Online TSP with Known Locations

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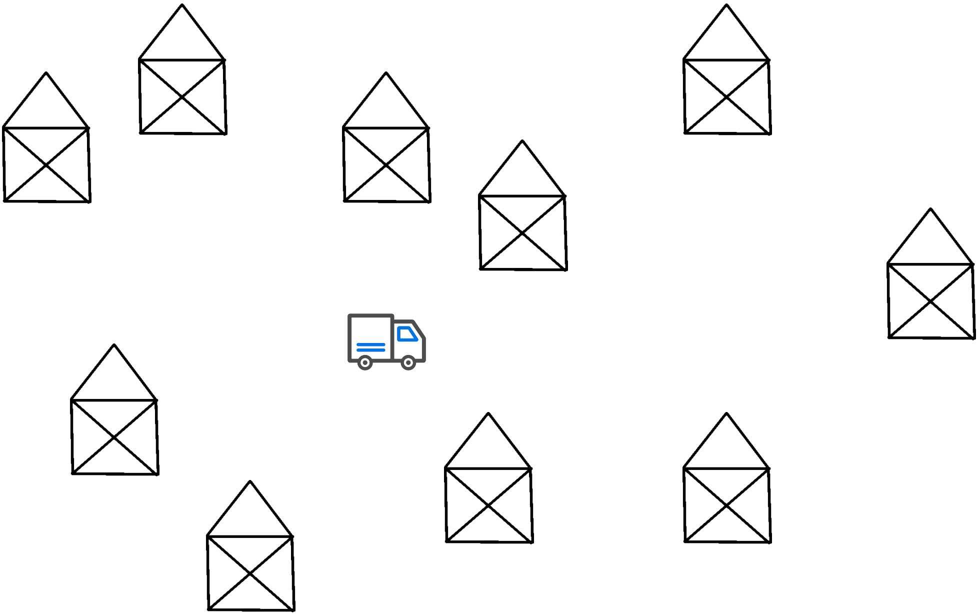
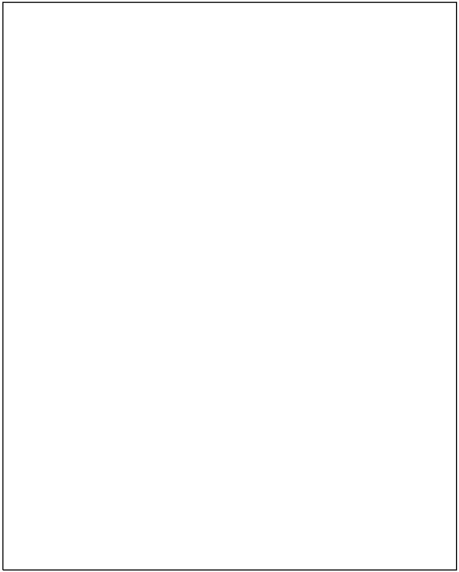
Traveling Salesperson Problem

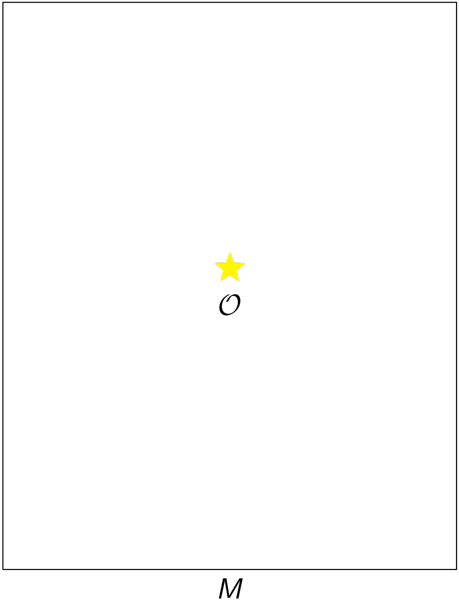


Model



M

- Metric Space $M = (X, d)$



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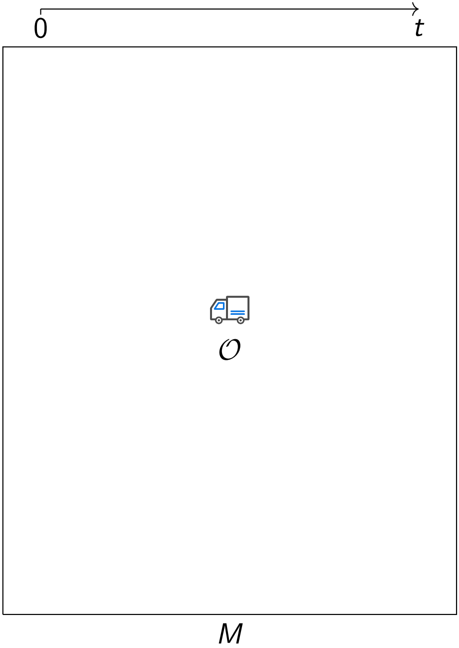
- Point $\mathcal{O} \in X$ ("Origin")



