Online Interval Scheduling with Predictions

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The Algorithms and Data Structures Symposium (WADS)

Outline

- **Interval Scheduling** and **Disjoint Path Allocation** problems.

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- Main results:
  - Disjoint Path Allocation problem
  - Interval Scheduling: competitive results, consistency/robustness tradeoffs, and experimental results

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  - Interval Scheduling: competitive results, consistency/robustness tradeoffs, and experimental results
- Implied results

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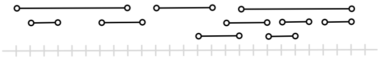
- Given a set of intervals, select a subset of non-overlapping intervals with maximum cardinality.
- In the offline setting, a simple greedy algorithm solves the problem optimally.



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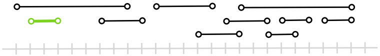

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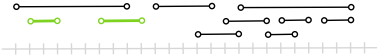

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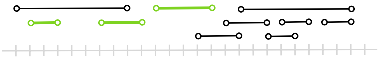

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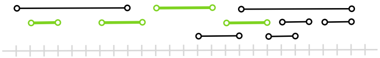

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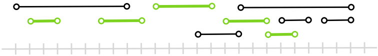

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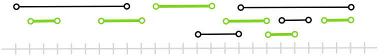

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- In the online setting, intervals appear one by one in any order, and an irrevocable decision must be made to accept to reject each item before the next ones appear.

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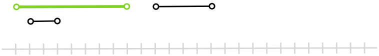

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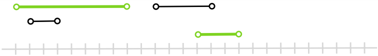

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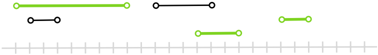

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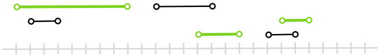

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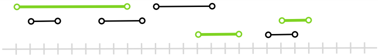

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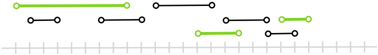

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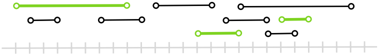

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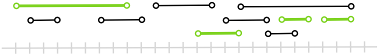

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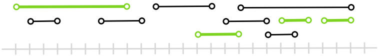

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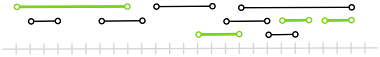

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- The best competitive ratio is $\Theta(1/m)$ and $\Theta(1/\log m)$ for deterministic and online algorithms, respectively, where m is the maximum interval length [Awerbuch et al., SODA'94, Lipton & Tomkins, SODA'94].

Disjoing Path Allocation Problem

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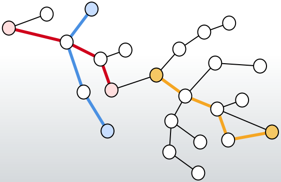
  - The goal is to accept a maximum number of pairs with disjoint paths between them.



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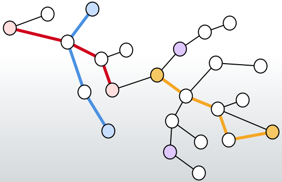

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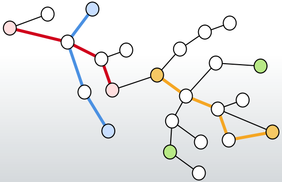

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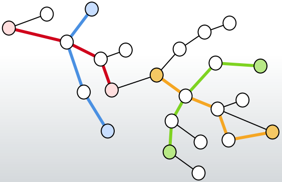

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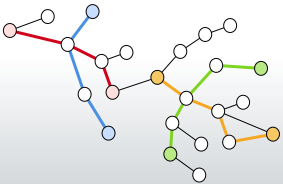
- Instead of intervals, the input is formed by pairs of vertices in a given graph.
  - The goal is to accept a maximum number of pairs with disjoint paths between them.
  - The problem is NP-hard for general graphs (even SP-graphs) [Even and Etai, 1976] and polynomial-time solvable for trees and outerplanar graphs [Garg, Vazirani, and Yannakakis, 1977], Wagner, 1995].



