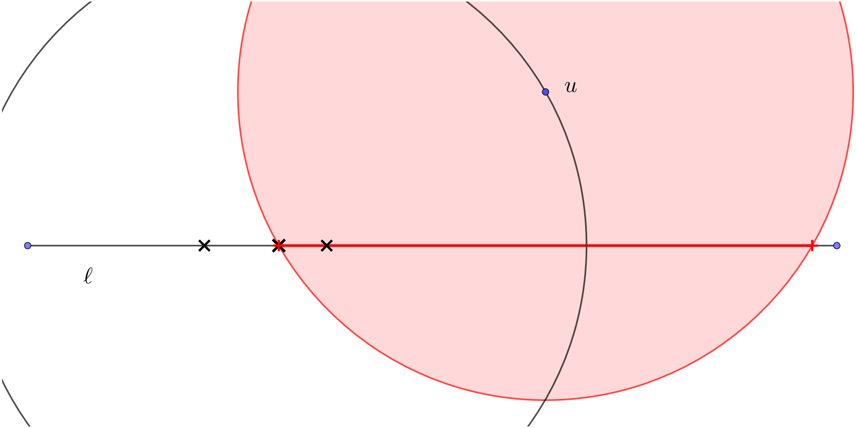
Search through parameter space looking for a "solution"

- decision algorithm to test candidates
- candidates need "monotonicity" property
- generic algorithm providing candidate solutions



Decision Algorithm

Disc of radius ρ



Decision Algorithm

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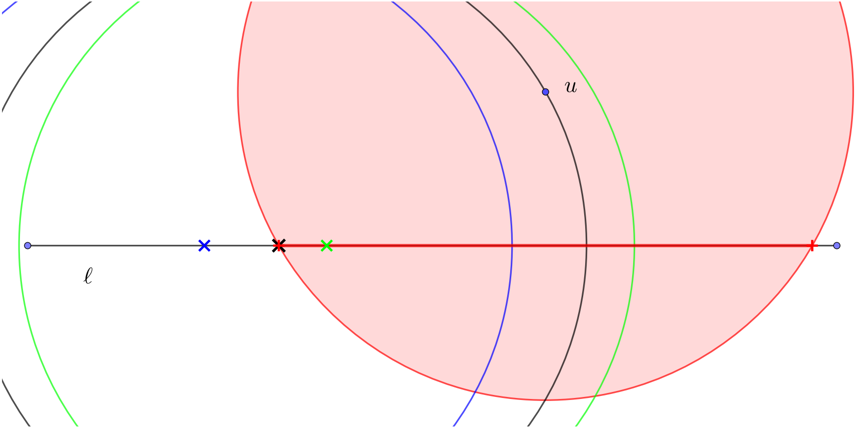
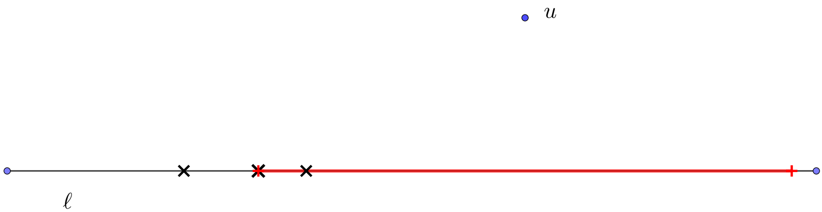


