Space-Efficient Functional Offline-Partially-Persistent Trees with Applications to Planar Point Location

Gerth S. Brodal Casper M. Rysgaard Jens Kristian R Schou Rolf Svenning

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Denmark

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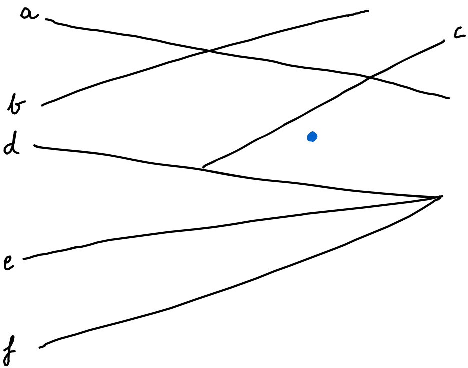

Sarnak, Tarjan 85

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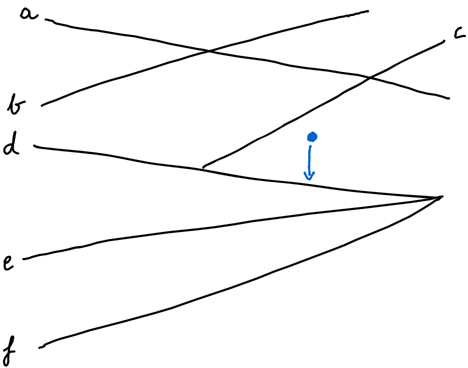

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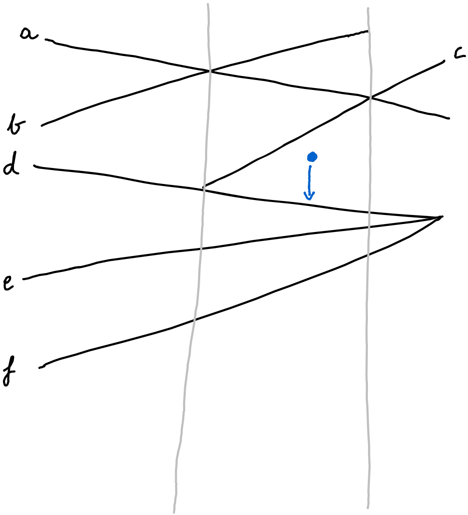

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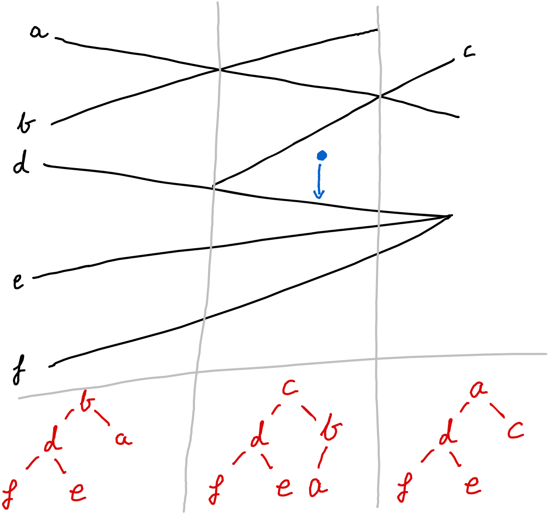

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- Persistent datastructure
  - Time x axis
  - Value "aboveness"

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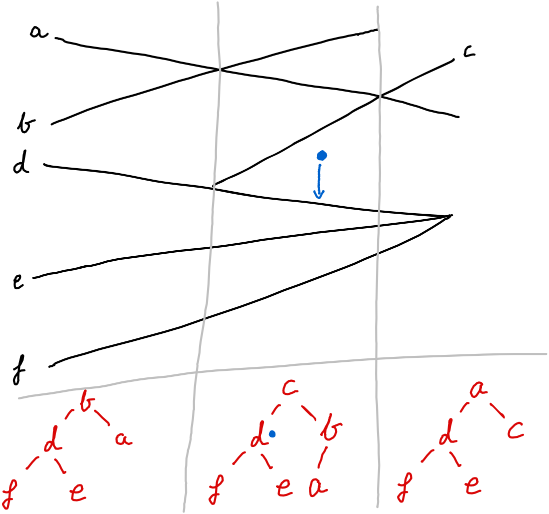