Online Algorithms with Prediction

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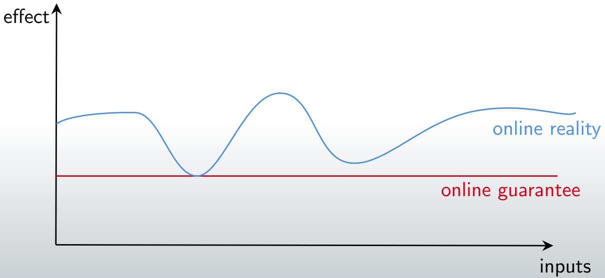

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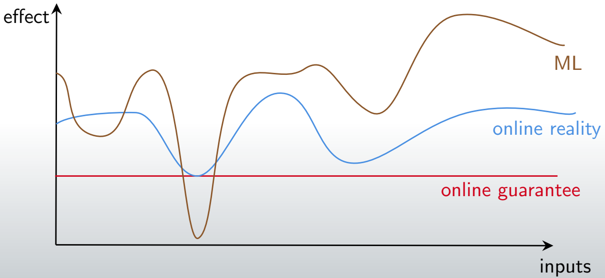

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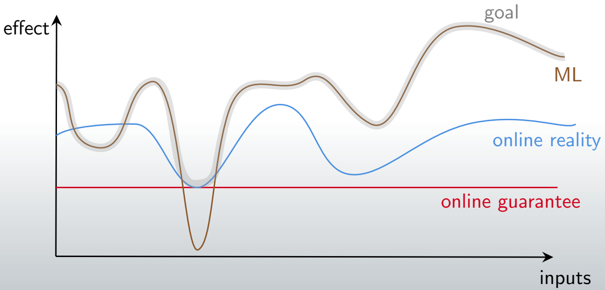

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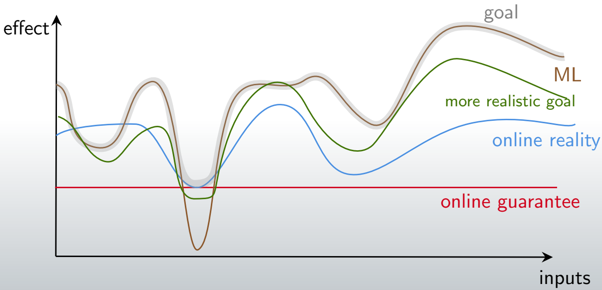
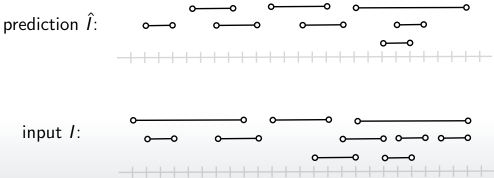


Online Algorithms with Prediction

- What prediction should be?
- How to measure error?
- Algorithm Design and Analysis

Interval Scheduling with predictions

- We consider predictions that concern membership in the input sequence.



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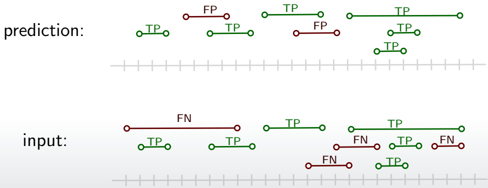

Interval Scheduling with predictions

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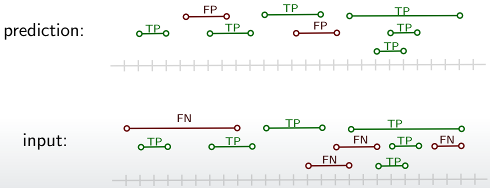
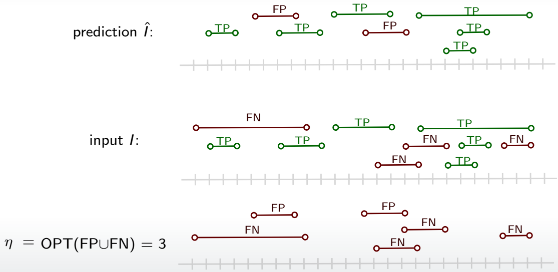
- Statistical predictions such as average input length are unlikely to help.