Decision Algorithm

Disc of radius ρ

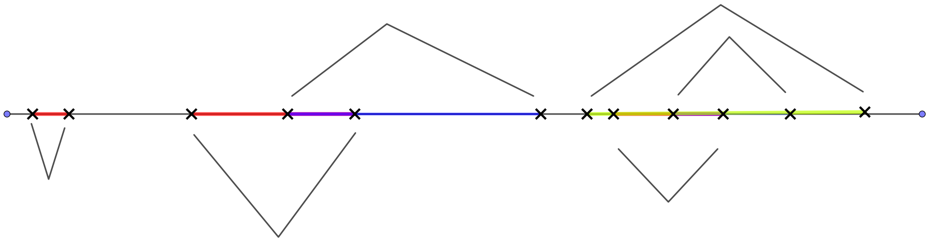


Decision Algorithm

Disc of radius ρ

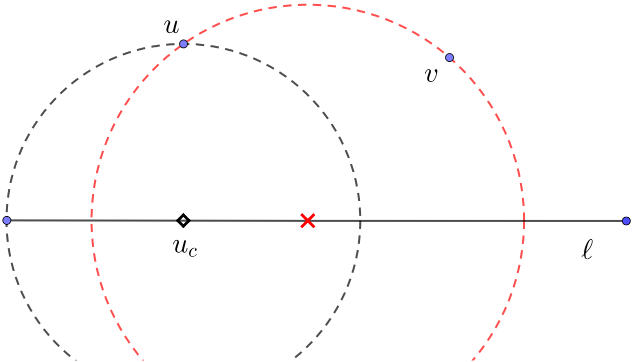
Count the depth of the overlapping intervals

$\implies O(n(\log r + \log n))$ time, $O(n + r)$ space



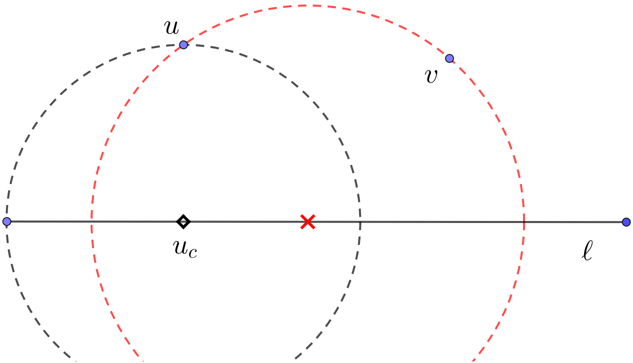
What Are Our Candidates?

- projections
- intersection of ℓ with geodesic bisector

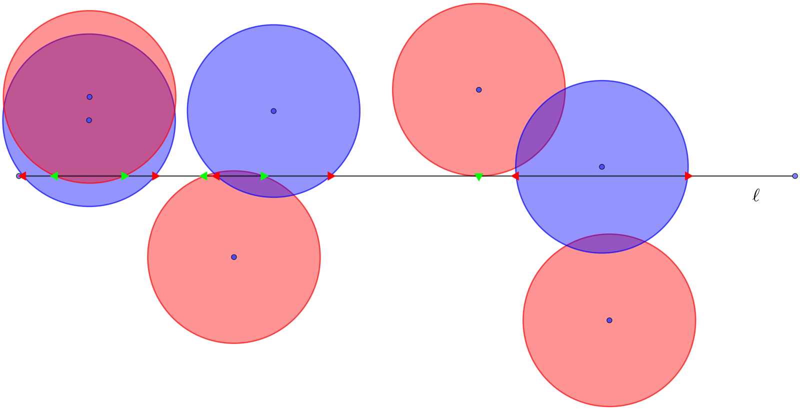


What Are Our Candidates?

- Test projections chosen with $O(n)$ time median selection algorithm (Blum et al., '73)
- $O(\log n)$ calls to decision algorithm



