Reconfiguration Algorithms

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Reconfiguration Algorithms

Naomi Mishimura's Introduction to Reconfiguration in *Algorithms,* 2018 states*:*

> "Reconfiguration is concerned with relationships among solutions to a problem instance where the reconfiguration of one solution to another is a sequence of steps such that each step produces an intermediate feasible solution".

Today, I would like to talk with you broadly about reconfiguration, wearing two hats:

- Fellow computational geometer
- Former member of the US National Science Board

Reconfiguration Algorithms

- Geometric Reconfiguration: “le pièce de resistance”

- Reconfiguration within the broader landscape of science: “le dessert”

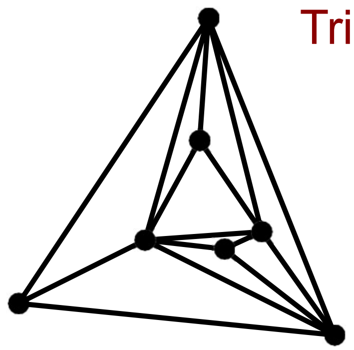
Reconfiguration Algorithms

- Geometric Reconfiguration: “le pièce de resistance”
  - Reviewing tools for mapping one configuration to another with examples
    - A basic reconfiguration step
    - A canonical configuration
- Reconfiguration of the broader landscape of science: “le dessert”

Reconfiguration Algorithms

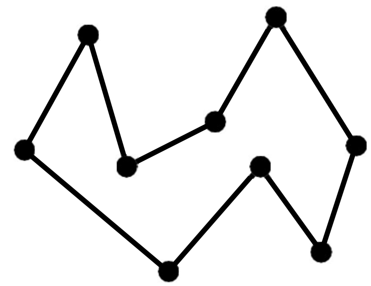
- Geometric Reconfiguration: “le pièce de resistance”
  - Reviewing tools for mapping one configuration to another with examples
    - A basic reconfiguration step
    - A canonical configuration
  - Reconfiguration of polygonal subdivisions as motivated by the challenges in electing a representative government.
- Reconfiguration in the broader landscape of science: “le dessert”

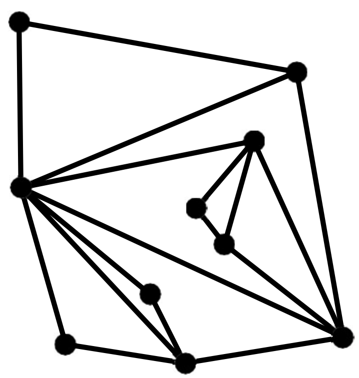
Reconfiguration of geometric graphs



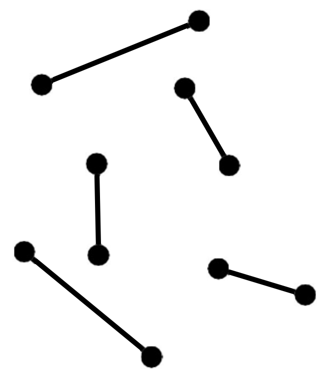
Triangulations

Polygons





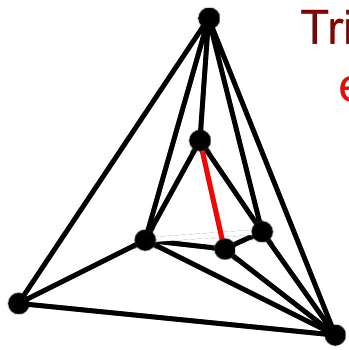
Pseudo-triangulations



Matchings

Reconfiguration of geometric graphs

Triangulations
edge flips



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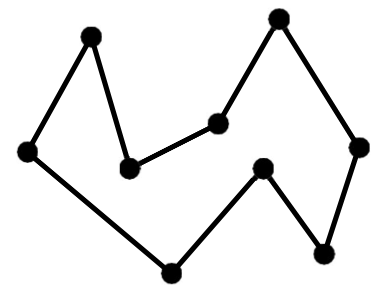

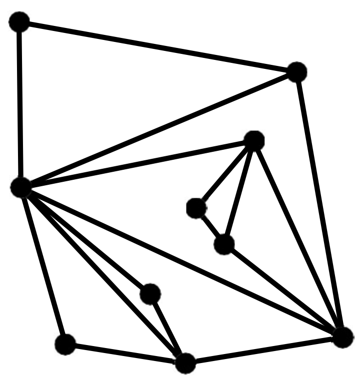

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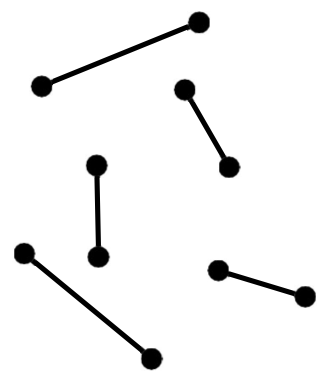

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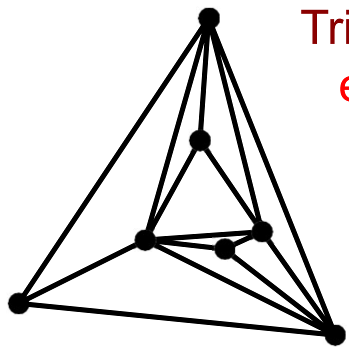

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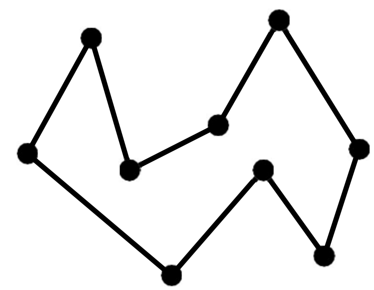

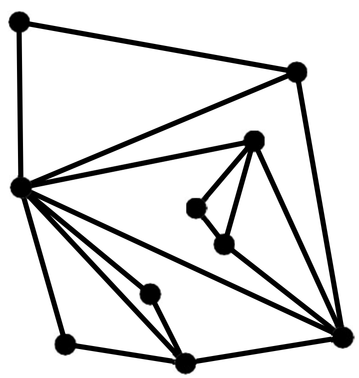

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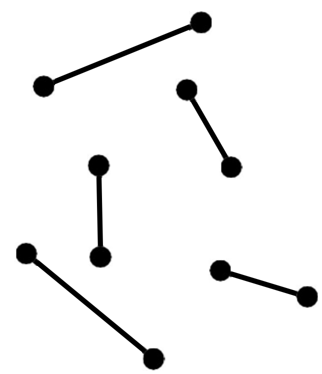

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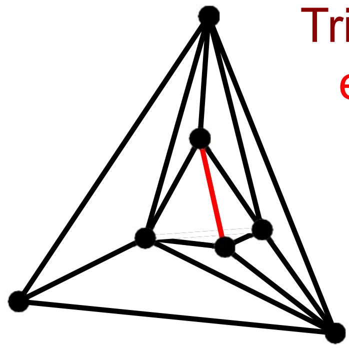