The Error Measure

- We use $\eta = Opt(FP \cup FN)$



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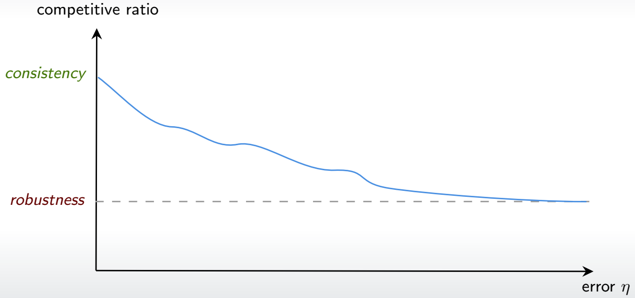
  - **Completeness:** the error is not "too large".
- We define **normalized error** $\gamma(\hat{I}, I) = \frac{\eta(\hat{I}, I)}{\textsc{Opt}(\hat{I}, I)}$

Algorithmic Goal

- Design an algorithm that is **consistent**, **robust**, and **smooth**.



Follow the Prediction

- The most obvious algorithm to try (first) just follows the prediction:

Algorithm Trust

From $\hat{I}$, compute an optimal solution, I^*
for all requests $r \in I$:
  if $r \in I^*$:
    accept
  else:
    reject

Follow the Prediction

- A positive result for general graphs:

Theorem

On any graph, $\textsc{Trust}(\hat{I}, I) \geq (1 - 2\gamma(\hat{I}, I))\ \textsc{Opt}(I)$

Proof.

$$\begin{aligned}
\textsc{Trust}(\hat{I}, I) &\geq \textsc{Opt}(\hat{I}) - \textsc{Opt}(FP) \\
&\geq \textsc{Opt}(I) - \textsc{Opt}(FN) - \textsc{Opt}(FP) \\
&\geq \textsc{Opt}(I) - 2\textsc{Opt}(FP \cup FN) \\
&= \textsc{Opt}(I) - 2\eta(\hat{I}, I) \\
&= (1 - 2\gamma(\hat{I}, I))\textsc{Opt}(I)
\end{aligned}$$

Follow the Prediction

- This is the best possible for the disjoint path allocation problem:

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On a star graphs S_8, $\textsc{Alg}(\hat{I}, I) \leq (1 - 2\gamma(\hat{I}, I))\ \textsc{Opt}(I)$

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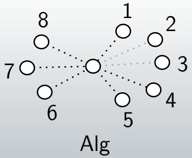

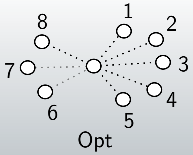

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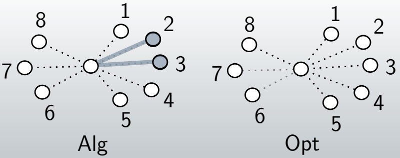

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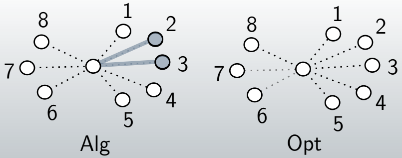

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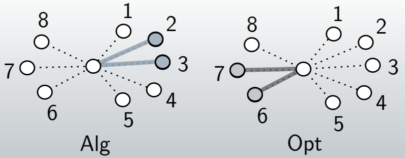

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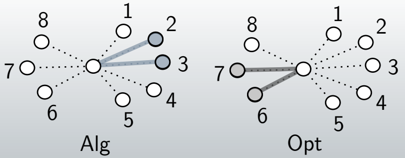

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- This is the best possible for the disjoint path allocation problem:

Theorem

On a star graphs S_8, $\text{Alg}(\hat{I}, I) \leq (1 - 2\gamma(\hat{I}, I))\ \text{Opt}(I)$

- Showing, equivalently, $\text{Alg}(\hat{I}, I) \leq \text{Opt}(\mathrm{I}) - 2\eta(\hat{I}, I)$
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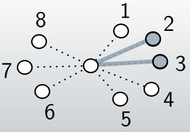

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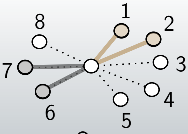

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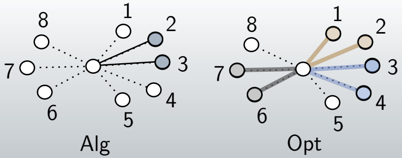