What to Sort



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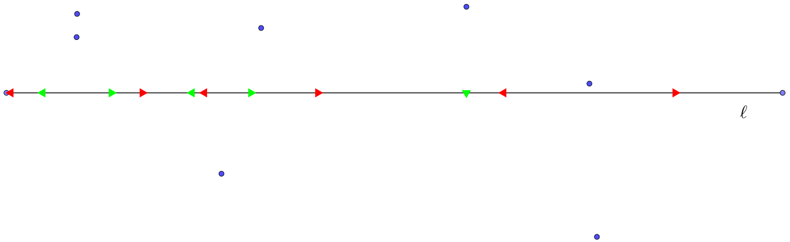
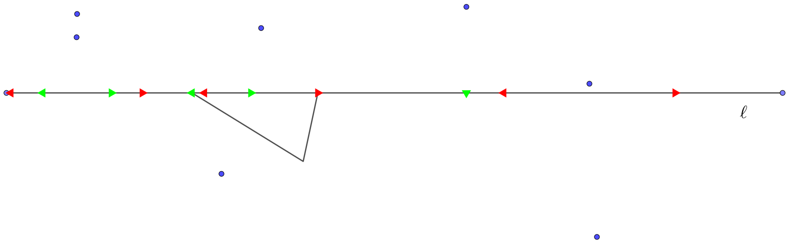
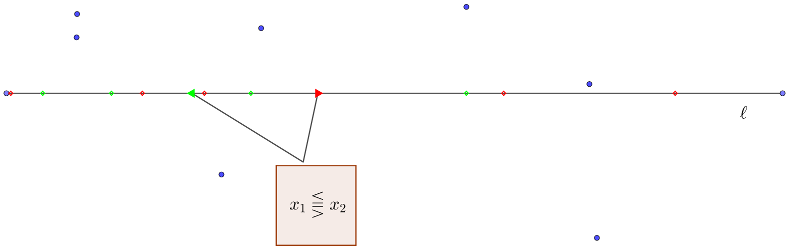


How to Compare



How to Compare

Assume the reflex vertices defining endpoint-equations are known



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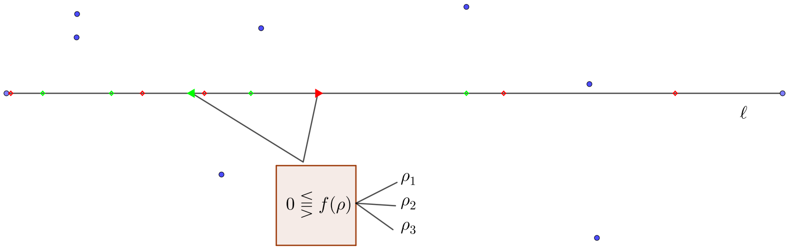

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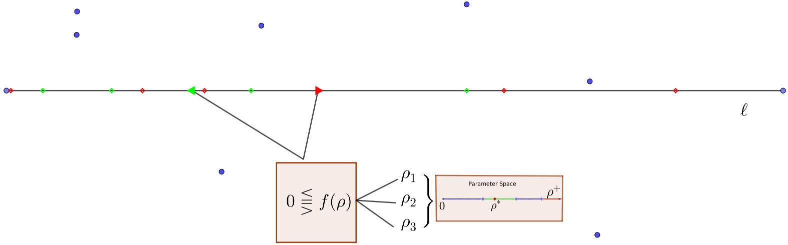
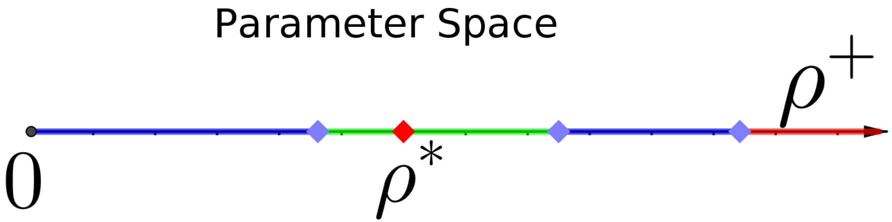
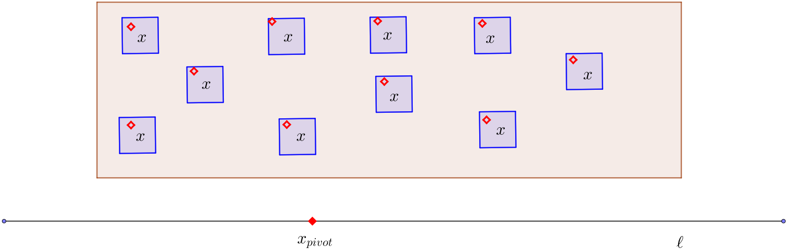