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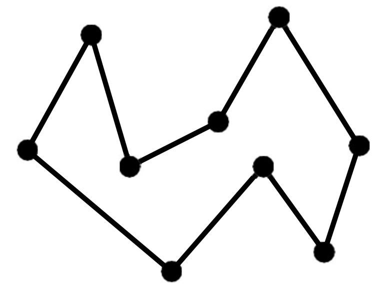

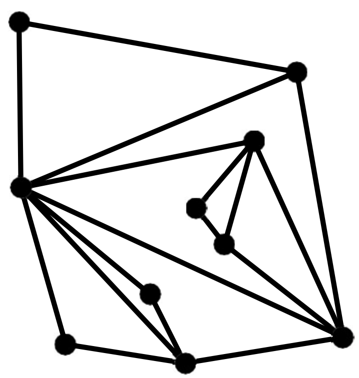

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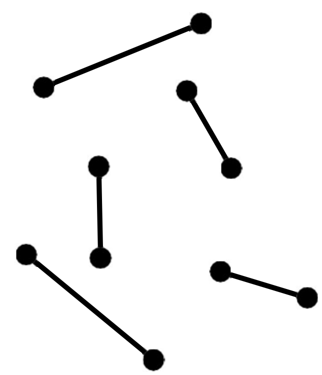

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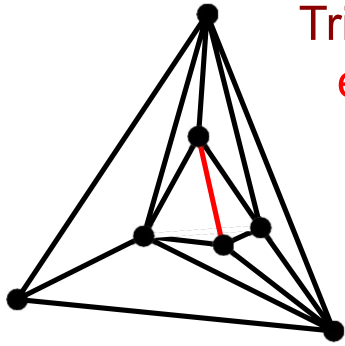

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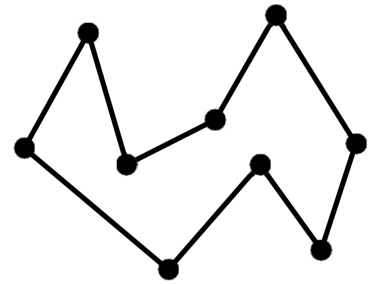

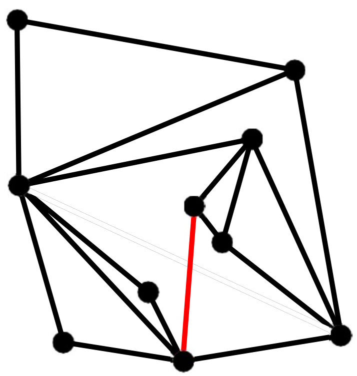

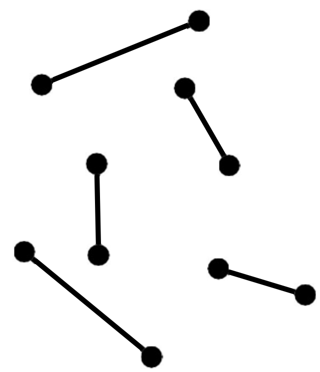

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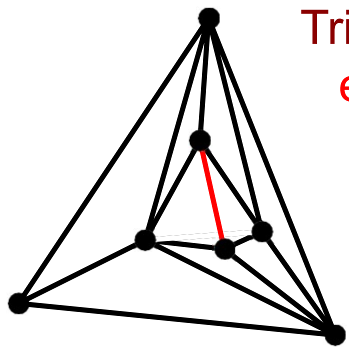

Triangulations
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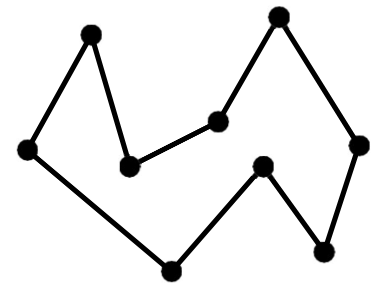
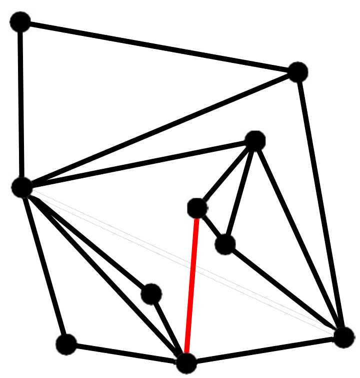
Polygons pops





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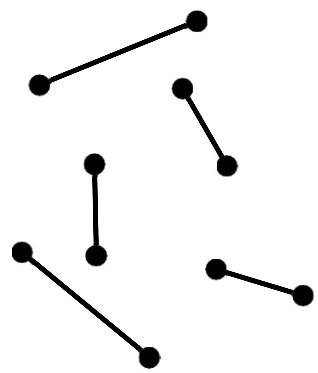

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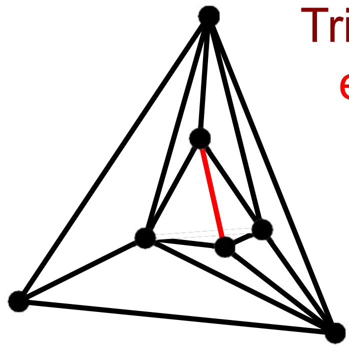

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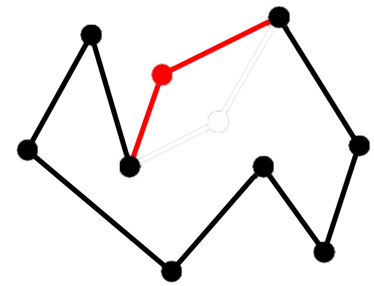

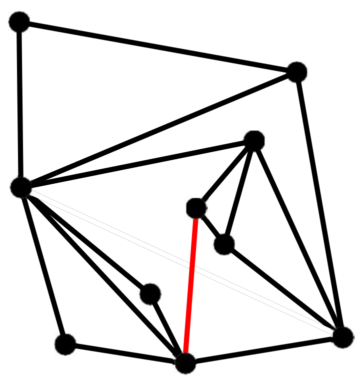

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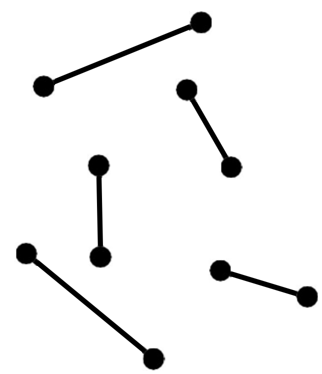

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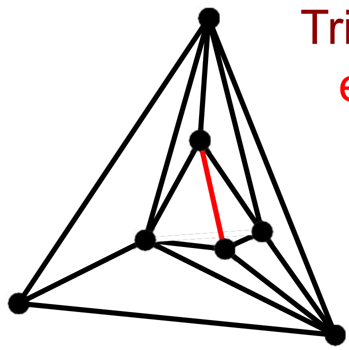

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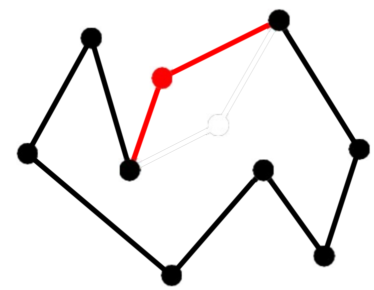

