Reconfiguration of Time-Respecting Arborescences

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with
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九州大学
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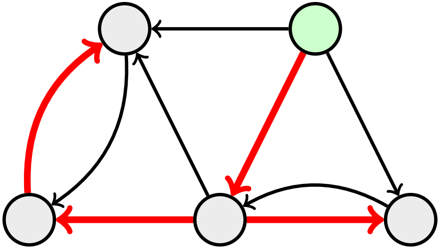
科研費
KAKENHI

Grant Number JP20H05795

Combinatorial Reconfiguration

Arborescence

Arborescence = Out-going directed tree in a digraph



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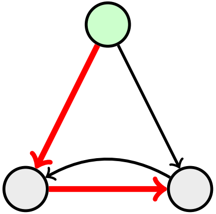

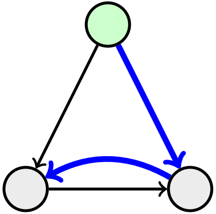

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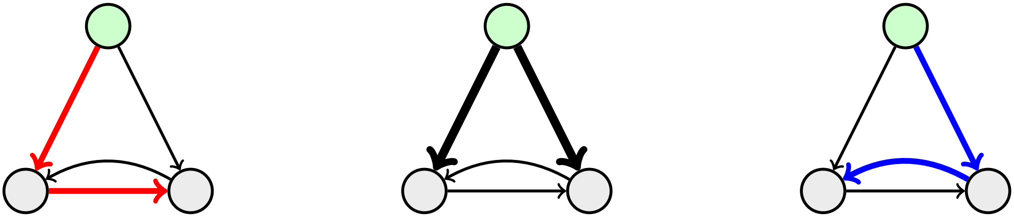

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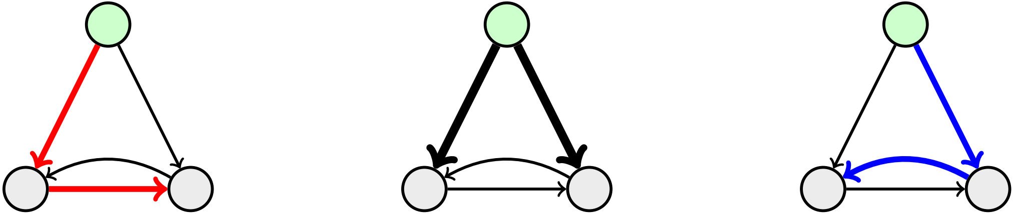

- [Ito et al. TCS 2023] Even when the roots are different
  - Checking reachability: **Always Yes**
  - Finding a shortest sequence: **P**

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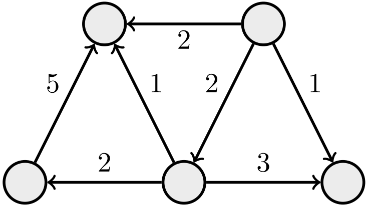
- An r-arborescence T in D is **time-respecting**

  $\overset{\text{def}}{\iff} \ \forall v \in V :$ the (r, v)-dipath in T is time-respecting

Time-Respecting Arborescence (Kempe et al. JCSS 2002)



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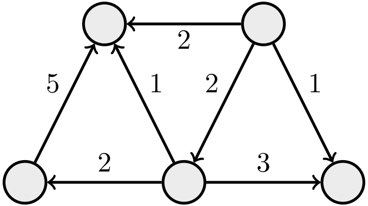

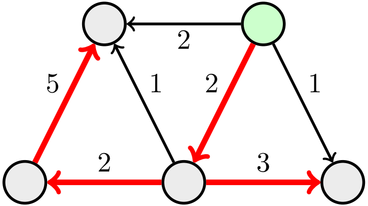

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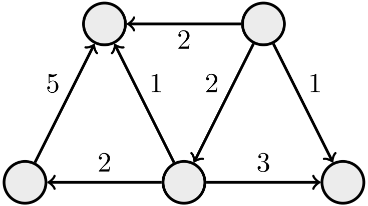