How to Compare



QuickSort

(van Oostrum, Veltkamp, '04)



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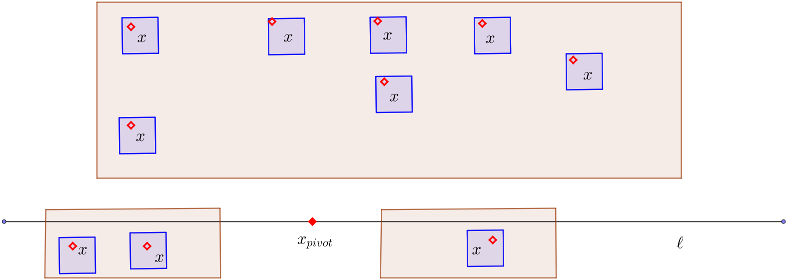
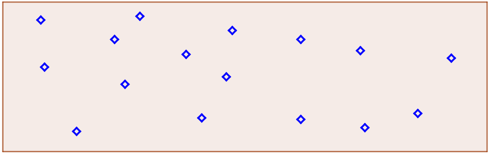


BoxSort

Boxsort: (Reischuk, '85)



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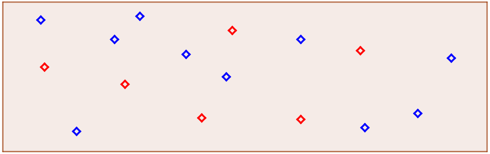

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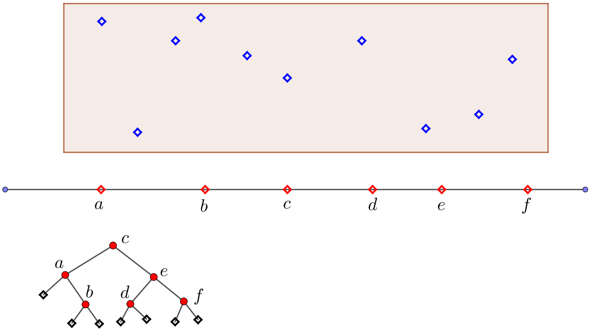

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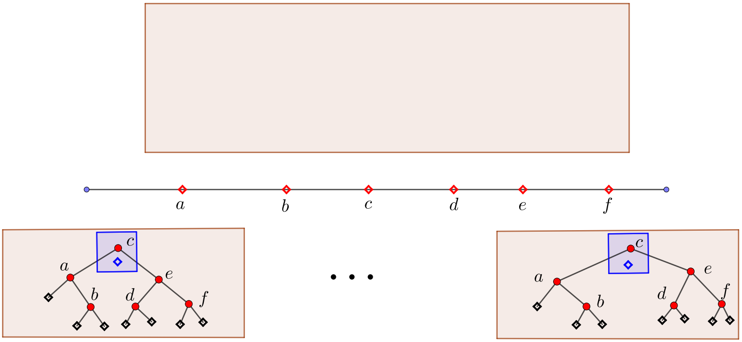

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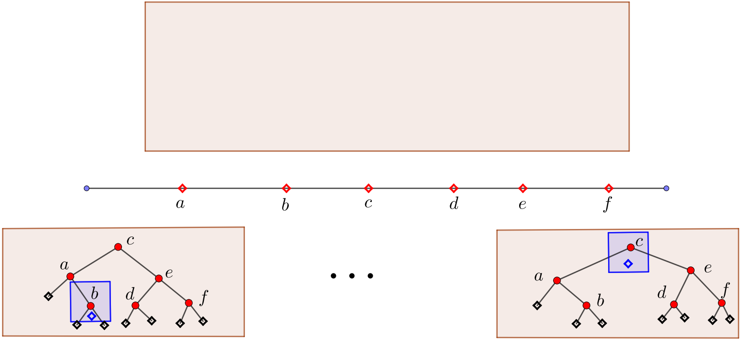
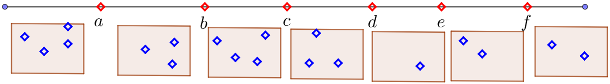