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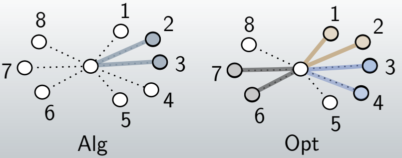

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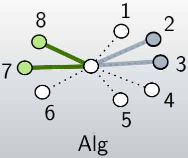

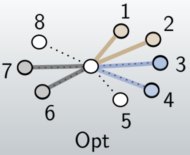

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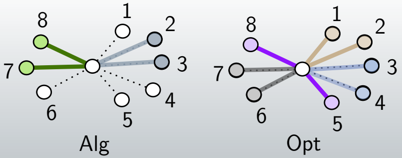

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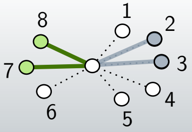

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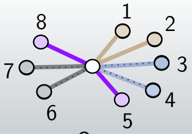

Opt

I = (2,3) , (6,7) , (1,2) , (3,4) , (7,8) , (5,8)

Opt(FN∪FP) = Opt({(4,5),(5,8)}) = 1

Conclusion for Disjoint Path Allocation

- As long as a graph class contains S_8, $\textsc{Trust}$ (follow-the-prediction) is the best possible.

Conclusion for Disjoint Path Allocation

- As long as a graph class contains S_8, TRUST (follow-the-prediction) is the best possible.
- This motivates us to focus on interval graphs (interval scheduling).

TrustGreedy Algorithm

- An improved algorithm for interval scheduling:

Algorithm TrustGreedy

From $\hat{I}$, compute a left-most optimal solution, I^*

for all requests $r \in I$:

if r does not overlap an accepted request **and**
(is in I^* **or**
does not overlap any I^*-requests **or**
overlaps exactly one I^*-request ending no earlier than r)

accept r

update I^* if necessary

else:

reject r

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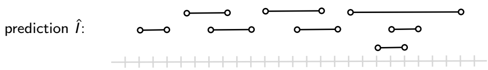

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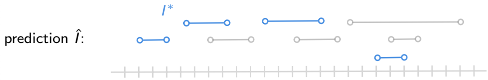

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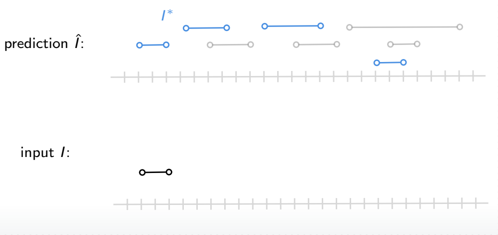

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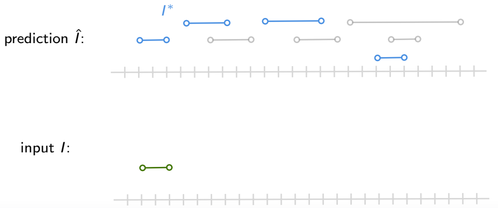

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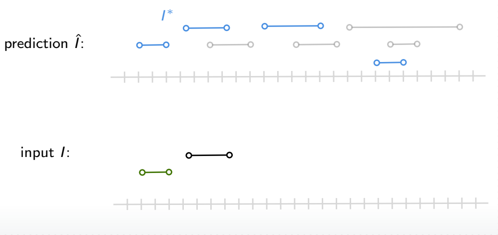

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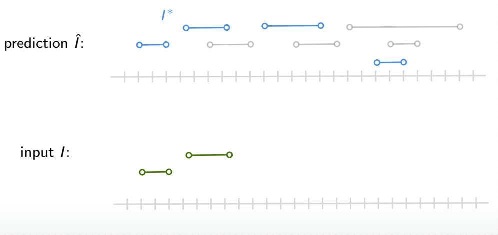

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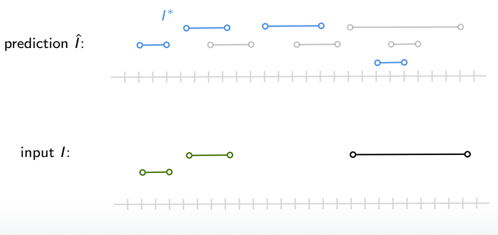