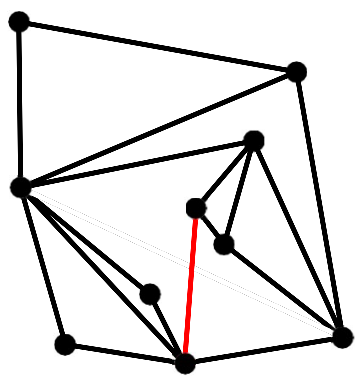

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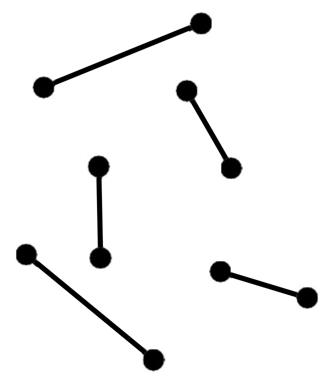
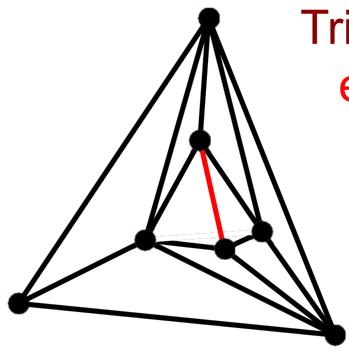


Matchings

compatible matchings

Reconfiguration of geometric graphs



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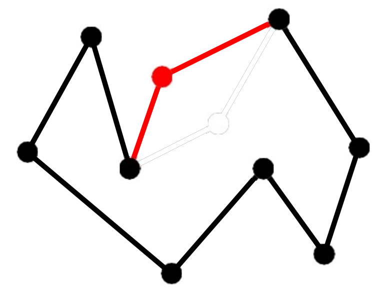

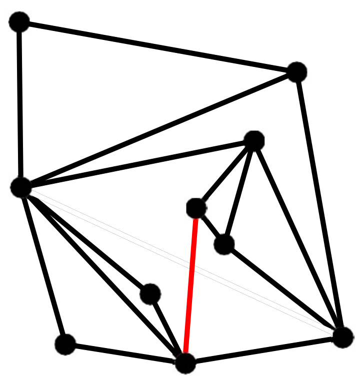

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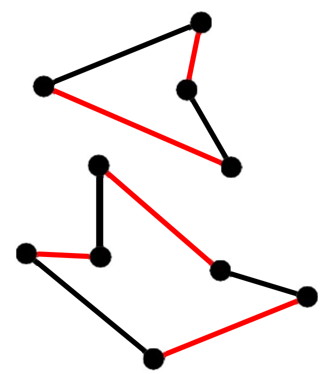

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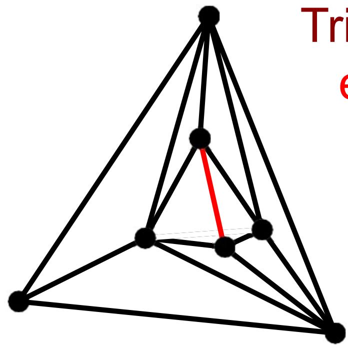

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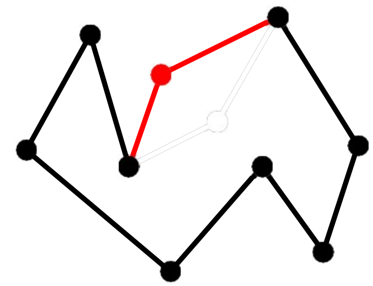

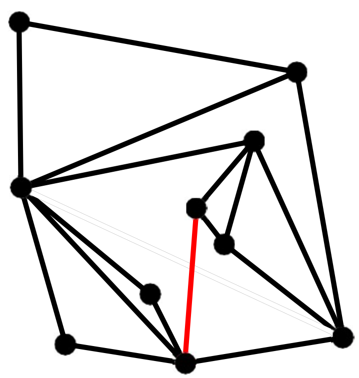

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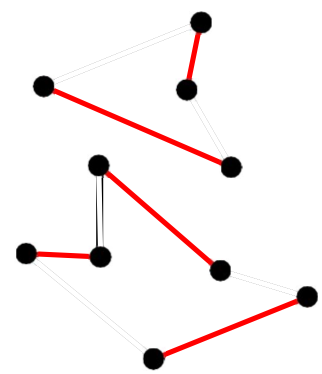

Matchings

compatible matchings

Reconfiguration of geometric triangulations

Given two geometric triangulations, T_1 and T_2 on a finite point set in the plane, reconfigure T_1 into T_2 through a sequence of edge flips.

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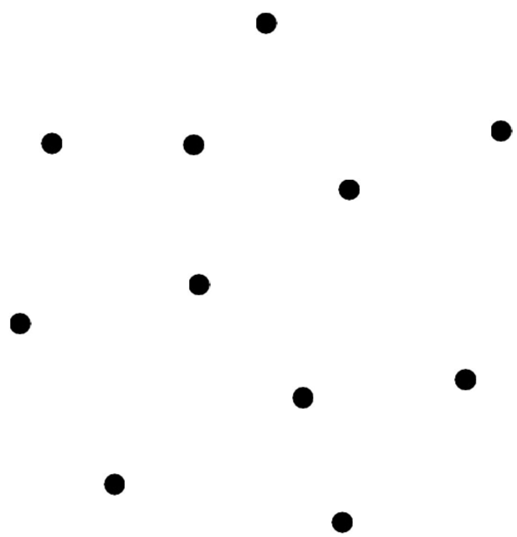

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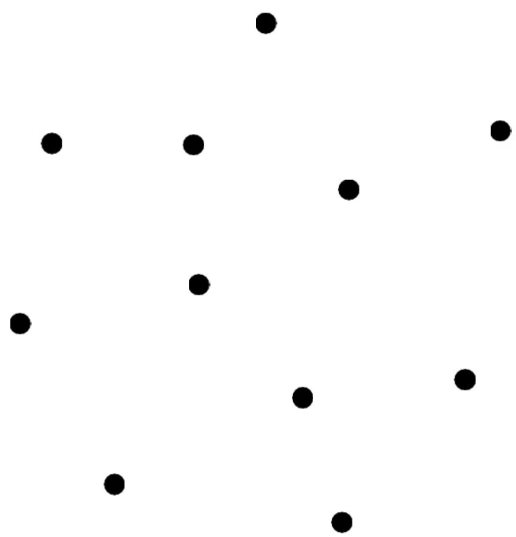