BoxSort

Boxsort: (Reischuk, '85)
Parm. search with boxsort: (Goodrich, Pszona, '13)
Weighted selection, $O(n)$ time: (Reiser, '78)



$O(\log n)$ calls to decision alg with high probability

CCOSKEG

Theorem

We compute a CCOSKEG disc in $O(n \log^2 n + m)$ *time with high probability* ($\geq 1 - e^{-\log^b n}$ *for some constant* $b > 0$) *using* $O(n \log r + m)$ *space.*

Comparing to Higher Order Geodesic VDs

Ignoring Polylogs

Can be solved exactly in the polygon with higher-order geodesic VDs in worst-case time $O(k^2 n + k^2 r + \min(kr, r(n-k)) + m)$.

Comparing to Higher Order Geodesic VDs

Ignoring Polylogs

CCOSKEG: $O(n + m)$
OKGVD:

Comparing to Higher Order Geodesic VDs

Ignoring Polylogs

CCOSKEG: $O(n+m)$

OKGVD:

- for $k \in \Theta(1)$: $O(n+m)$
- for $k \in \Omega(n)$, $k < n-1$: $O(\text{more than } n^3)$ time

Great animation:

Ilinkin, SOCG '13, **DOI:** 10.1145/2462356.2462359

Notes: Michiel Smid

"Solving Geometric Optimization Problems Using Parametric Search"

The End

Related Results

- Coverings/packing simple polygon with geodesic discs [11, 13]
- Geodesic centre, simple polygon [1, 3, 5, 10, 12]
  Geodesic 2-centre, simple polygon [9, 13]
- Geodesic centre, n points in simple m-gon: $O(m + n\log(mn))$ [2, 7, 12]
  Geodesic 2-centre, n points in simple m-gon: $O(n(m+n)\log^3(m+n))$ [8]
- Simple m-gon, n points, all geodesic discs of radius ρ that contain at least k points [4]: for output size $Y \in O(nm)$
  (ignoring polylogs)
  $O(m + (Ym)^{2/3} + Y + n^2)$
- Geodesic k-Nearest Neighbour Queries (static) [6]:
  built in $O(n * \text{polylog})$ expected time
  queries in $O(k * \text{polylog})$ expected time
- 2-approximation SKEG disc : $O(n\log^2 n\log r + m)$
  (Bose, D'Angelo, Durocher, WADS '23)

2-SKEG Depth

- $\rho \implies$ 2-SKEG radius

2-SKEG Depth

- $\rho \implies$ 2-SKEG radius
- depth $=?$

2-SKEG Depth

- $\rho \implies$ 2-SKEG radius
- depth $= \Theta(\min(kr, n))$

Can we do better?

2-SKEG Depth

2-SKEG depth

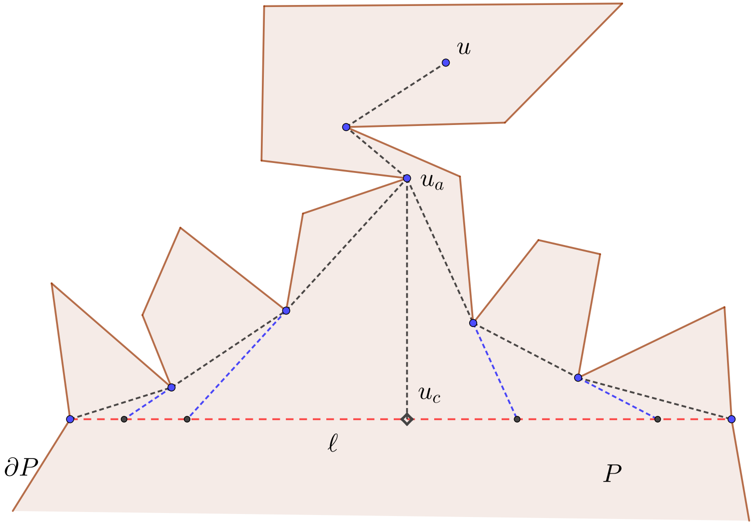
- In $O(nr \log^2 n + nr \log^2 r + m)$ expected time we compute a radius ρ using $O(n \log r + m)$ expected space that is a 2-approximation to the radius for a SKEG disc such that depth$(\rho) \leq 10k$.