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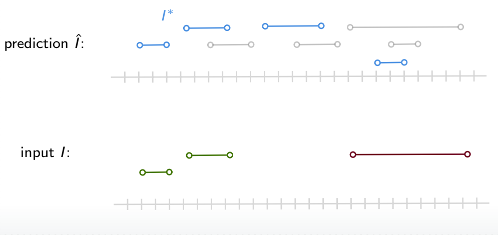

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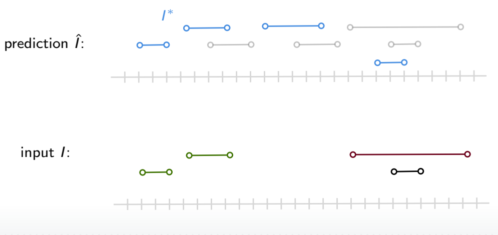

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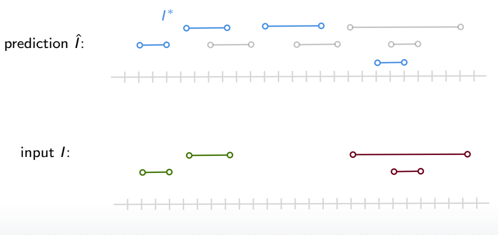

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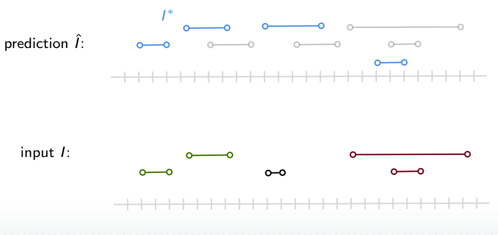

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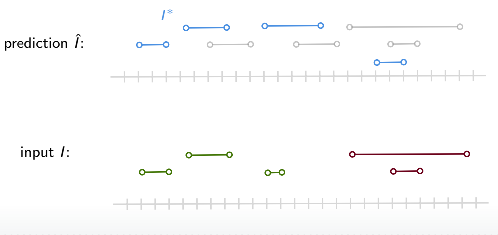

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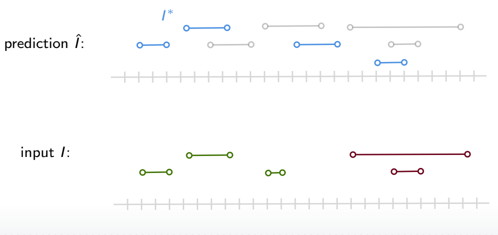

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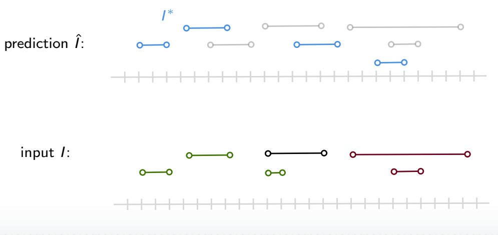

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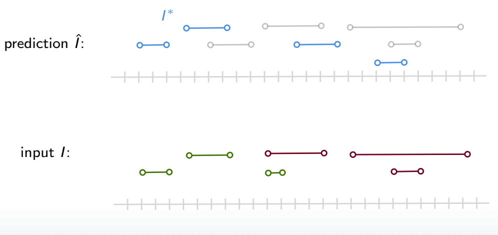

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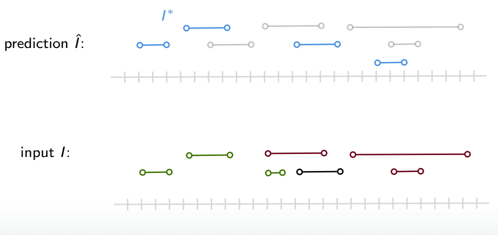

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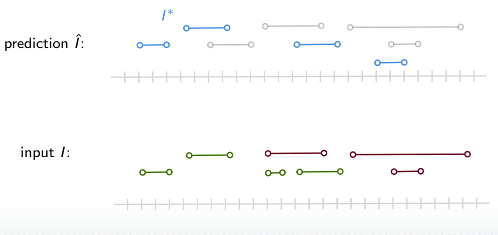

TrustGreedy Algorithm

Theorem

For any prdeiction $\hat{I}$ and input sequenec I, $\textsc{TrustGreedy}(\hat{I}, I) \geq (1 - \gamma(\hat{I}, I))\ \textsc{Opt}(I)$

- An improvement over the competitive ratio $1 - 2\gamma(\hat{I}, I)$ of $\textsc{Trust}$.