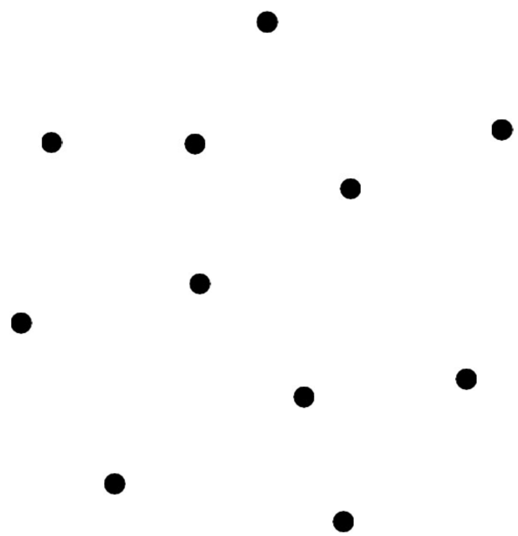

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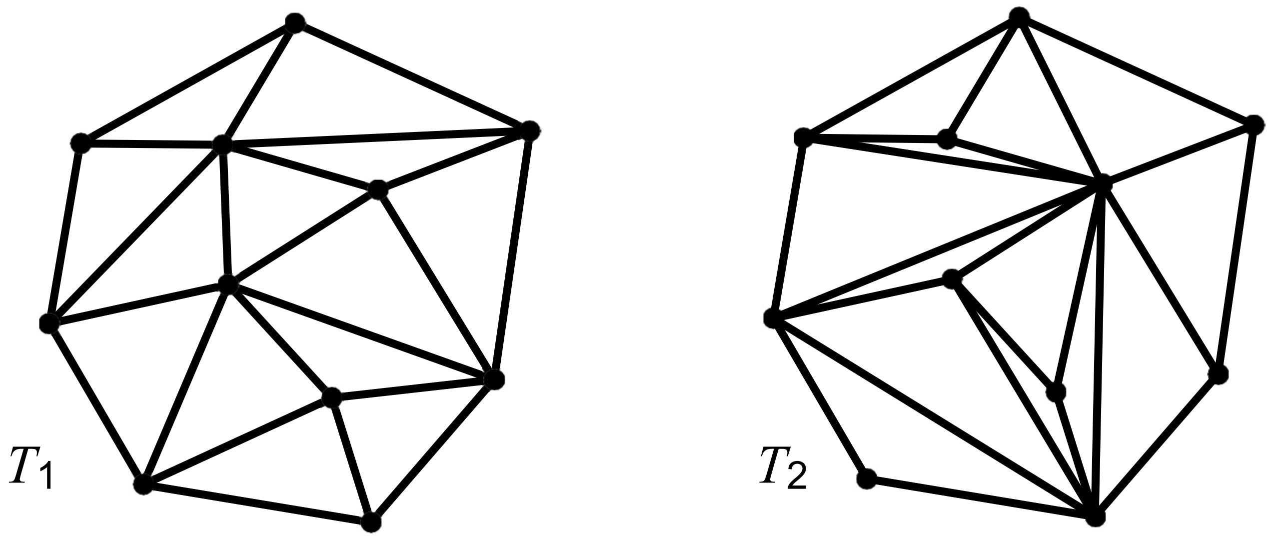

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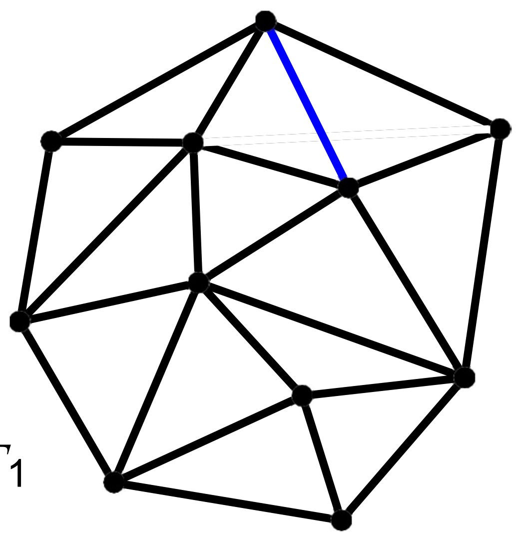
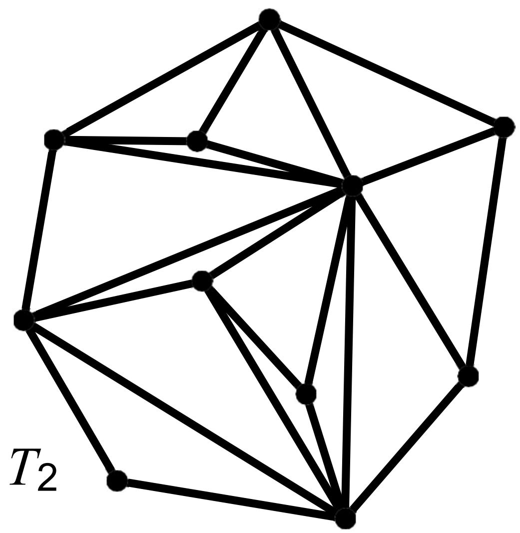


T_1



T_2

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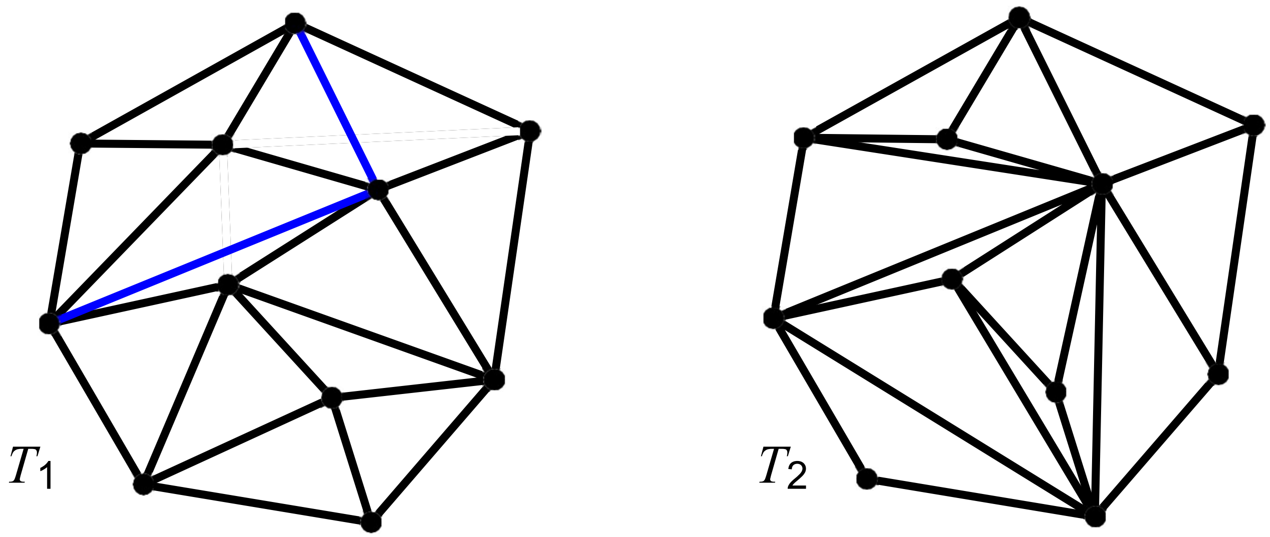