2-SKEG Depth

2-SKEG depth

- In $O(nr\log^2 n + nr\log^2 r + m)$ expected time we compute a radius ρ using $O(n\log r + m)$ expected space that is a 2-approximation to the radius for a SKEG disc such that depth(ρ) $\leq 10k$.
- If we use $O(nr\log^2 n + nr\log^2 r + nk + m)$ expected time and $O(n\log r + k^2 + m)$ expected space, we can improve ρ such that depth(ρ) $\leq 4k$.

Endpoint-Equation Reflex Vertices



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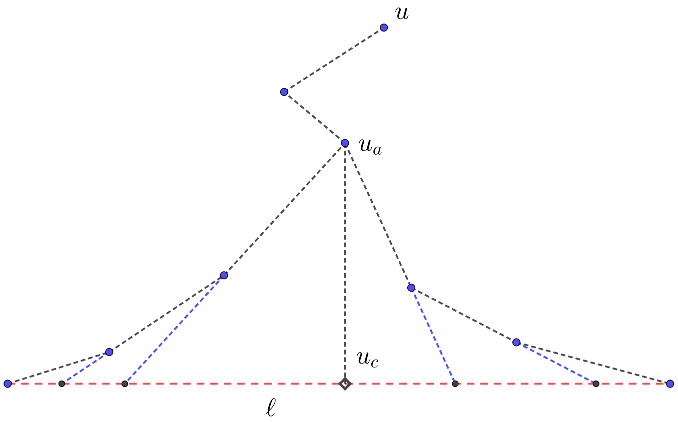

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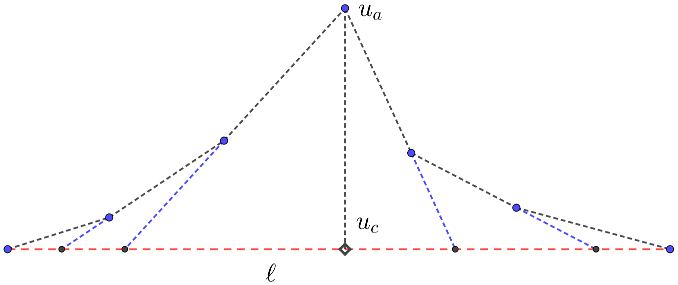

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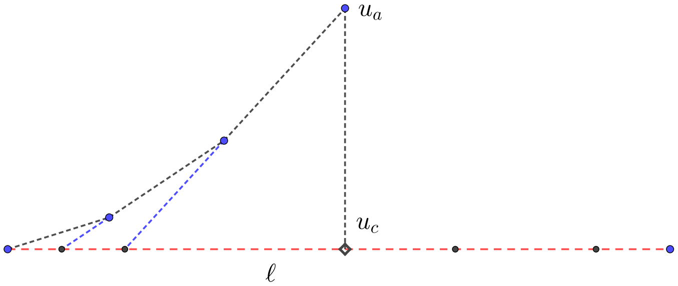

