OPEN PROBLEMS FROM CCCG 2023

CCCG 2023

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1 Convex Cap Cover

Joseph O'Rourke (Smith College) with attribution to Dmitri Burago (Penn State University)

A *convex cap* (example is shown below) is a convex body C such that ∂C is planar, and points of C project 1-to-1 into a planar base.

Open Problem 1. What is the largest number of *convex caps* needed to cover the surface of a smooth convex body in $\mathbb{R}^3$?

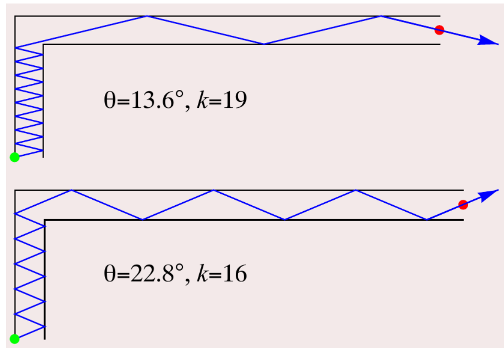
2 Minimum Reflection Path in an Orthogonal Polygon

Joseph O'Rourke (Smith College)

Open Problem 2. Given a simple orthogonal polygon P (scaled to fit in a 1×1 box), and two points s and t, provide an algorithm to find the minimum reflection path from s to t.

Bound its complexity in some geometric measure, perhaps the min distance between parallel edges that can see one another.

Example of light rays in corridors is shown below.



3 Minimum Volume Cube

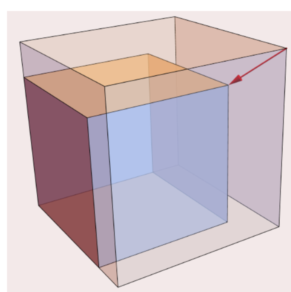
Joseph O'Rourke (Smith College)

Open Problem 3. Construct an algorithm to find a minimum volume cube circumscribing a convex polyhedron P in $\mathbb{R}^3$. Is it possible to compute it in $O(n^3)$? Perhaps, even $O(n^2)$ is possible.

This problem came from practice. Even the 2*D* version is not trivial (David Eppstein suggested an approach based on fixing two perpendicular calipers and rotating them).

A couple of observations:

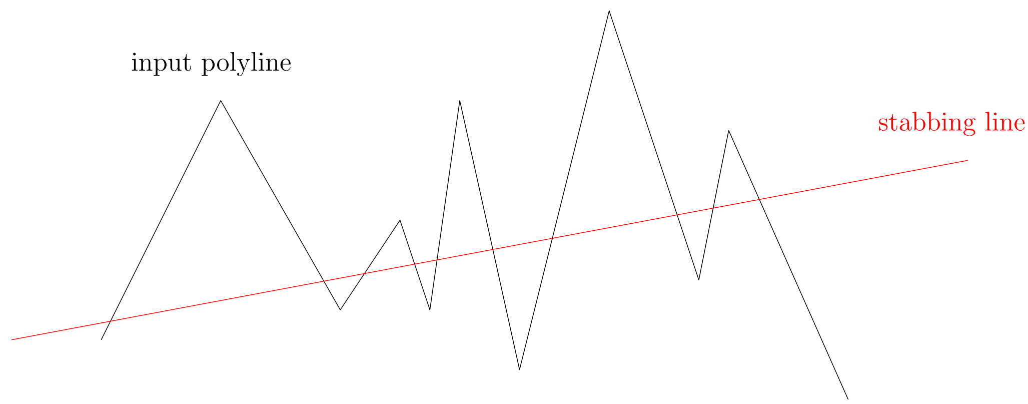
- If a cube circumscribing a convex polyhedron has a free corner then the cube is not minimal (see figure below).
- If opposite faces of the cube touch P then no corner is free.



4 Monotone Polyline Stabbing by a Line

Stephane Durocher (University of Manitoba) and Myroslav Kryven (University of Manitoba)

Given a monotone polyline (aka polygonal chain) in $\mathbb{R}^2$, the goal is to find a line that intersects maximum number of segments of the polyline. An example is shown below.



First observation is that if we consider a stabbing line then if it does not pass through two vertices of a polyline then we can move it around without changing which line segments this stabbing line intersects. This shows that there is a solution passing through two vertices of the input polyline. This gives rise to a brute force algorithm based on enumerating all pairs of vertices, considering a stabbing line through those two points, and counting how many line segments it intersects. The line giving maximum number of intersections is the answer. This is an $O(n^3)$ algorithm.

Stephane described an $O(n^2)$ time algorithm that is based on the observation that a polyline can be projected along a line of slope θ to obtain an interval graph (line segments defining a polyline become line segments of different length and position depending on θ in $1D$ after the projection). Finding a stabbing line that intersects maximum number of segments is then equivalent to finding a θ that results in a largest clique number of the interval graph obtained after the projection. With this representation, it is possible to maintain clique number while varying θ and solve the problem in total time $O(n^2)$. David Eppstein mentioned another $O(n^2)$ algorithm based on a projective dual, where lines turn into points and segments turn into double wedges, then one can walk over the points in the projective dual.

Open Problem 4. Is this problem solvable in $o(n^2)$ time?

5 Advice Complexity of the Online Non-Crossing Matching Problem

Ali Lavasani (Concordia University)

Consider the following online problem: $2n$ points in $\mathbb{R}^2$ arrive one by one; for each arrival, an algorithm can match the newly arrived point to a previously arrived and yet unmatched point via a straight-line segment. The algorithm can also decide to leave an arriving point unmatched. There is a non-crossing constraint, which means that line segments from the matching are not allowed to intersect. The goal is to maximize the number of matched points subject to the non-crossing constraint. It is easy to see that it is always possible to match all $2n$ points when the entire input is given in advance. In the online setting, it is impossible to match all the points without any advice. The question is how many bits of advice are needed in the online setting to match all the points. This version of the problem is known as monochromatic non-crossing matching (MNM). In another version, called bichromatic non-crossing matching (BNM), n blue points are given in advance, while n red points arrive online. An algorithm can match a red point to a blue point subject to the non-overlapping constraint. The goal, as before, is to maximize the number of points that are matched. The following table summarizes what is known about advice complexity to achieve perfect matching:

Problem	Lower Bound	Upper Bound
MNM, points on a circle	$n/3$	$\log C_n$
MNM, general position	$n/3$	$3n$
BNM, points on a circle	$\log C_n$	$\log C_n$
BNM, general position	$2n$	$n \log n$

In the above table, C_n refers to the n^{th} Catalan number

Open Problem 5. Close the gaps in the above table.

Shahin Kamali (York University) mentioned another interesting question:

Open Problem 6. What is the advice complexity of the fully online BNM? In the fully online BNM, both red and blue points arrive online.

6 Existence of Overlapping Edge Unfolding for an n-Sided Prism with Height h

Takumi Shiota (Kyushu Institute of Technology)

Consider a regular n-sided prism of height h: the bases are regular n-gons of side length 1, and sides are rectangles of size $1 \times h$. We consider the question of the existence of overlapping edge unfoldings. The following results are known (see the Master's thesis of Takumi):

- If $h = 1$ and $3 \leq n \leq 23$ then an overlapping edge unfolding does not exist.
- If $h = 1$ and $n \geq 24$ then an overlapping edge unfolding exists.

The behavior is different when h is different. For example, when $h = 0.2$ and $n = 15$ an overlapping edge unfolding exists.

Open Problem 7. Characterize existence of overlapping edge unfolding of a regular n-sided prism of height h in terms of both h and n.