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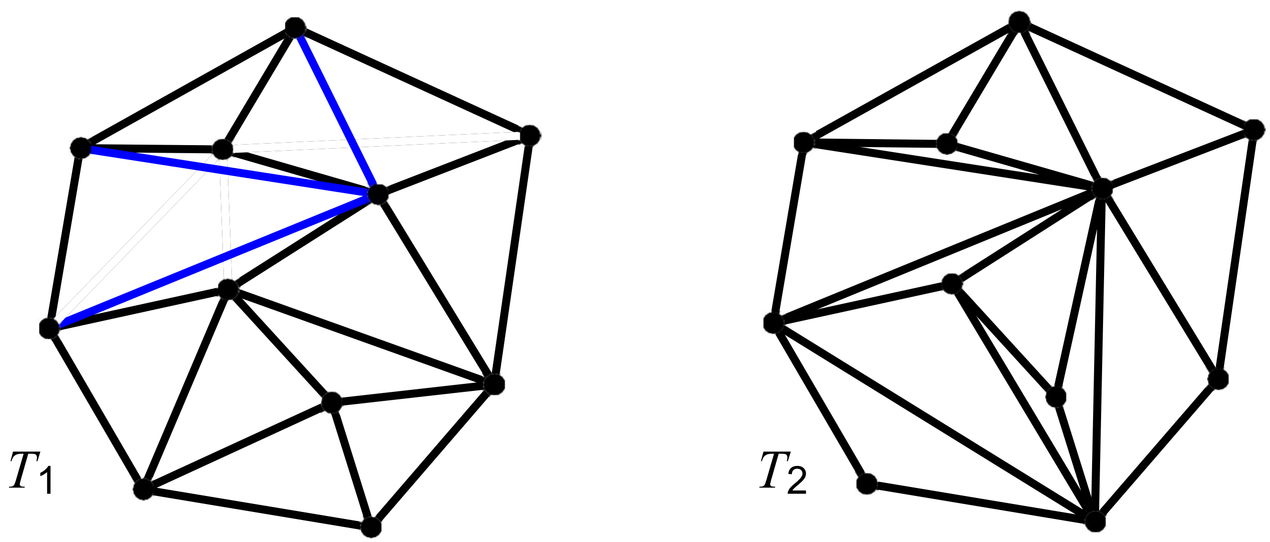

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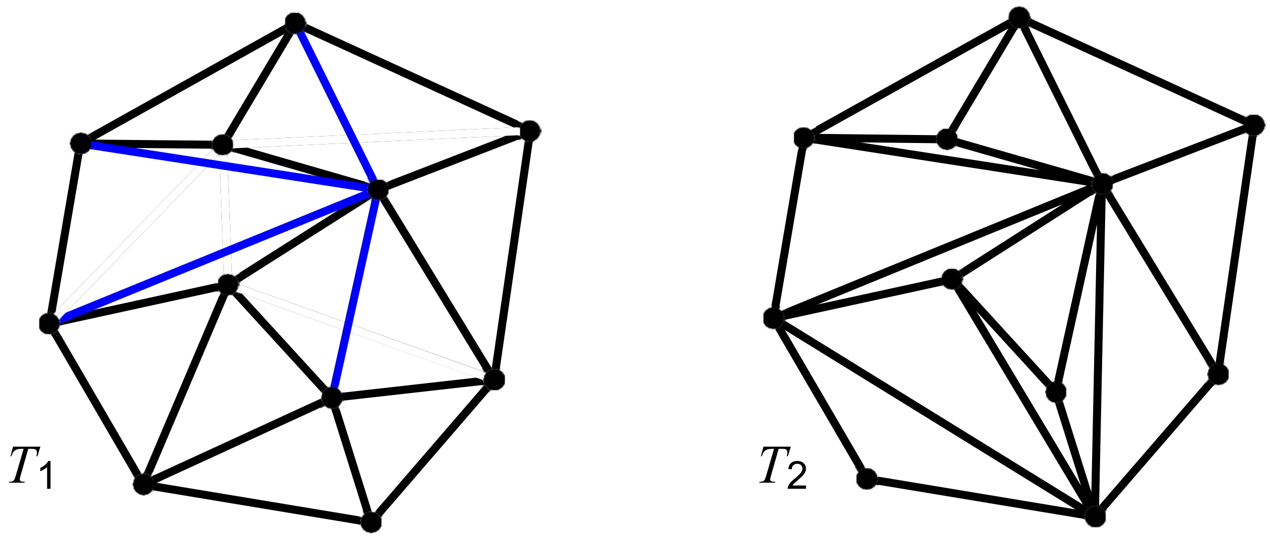

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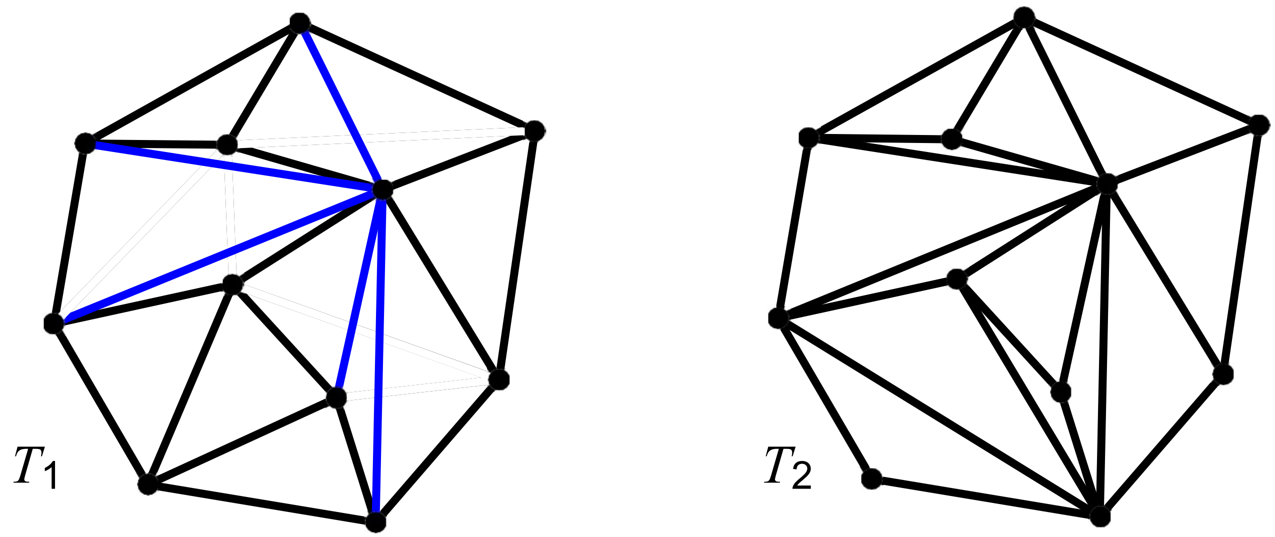