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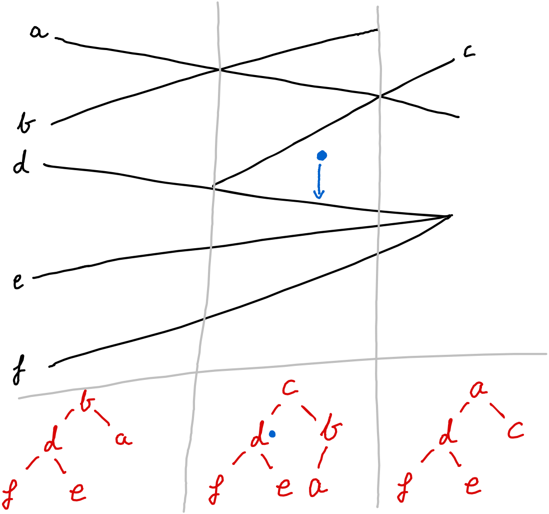

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  - Query past versions
    - ◻ Stored as a graph

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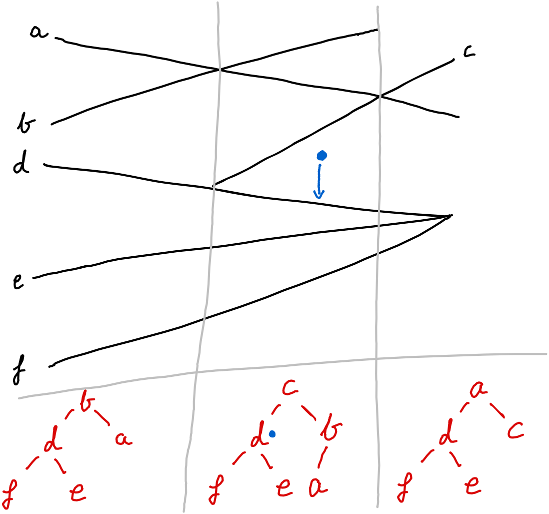

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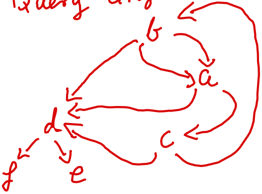
Query Graph



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- Persistent datastructure
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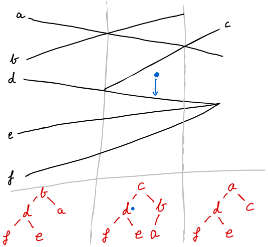
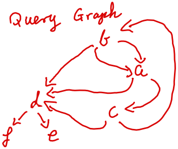
    - ▫ Stored as a graph
- Offline Partial Persist
  - All updates before queries





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Our contribution

	Construction	Space	Query	Model
ST85	n log n	n	log n	Imperative
ST85	n log n	n log n	log n	Functional
New	n log n	n	log n	Functional

Sarnak, Tarjan 85

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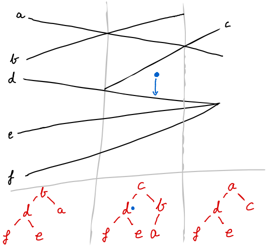