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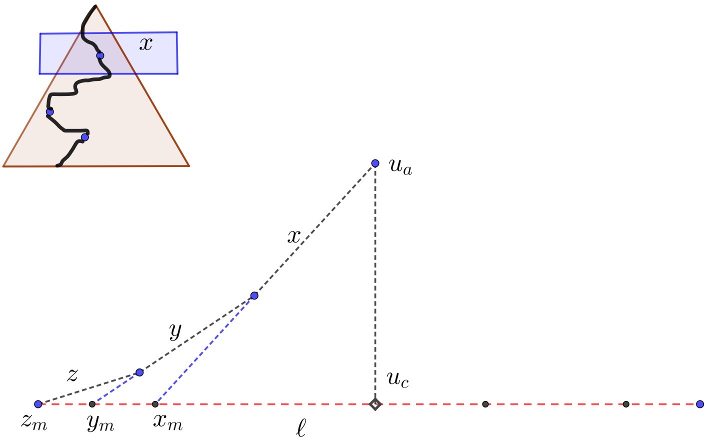

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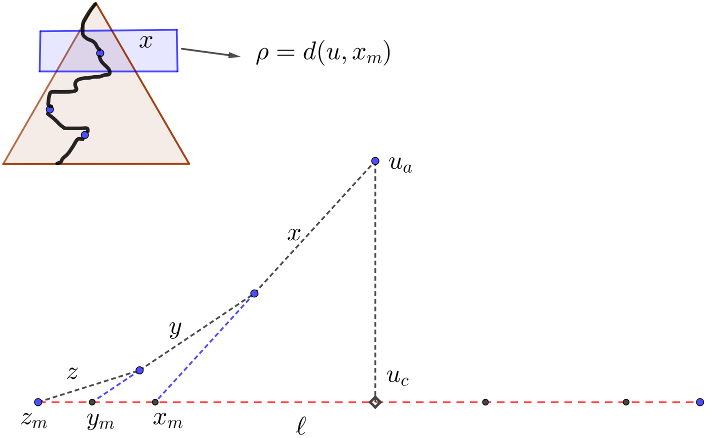

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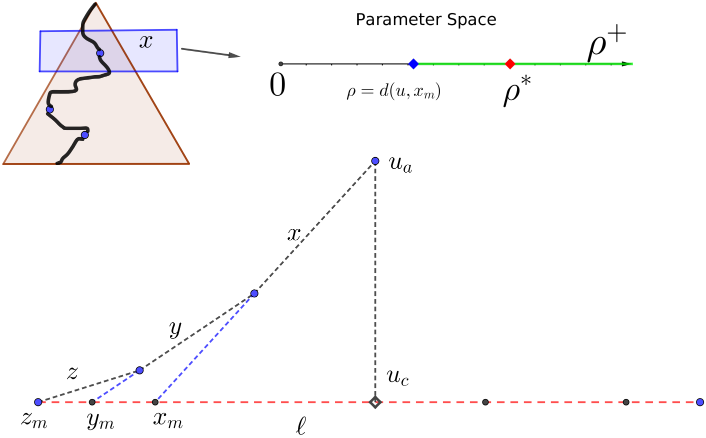

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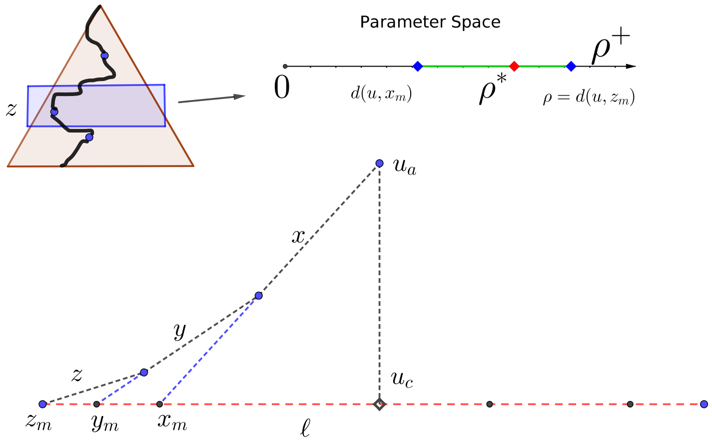