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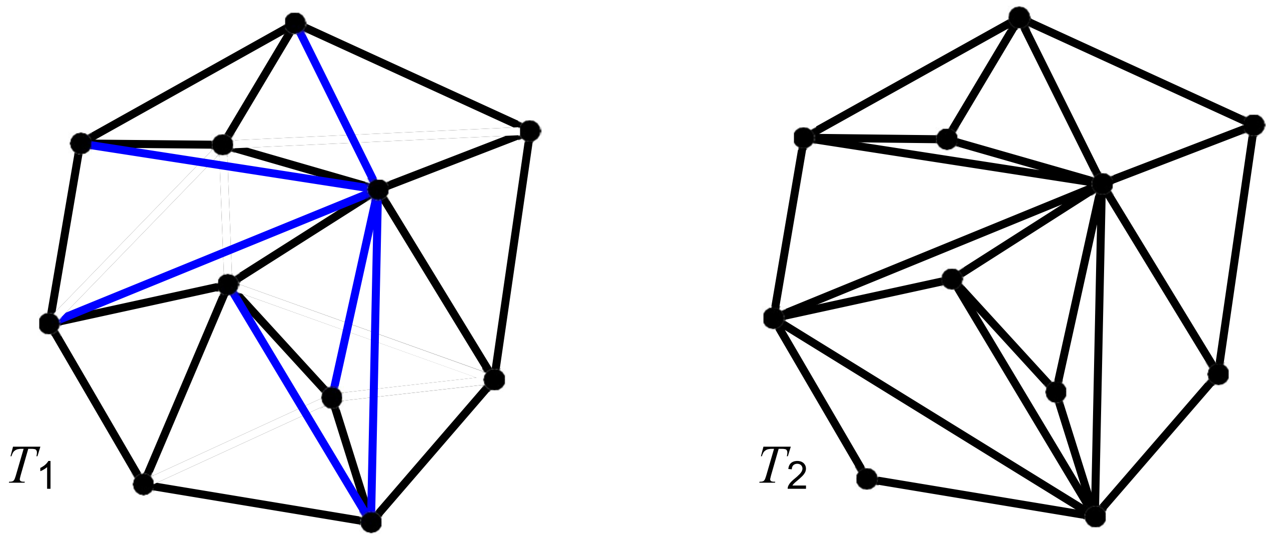

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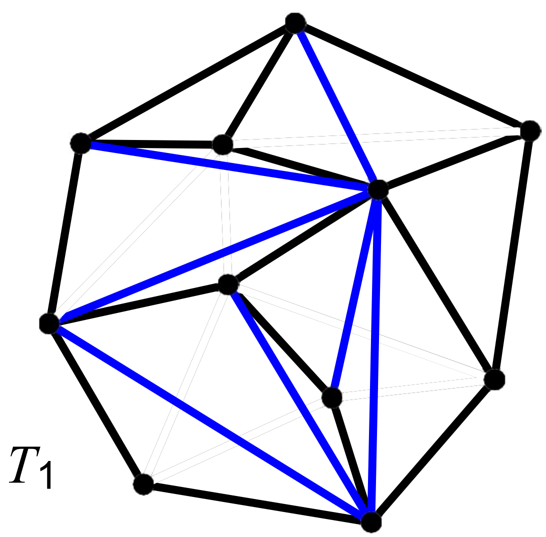

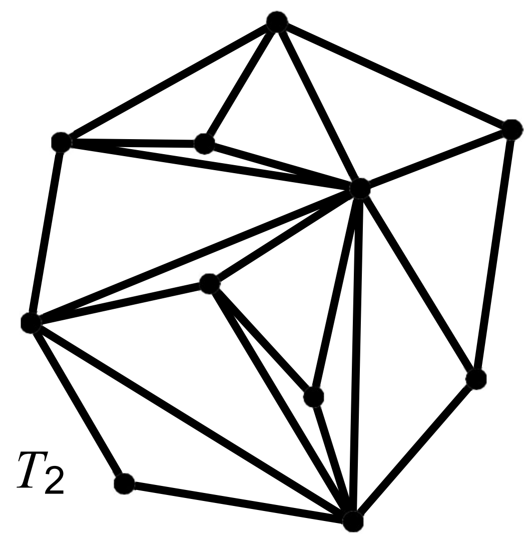

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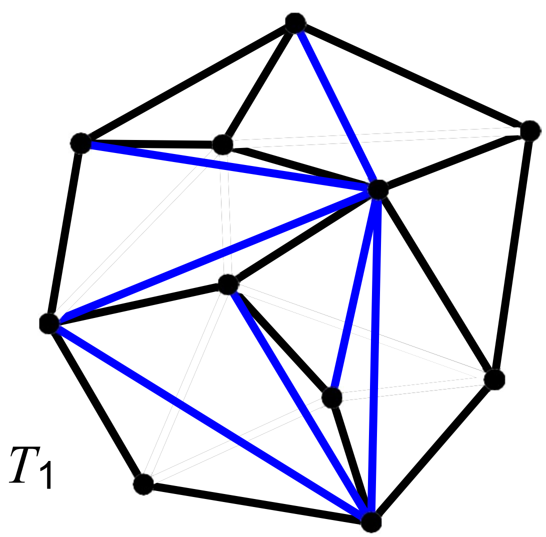
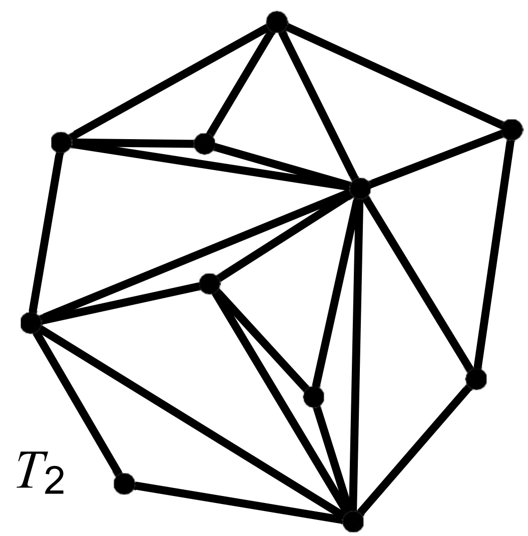




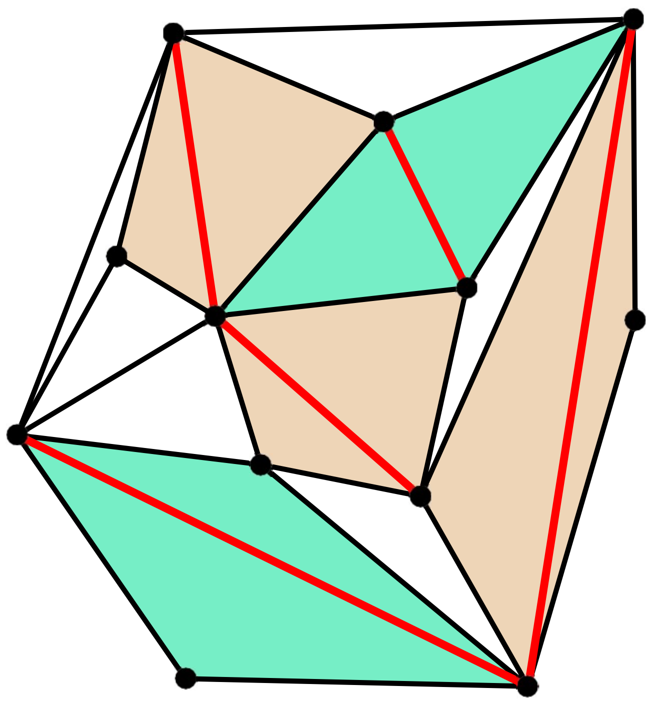
$O(n^2)$ edge flips are always sufficient (Lawson, 1972).

