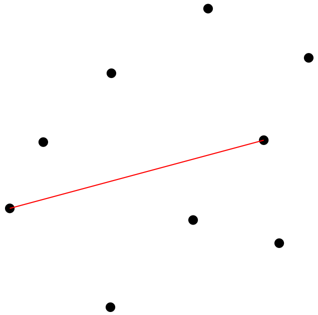
Approximating the discrete center line segment in linear time

Joachim Gudmundsson and Yuan Sha

University of Sydney

The problem

The discrete center segment of a point set P



The red segment is the discrete center segment

Computing the discrete center segment

- Previous result: an $O(n^2)$ time, $O(n^2)$ space exact algorithm by Daescu and Teo.

Computing the discrete center segment

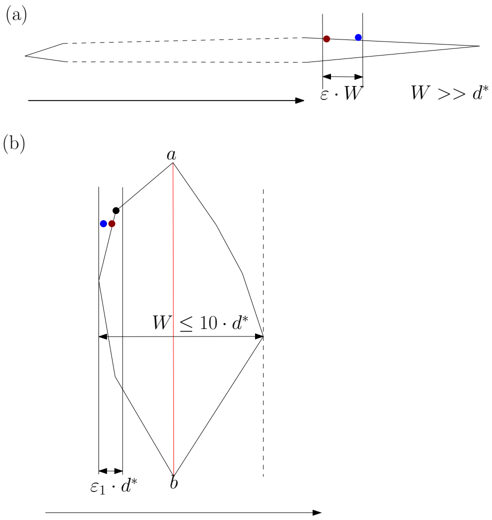
- Use approximation: $(1+\varepsilon)$-approximation algorithm that runs in $O(n + \frac{1}{\varepsilon^4} \log \frac{1}{\varepsilon})$ time and uses linear space.

Approach

- compute an approximate convex hull
  $(1+\varepsilon)$-approximate convex hull has $O(1/\varepsilon)$ vertices
- reduce the number of candidate center segment: the diagonals of $CH(\tilde{P})$, grids

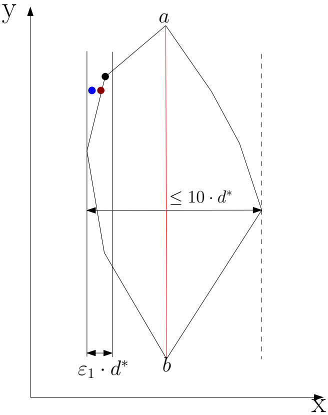
$O(n + \frac{1}{\varepsilon^7})$ time algorithm.

Find an orientation



- find an orientation for constructing the apprx convex hull: $(1+\varepsilon)$-approximate diametral point pair

Approximate point set



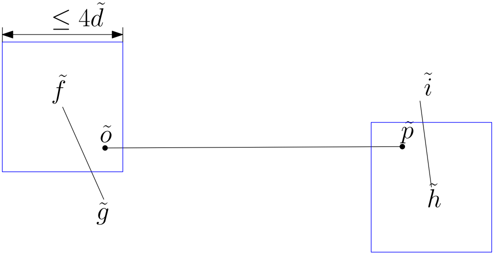
The blue point is in P and lies outside $ACH(P)$. We shift it until it lies on the boundary of $ACH(P)$. The red point is the shifted blue point.

Let $\tilde{P}$ be the approximate point set, a $(1 + \varepsilon)$ center segment of $\tilde{P}$ corresponds to a $(1 + \varepsilon)$ center segment of P.

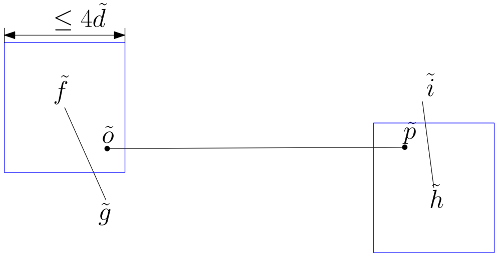
Computing a $(1+\varepsilon)$ center segment of $\tilde{P}$

Reduce the number of candidate center segment

- use the diagonals of $CH(\tilde{P})$ to estimate $\tilde{d}$
- lay a grid



$(1+\varepsilon)$ center segment of $\tilde{P}$



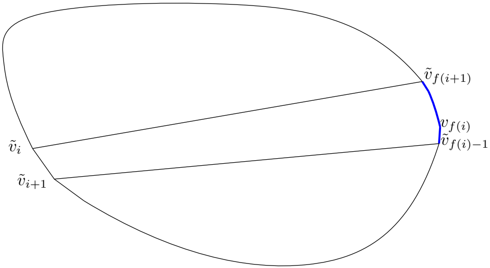
- Lay a grid. Consider square grids.
- $O(\frac{1}{\varepsilon^2})$ pairs of square grids. $O(\frac{1}{\varepsilon^4})$ segments for each pair.
- $O(n + \frac{1}{\varepsilon^7})$ time algorithm.

Further refinement

- only $O(\frac{1}{\varepsilon})$ pairs of vertex grids – monotone properties
- only $O(\frac{1}{\varepsilon^3})$ segments for a pair of square grids – build tables
- Query a half-plane farthest point in $O(\log \frac{1}{\varepsilon})$ time – build data structure

Further refinement

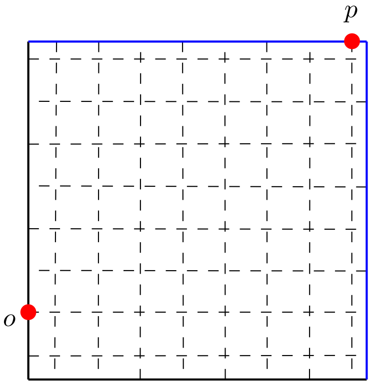
- only $O(\frac{1}{\varepsilon})$ pairs of vertex grids – monotone properties



For $\tilde{v}_i\tilde{v}_{i+1}$, only consider $\tilde{v}_{f(i)-1}\tilde{v}_{f(i)}, \ldots, \tilde{v}_{f(i+1)-1}\tilde{v}_{f(i+1)}$

Further refinement

- only $O(\frac{1}{\varepsilon^3})$ segments for a pair of vertex grids – build tables



a table entry for a pair of grid corners on the boundary of a square grid

Further refinement

- Query a half-plane farthest point in $O(\log \frac{1}{\varepsilon})$ time – build data structure
- $O(n + \frac{1}{\varepsilon^4} \log \frac{1}{\varepsilon})$ time algorithm

Thank you!