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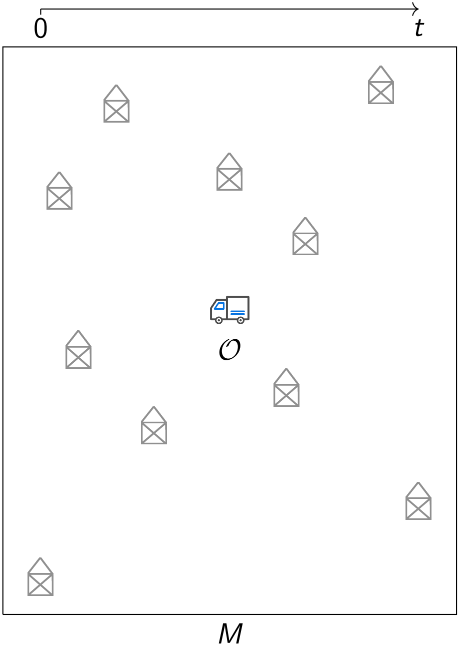
- Point $\mathcal{O} \in X$ ("Origin")
- The server $\mathcal{S}$ starts at $t = 0$ at $\mathcal{O}$ and moves with (up to) unit speed



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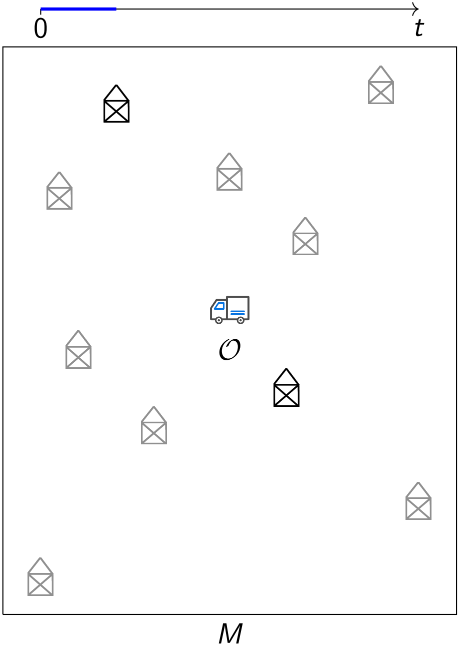

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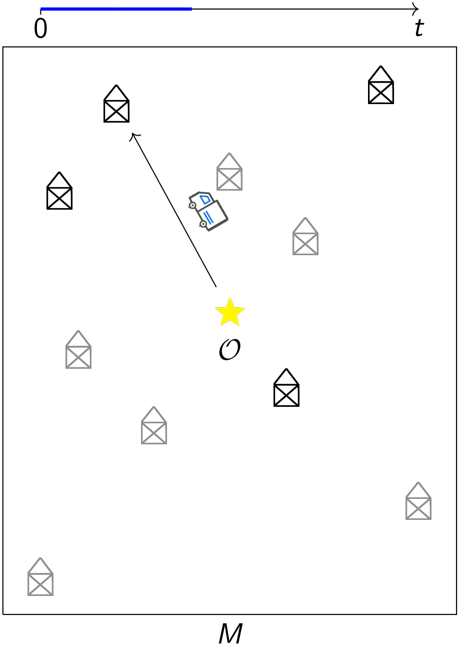

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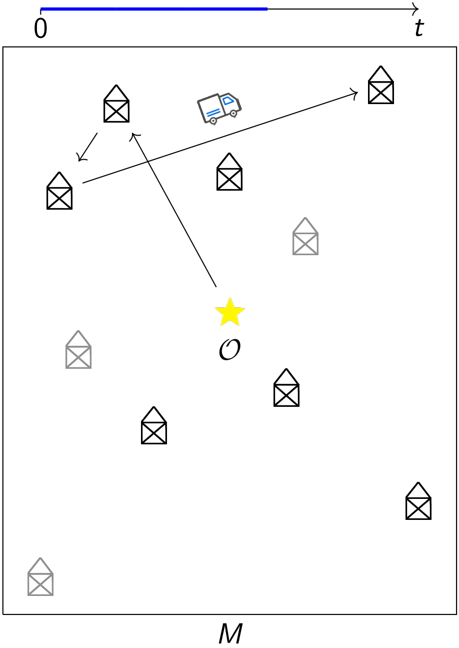