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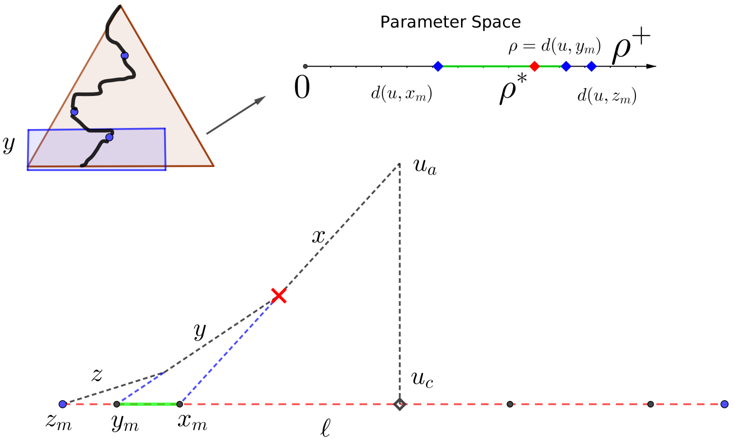

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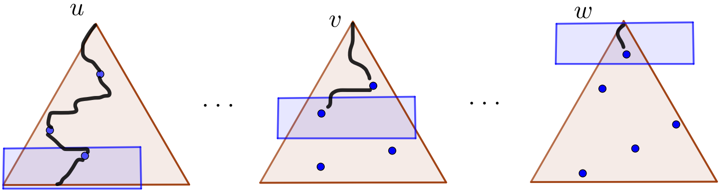

$O(\log n + \log r)$ calls to decision alg with high probability

References I

[1] H. Ahn, L. Barba, P. Bose, J. D. Carufel, M. Korman, and E. Oh. A linear-time algorithm for the geodesic center of a simple polygon. *Discrete & Computational Geometry*, 56(4):836–859, 2016.

[2] B. Aronov, S. Fortune, and G. T. Wilfong. The furthest-site geodesic voronoi diagram. *Discrete & Computational Geometry*, 9:217–255, 1993.

[3] T. Asano and G. Toussaint. Computing the geodesic center of a simple polygon. In *Discrete Algorithms and Complexity*, pages 65–79. Elsevier, 1987.

[4] M. G. Borgelt, M. J. van Kreveld, and J. Luo. Geodesic disks and clustering in a simple polygon. *Int. J. Comput. Geometry Appl.*, 21(6):595–608, 2011.

References II

[5] P. Bose and G. T. Toussaint. Computing the constrained euclidean geodesic and link center of a simple polygon with application. In *Computer Graphics International*, pages 102–110. IEEE Computer Society, 1996.

[6] S. de Berg and F. Staals. Dynamic data structures for k-nearest neighbor queries. *Computational Geometry*, 111:101976, 2023.

[7] L. J. Guibas and J. Hershberger. Optimal shortest path queries in a simple polygon. *Journal of Computer and System Sciences*, 39(2):126–152, 1989.

[8] E. Oh, S. W. Bae, and H. Ahn. Computing a geodesic two-center of points in a simple polygon. *Comput. Geom.*, 82:45–59, 2019.

[9] E. Oh, J. D. Carufel, and H. Ahn. The geodesic 2-center problem in a simple polygon. *Comput. Geom.*, 74:21–37, 2018.

[10] R. Pollack, M. Sharir, and G. Rote. Computing the geodesic center of a simple polygon. *Discrete & Computational Geometry*, 4:611–626, 1989.

References III

[11] G. Rabanca and I. Vigan. Covering the boundary of a simple polygon with geodesic unit disks. *CoRR*, abs/1407.0614, 2014.

[12] G. Toussaint. Computing geodesic properties inside a simple polygon. *Revue D'Intelligence Artificielle*, 3(2):9–42, 1989.

[13] I. Vigan. Packing and covering a polygon with geodesic disks. *CoRR*, abs/1311.6033, 2013.