$\Omega(n^2)$ edge flips are sometimes necessary (Hurtado, Noy, Urrutia, 1999)

NPcomplete to fInd flip d istance (Lubiw, Pathak,2015)

Geometric triangulations and edge flips

A set of edges *E* is *simultaneously flippable* if the edges in *E* are diagonals of pairwise interior-disjoint convex quadrilaterals.



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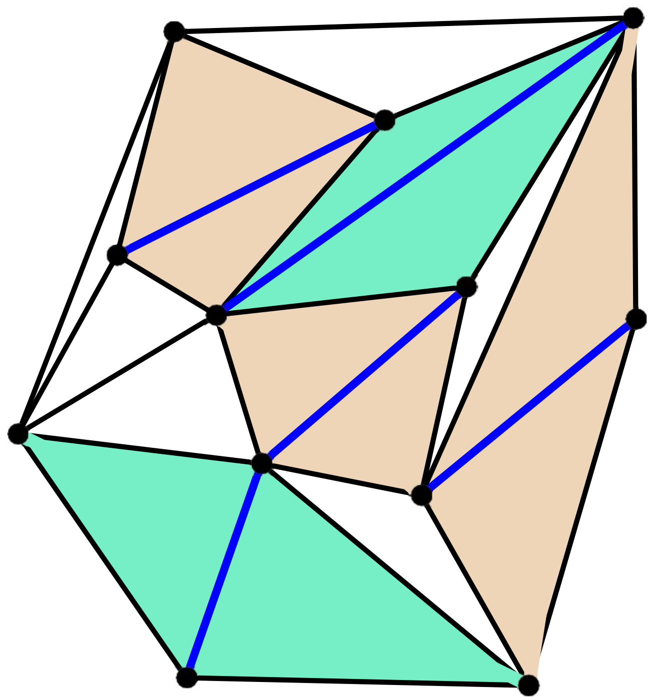
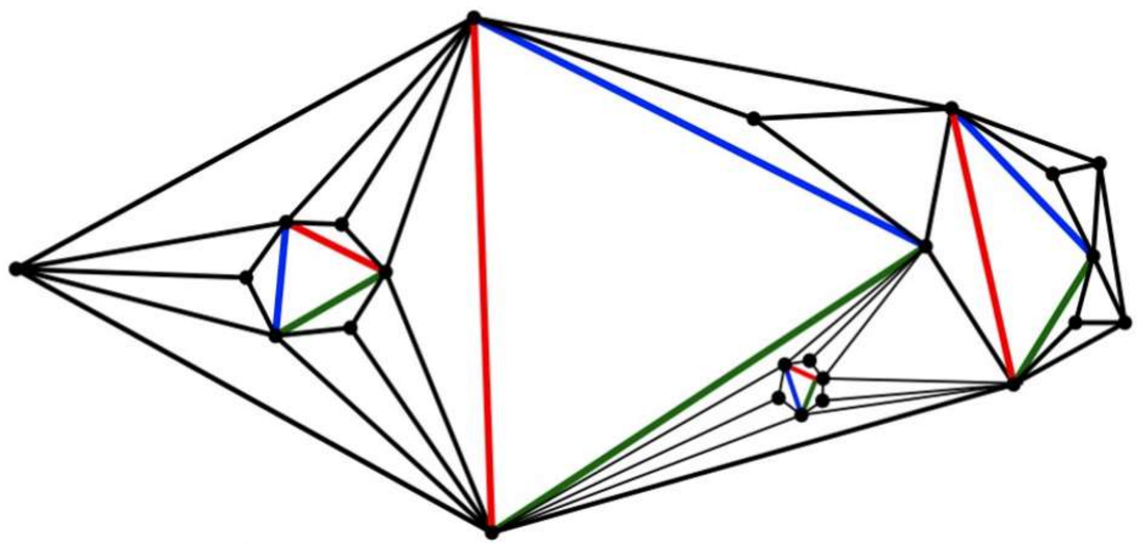


Geometric triangulations and edge flips

Galtier, Hurtado, *et al.* (2003): There are triangulations on *n* vertices that contain **at most** (*n* - 4) / 5 simultaneous flippable edges.