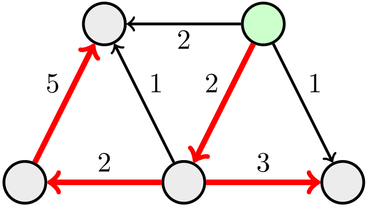

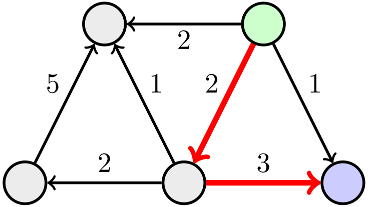

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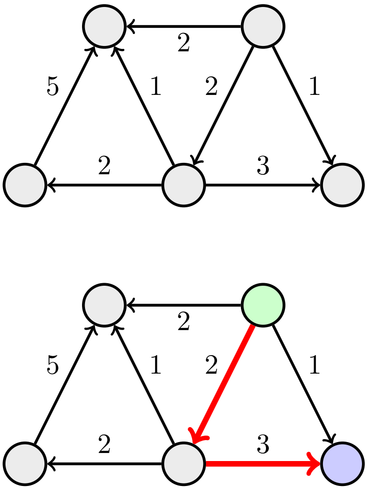

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  $\overset{\mathrm{def}}{\Longleftrightarrow}$ $T^0 = T_{\mathrm{o}}$ and $T^\ell = T_{\mathrm{t}}$,

  $T^0, \ldots, T^\ell$ are time-respecting arborescences in D, and

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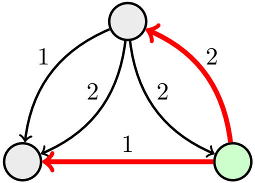
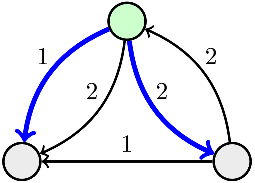
Our problem:

- **Input:** Time-respecting arboresnces T_1, T_2
- **Question:** Is there a reconfiguration sequence between T_1, T_2?

Yes instance:





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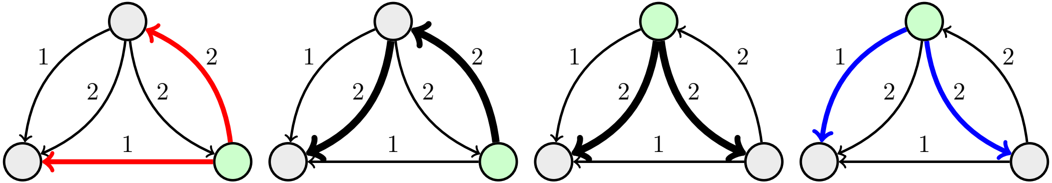

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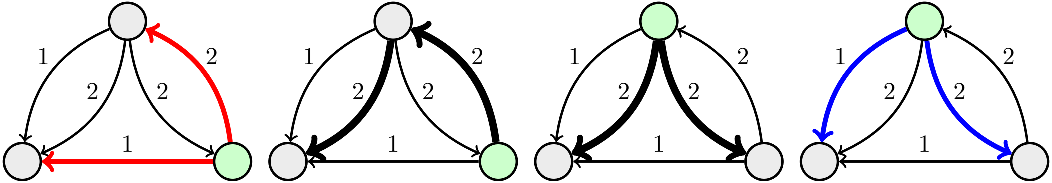

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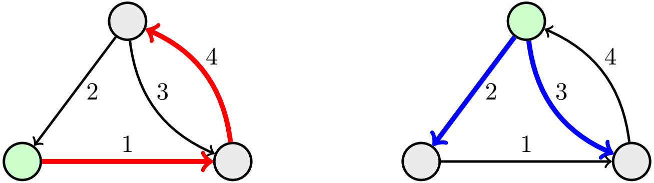

Results

- (without time labels) Identical/Non-identical roots case: Reachability: Always Yes, and Shortest sequence: P