TrustGreedy Algorithm Optimality

Theorem

For any deterministic algorithm $\textsc{Alg}$*, there are input sequences and predictions* I *and* $\hat{I}$*, so* $\textsc{Alg}(\hat{I}, I) \leq (1 - \gamma(\hat{I}, I))\ \textsc{Opt}(I)$

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  - If $\textsc{Alg}$ rejects $(0,2)$, $\textsc{Opt}$ accepts it and input ends, so $(0,1) \in FP$, and $\eta = 1$.

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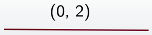
  - If $\textsc{Alg}$ accepts $(0,2)$, then I continues with $(0,1)$ and $(1,2)$ that $\textsc{Opt}$ accepts, so $(1,2) \in FN$ and $\eta = 1$.



Alg $= 0$ Opt $= 1$

$\eta = 1$

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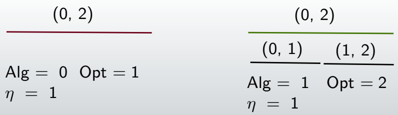

Consistency/Robustness Tradeoffs

- **Consistency** refers to the competitive ratio when the predictions are correct, and **robusteness** is the competitive ratio when predictions are adversarial.

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- Starting with a negative result:

Theorem

If a (possibly randomized) algorithm $\textsc{Alg}$ *is both* α*-consistent, then its robustness is at most* $\beta = \frac{2(1-\alpha)}{\lfloor \log m \rfloor - 1}$.

Consistency/Robustness Tradeoffs

- For a positive result, we define $\textsc{Robust-Trust}(\alpha)$ as follows:

Algorithm RobustTrust (α)

Draw probablity p uniformly at random
if $p < \alpha$:
    apply algorithm TrustGreedy
else
    apply algorithm Classify-and-Randomly-Select

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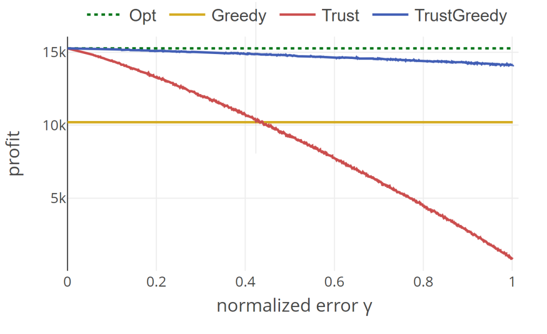
Theorem

$\textsc{Robust-Trust}(\alpha)$ *has consistency at least* α *and robustness at least* $\frac{1-\alpha}{\lceil \log m \rceil}$

- Robust-Trust(α) asymptotically Pareto optimal.

Experimental Result

- Consider Trust, TrustGreedy, Greedy, and Opt on real-world scheduling data on parallel machines [Chapin et al. IPPS/SPDP, 1999]



Implied Results

- The negative result on star graphs implies a negative result for **matching** in general graphs (even if restricted to planar graphs).

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- The negative results on matching implies a negative result for **indepenedent set** in general graphs

Summary

- For disjoint path allocation problem, $\textsc{Trust}$ has a competitive ratio of $1 - 2\gamma(\hat{I}, I)$, which is optimal.
- For interval scheduling, $\textsc{TrustGreedy}$ has a competitive ratio of $1 - \gamma(\hat{I}, I)$, which is optimal.
- For consistency/robustness tradeoff, $\textsc{RobustTrust}(\alpha)$ is α-consistent and $(1 - \alpha)/\lceil \log m \rceil$-robust, which is asymptotically Pareto-optimal.