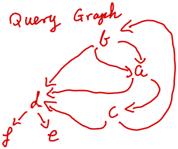

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Imperative OPP algorithm

- On edge insertion, at time t_1
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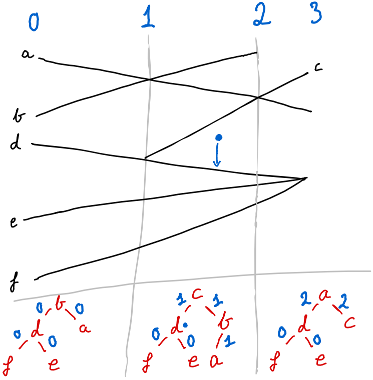
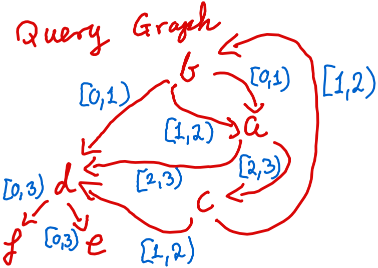
- On edge deletion, at time t_2
  - insert edge in Query graph with "lifespan" $[t_1, t_2)$





Space-Efficient Functional Offline-Partially-Persistent Trees with Applications to Planar Point Location

- Functional data structures
  - Immutable
  - Inheritantly persistent
  - Change by addition
  - No cycles in graphs

Space-Efficient Functional Offline-Partially-Persistent Trees with Applications to Planar Point Location

- Functional data structures
  - Immutable
  - Inheritantly persistent
  - Change by addition
  - No cycles in graphs
- No side effects
- Excellent for recursive data structures
- Reduced model checking complexity

Space-Efficient Functional Offline-Partially-Persistent Trees with Applications to Planar Point Location

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Functional OPP algorithm ?

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Functional challenge

1 no cycles
  - Solution: Query DAG

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Functional challenge

1 no cycles
  - Solution: Query DAG

2 Functional DAG insertion is $\Omega(n)$
  - Solution: Topological Sort

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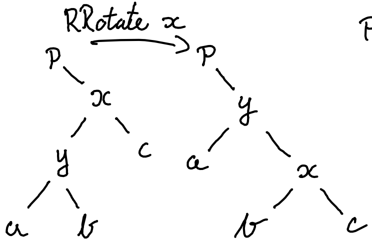
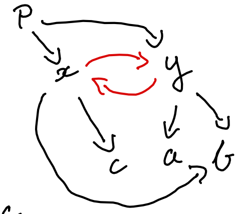
Breaking Cycles

Red-Black tree



Query Graph



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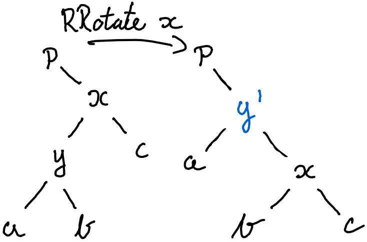
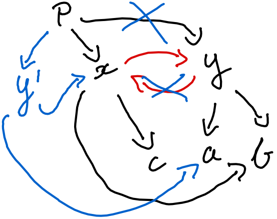
Breaking Cycles (new)

Red-Black tree



Query DAG



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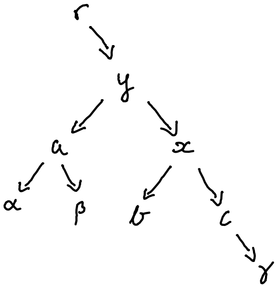
Functional BST insertion
via path copying

insert $\delta \in [b, x]$



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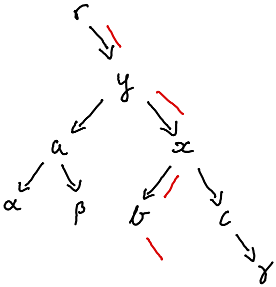

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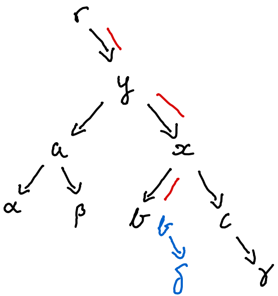

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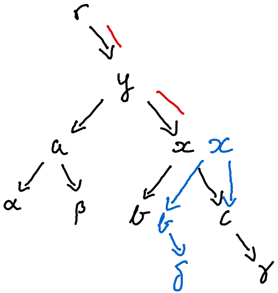