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Results

- **Identical roots case:**
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- **Non-identical roots case:**
  - Checking reachability: **P**
  - Finding a shortest sequence: **NP-complete** (Decision version)

- (without time labels) Identical/Non-identical roots case:

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- TR := time-respecting
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- T_{o} (resp. T_{t}) := initial (resp. target) TR-arboresncence

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  ii) There are $T_{\mathrm{o}} \rightsquigarrow T^*$ and $T^* \rightsquigarrow T_{\mathrm{t}}$

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- **Sketch of Proof:**

  i) There is a **minimal TR arborescence** T^*

  ii) There are $T_{\rm o} \rightsquigarrow T^*$ and $T^* \rightsquigarrow T_{\rm t}$

  iii) Length of $T_{\rm o} \rightsquigarrow T^* \rightsquigarrow T_{\rm t} = |A(T_{\rm o}) \setminus A(T_{\rm t})|$

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- For each $v \in V$, we define $d(v)$ by

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- We can find a minimal TR arborescence in poly-time

  (Also, $T_{\mathrm{o}} \rightsquigarrow T^* \rightsquigarrow T_{\mathrm{t}}$ can be found in poly-time)

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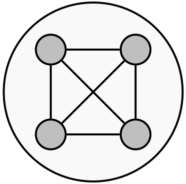
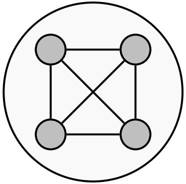
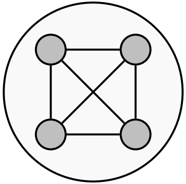
root $= u$

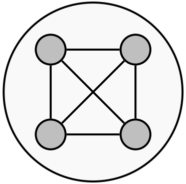


root $= w$





root $= v$



root $= r$

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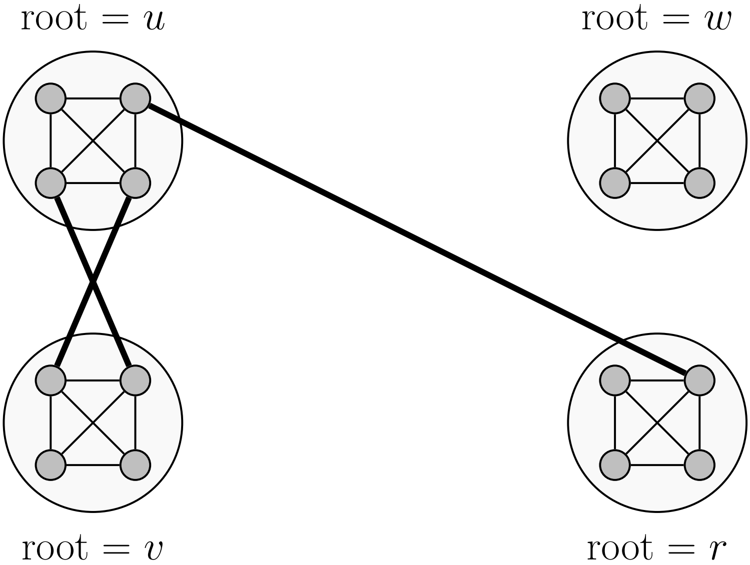

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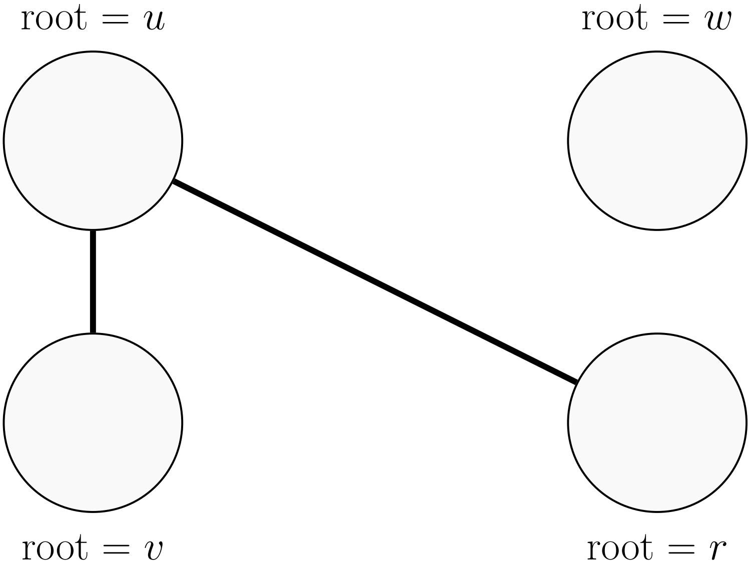