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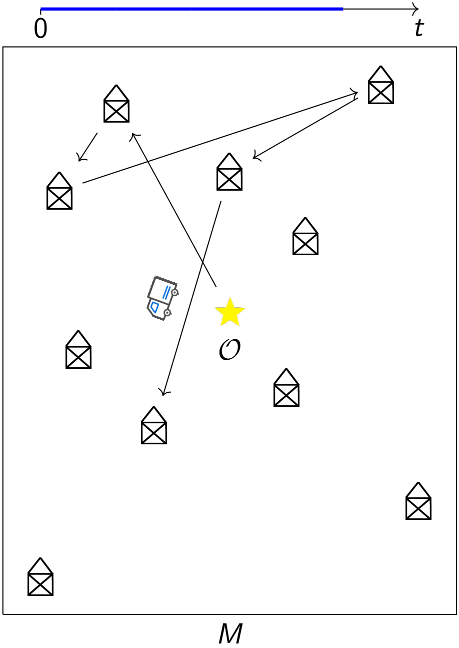

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- Objective: Minimize the completion time of the tour
- Open variant: $\mathcal{S}$ finishes when reaching the final request
  Closed variant: $\mathcal{S}$ has to return to $\mathcal{O}$

Competitive Ratio

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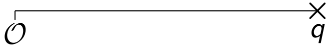
Example

Open variant, single request q with distance 1 and release time 1



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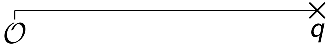

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Classic Online TSP

Tight bound of 2 for the general case in the closed variant [Ausiello et al. '01]

Tight bound of 2.04 for the open variant on the line, 2.5 general upper bound [Bjelde et al. '20, Ausiello et al. '01]

Observation

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Algorithm for General Metrics (1)

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Theorem

The algorithm for general metrics achieves a competitive ratio of $3/2$ in both the open and the closed variant of online TSP with known locations.

Main idea:

- Wait in $\mathcal{O}$ until time t s.t. two conditions are satisfied for a single tour σ:
  1. The first half of σ (w.r.t. its length ℓ_σ) can be traversed without encountering an unreleased request

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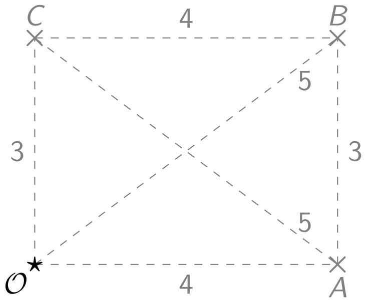

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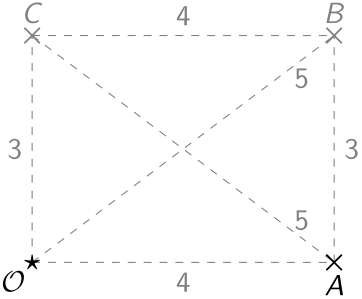

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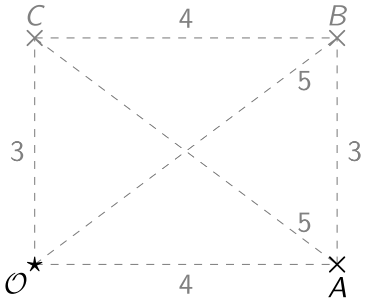

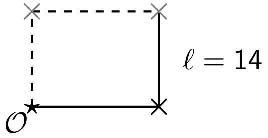

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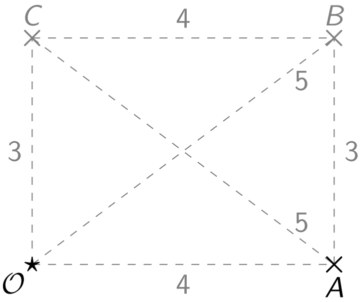

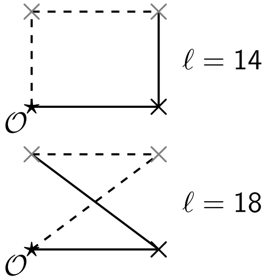

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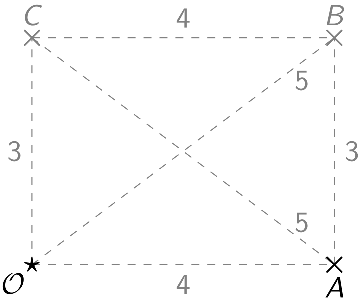