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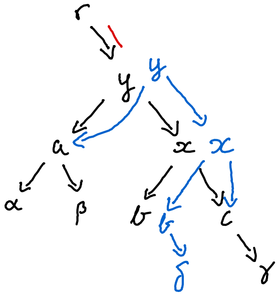

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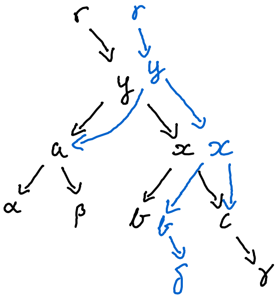

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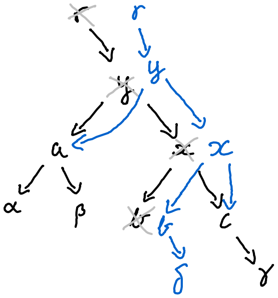

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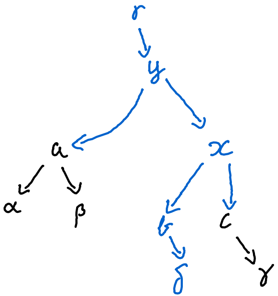

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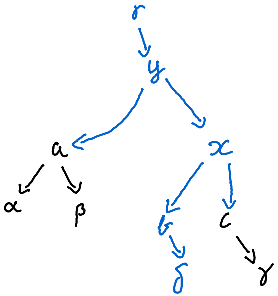

$O(\log n)$ time
$O(1)$ space

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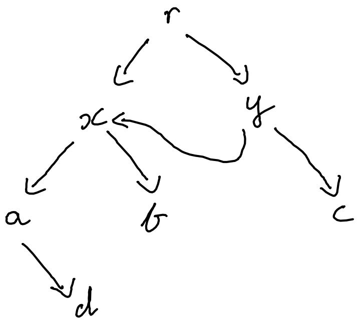
Functional Query DAG insertion
via path copying

insert edge b -> d



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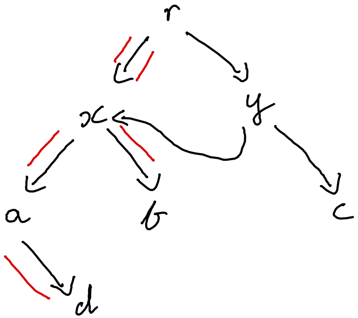

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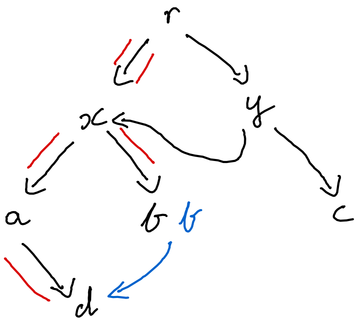

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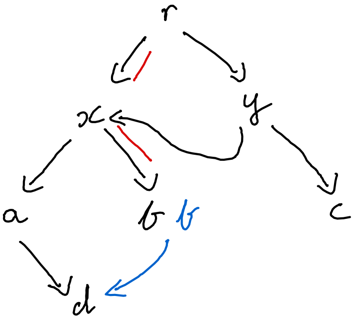

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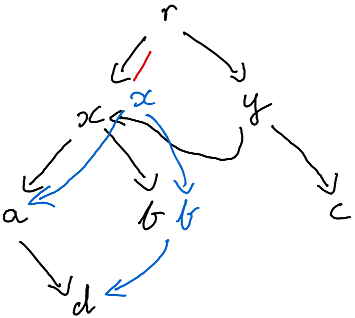