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- Suppose T_{o} is rooted at r_{o} and T_{t} is rooted at r_{t}
- We check the reachability between $r_{\mathrm{o}}, r_{\mathrm{t}}$ in $\mathcal{G}$

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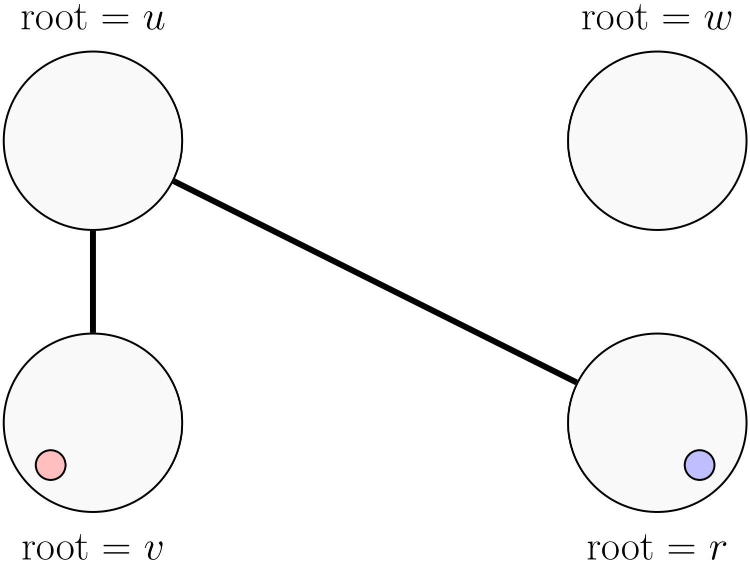

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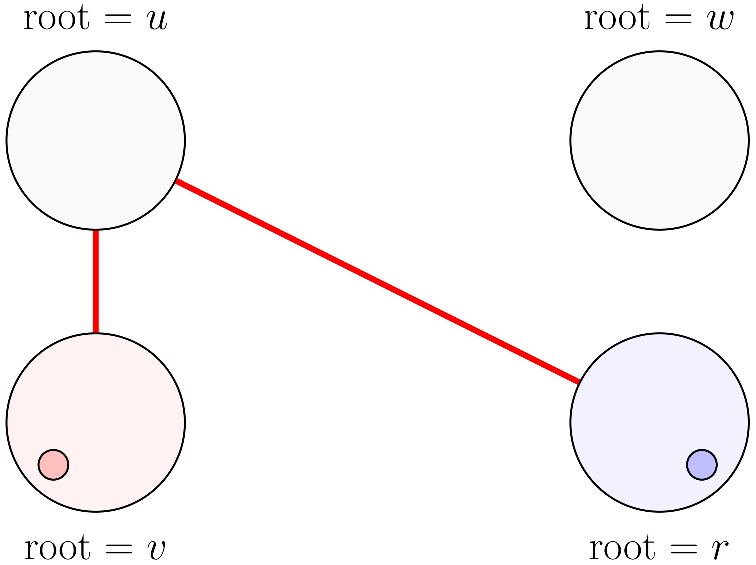

Conclusion

- We consider reconfiguration of time-respecting arborescences
- **Identical roots case:**
  - Checking reachability: **Always Yes**
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Thank you for your attention!!