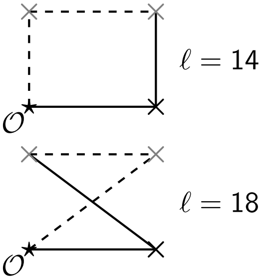

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- Choose a tour σ' that minimizes $\max\{\ell_{\sigma'}/2, \text{unreleased length}(\sigma')\}$ and follow it without deviating, waiting at any unreleased request

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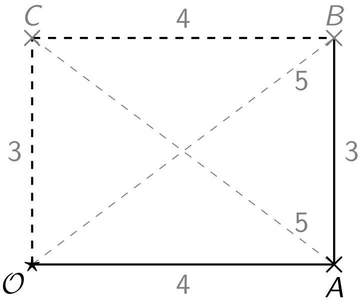

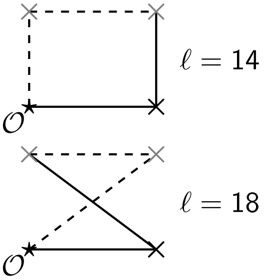

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Competitive ratio at least $3/2$:

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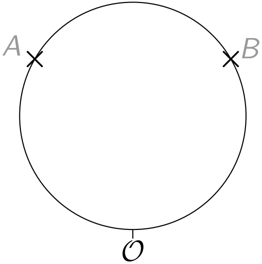

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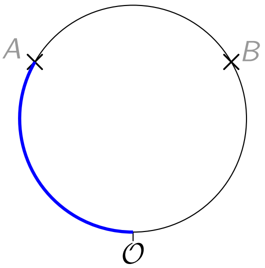

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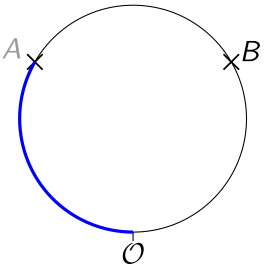

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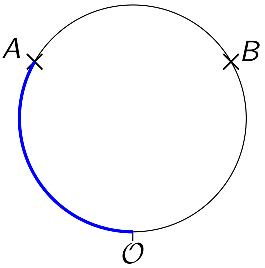

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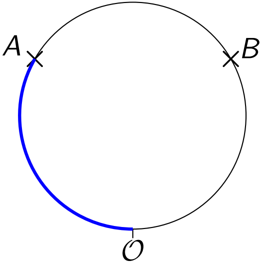

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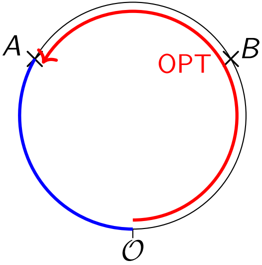

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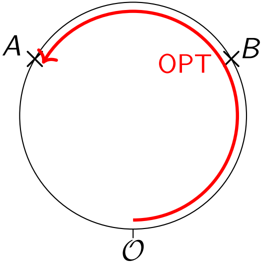

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An example on the line suffices for a lower bound of $3/2$ [Gouleakis et al. '23]

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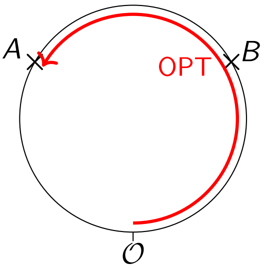

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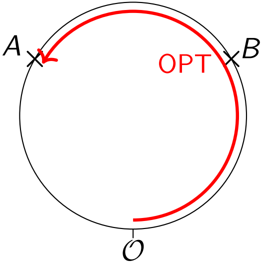

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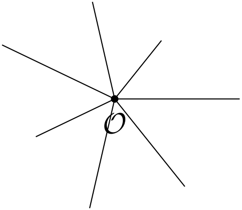
$\rightsquigarrow$ The general algorithm is optimal, **but** it is not poly-time

Stars:



Poly-time algorithm which is $(7/4 + \varepsilon)$ competitive (closed).

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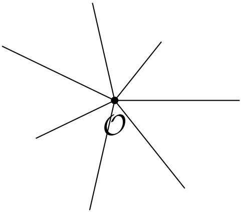

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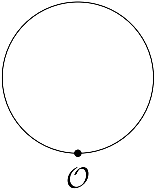

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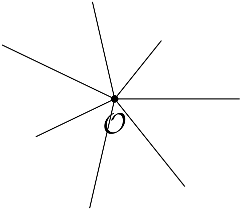

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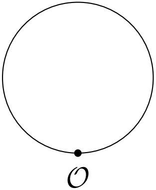

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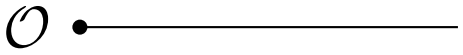
Semi-line:



Poly-time algorithms which are 1 (closed) and $13/9$ competitive (open).