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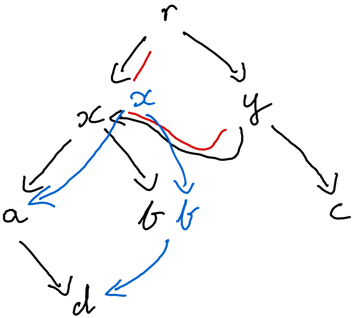

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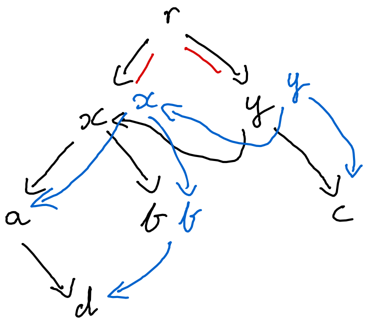

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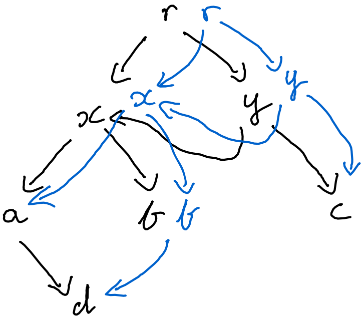

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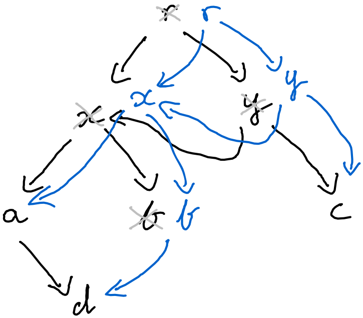

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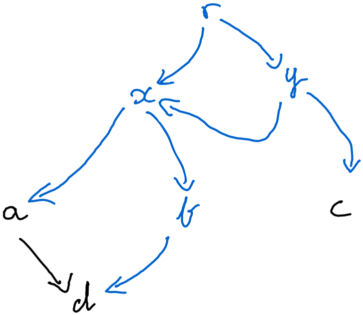

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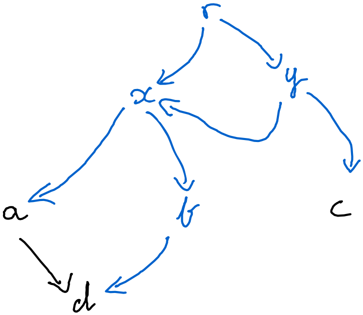
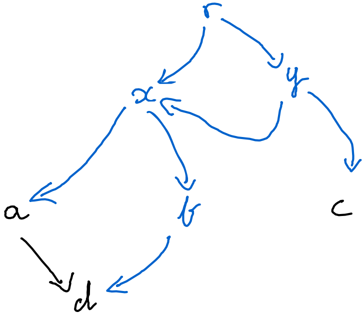


$\Omega(n)$ time (worst case)

$O(1)$ space

Space-Efficient Functional Offline-Partially-Persistent Trees with Applications to Planar Point Location

Functional Query DAG
without path copying
given all edges E
and nodes V



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Functional Query DAG
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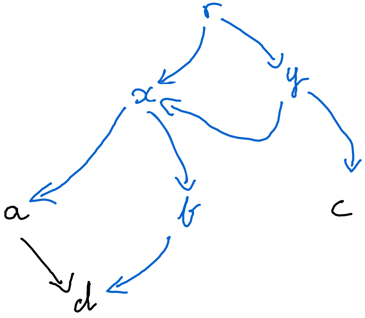
Algorithm

1 Topologically Sort (V, E)
[Kahn 62]

2 Construct Query DAG
bottom up



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Functional OPP algorithm

- On edge insertion, at time t_1
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  - insert edge in Freezer with "lifespan" $[t_1, t_2)$
- After all updates
  - Construct Functional Query DAG from edges in the Freezer

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Efficient for

- Random access arrays [Okasaki 95]
- Treaps [Aragon Seidel 89]
- Red-Black trees [Bayer 72]
- AVL-trees [Adel'son-Vel-skii, Landis 62]