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   - Then, serve the requests in R and return to $\mathcal{O}$.

2. Wait in $\mathcal{O}$ until all requests are released. Afterwards, serve the unserved requests in an optimal manner.

- The algorithm achieves a competitive ratio of $7/4$ with an optimal set R

Stars (2)

- The algorithm achieves a competitive ratio of $7/4$ with an optimal set R
- Finding an optimal set R constitutes solving a knapsack problem
  $\rightsquigarrow$ FPTAS to find R gives us
  for every $\varepsilon > 0$, a $(7/4 + \varepsilon)$ competitive poly-time algorithm

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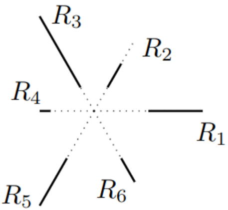
  $\rightsquigarrow$ FPTAS to find R gives us
  for every $\varepsilon > 0$, a $(7/4 + \varepsilon)$ competitive poly-time algorithm

Example

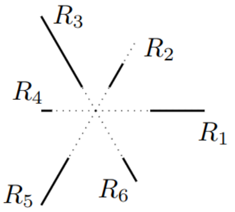


Ray	R_1	R_2	R_3	R_4	R_5	R_6
Length	0.2	0.15	0.2	0.1	0.2	0.15
Released	0.1	0	0.15	0.02	0.1	0.05

Stars (2)

- The algorithm achieves a competitive ratio of $7/4$ with an optimal set R
- Finding an optimal set R constitutes solving a knapsack problem
  $\rightsquigarrow$ FPTAS to find R gives us
  for every $\varepsilon > 0$, a $(7/4 + \varepsilon)$ competitive poly-time algorithm

Example



Ray	R_1	R_2	R_3	R_4	R_5	R_6
Length	0.2	0.15	0.2	0.1	0.2	0.15
Released	0.1	0	0.15	0.02	0.1	0.05

$\rightsquigarrow$ R : rays R_1, R_3 and R_4 (or R_3, R_4, R_5) with a combined length of $1/2$ and released length of $\ell = 0.27$

	Open OLTSP-L		Closed OLTSP-L		
	Lower Bound	Upper Bound	Lower Bound	Upper Bound	
Semi-line	4/3	13/9*	**1**	**1***	
Star	13/9	3/2	**3/2**	**3/2**	$(7/4+\varepsilon)$*
Ring	**3/2**	**3/2**	**3/2**	**3/2**	5/3*
General	**3/2**	**3/2**	**3/2**	**3/2**	

Poly-time algorithms denoted by *

Outlook

	Open OLTSP-L		Closed OLTSP-L	
	Lower Bound	Upper Bound	Lower Bound	Upper Bound
Semi-line	4/3	13/9*	**1**	**1***
Star	13/9	3/2	**3/2**	**3/2** (7/4+ε)*
Ring	**3/2**	**3/2**	**3/2**	**3/2** 5/3*
General	**3/2**	**3/2**	**3/2**	**3/2**

Poly-time algorithms denoted by *

Open Questions

- Improving running time (general)
- Improving the Bounds
- Predictions on locations

	Open OLTSP-L		Closed OLTSP-L	
	Lower Bound	Upper Bound	Lower Bound	Upper Bound
Semi-line	4/3	13/9*	**1**	**1***
Star	13/9	3/2	**3/2**	**3/2** (7/4+ε)*
Ring	**3/2**	**3/2**	**3/2**	**3/2** 5/3*
General	**3/2**	**3/2**	**3/2**	**3/2**

Poly-time algorithms denoted by *

Open Questions

- Improving running time (general)
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Outlook

	Open OLTSP-L		Closed OLTSP-L	
	Lower Bound	Upper Bound	Lower Bound	Upper Bound
Semi-line	4/3	13/9*	**1**	**1***
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Ring	**3/2**	**3/2**	**3/2**	**3/2** 5/3*
General	**3/2**	**3/2**	**3/2**	**3/2**

Poly-time algorithms denoted by *

Open Questions

- Improving running time (general)
- Improving the Bounds
- Predictions on locations

$\rightsquigarrow$ Learning-Augmented Online TSP on Rings, Trees, Flowers and (almost) Everywhere Else

Thank you!