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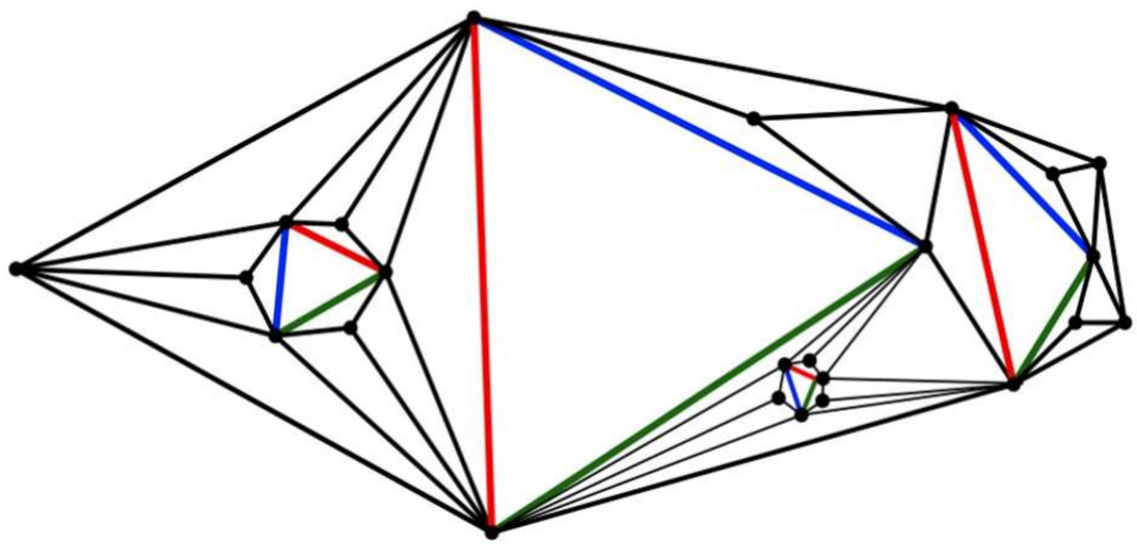
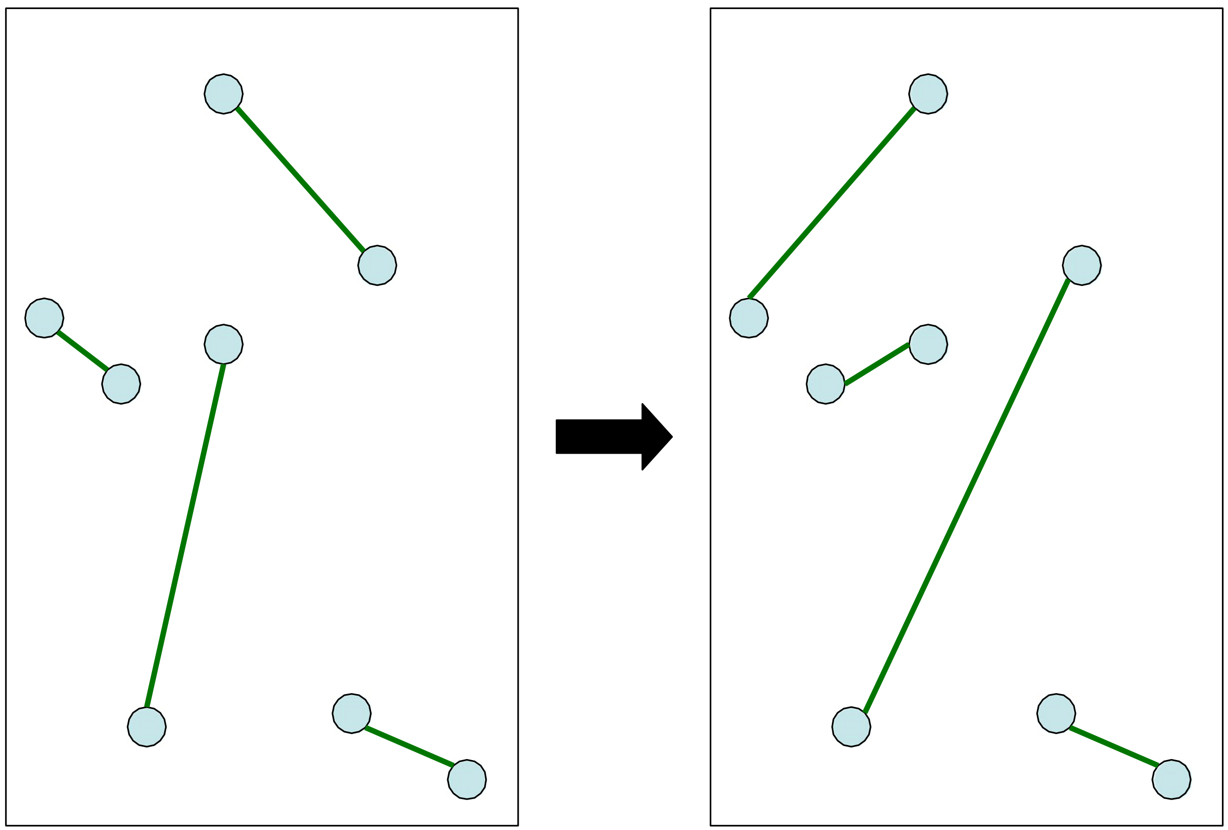
Galtier, Hurtado, *et al.* (2003): There are triangulations on *n* vertices that contain **at most** (*n* - 4) / 5 simultaneous flippable edges.

Souvaine, Tóth, and Winslow. (2011): Every geometric triangulation on *n* vertices contains **at least** (*n* - 4) / 5 simultaneous flippable edges.



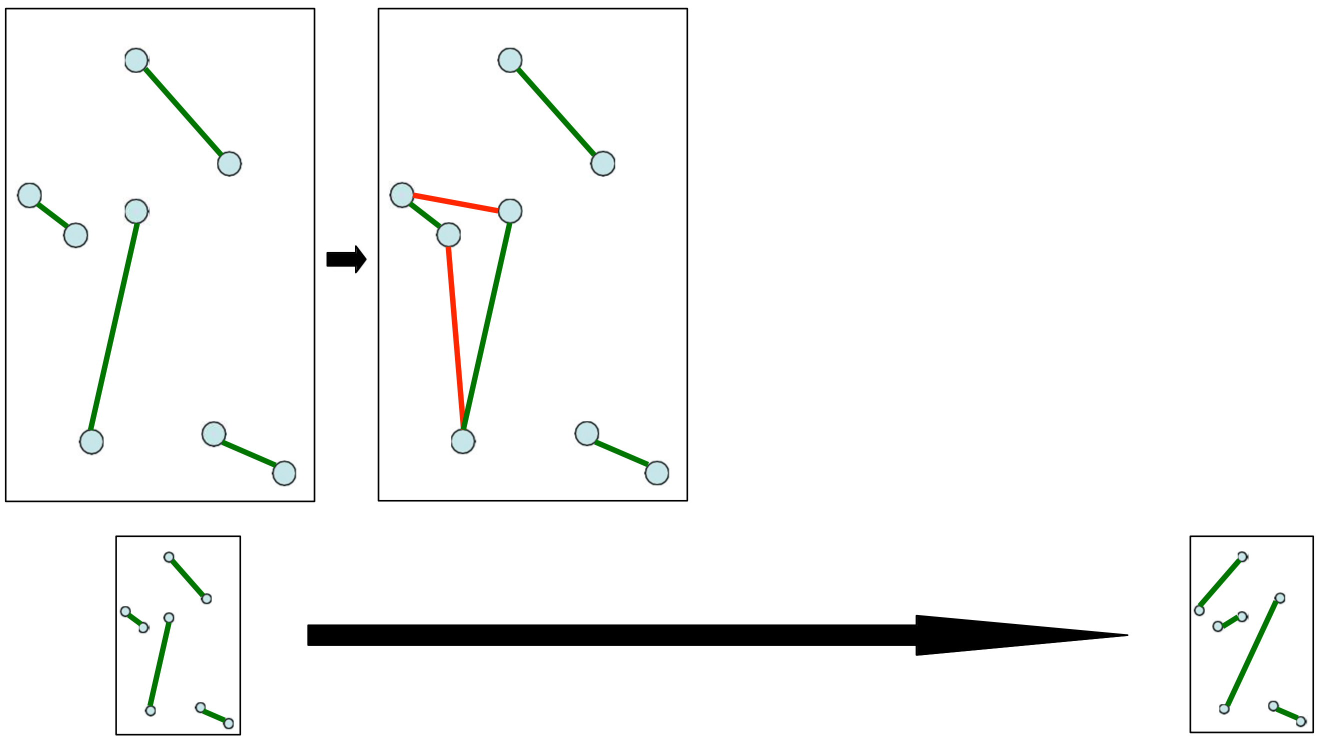
Reconfiguring matchings

Transforming a perfect matching M into another perfect matching M' on the same vertex set S where |S| is even.



Reconfiguring matchings

A transformation between M and M' of length k is a sequence M_0, M_1, ..., M_k = M' of perfect matchings of S such that each matching M_i is compatible with M_{i+1} for i in {0, 1, ..., k-1}.



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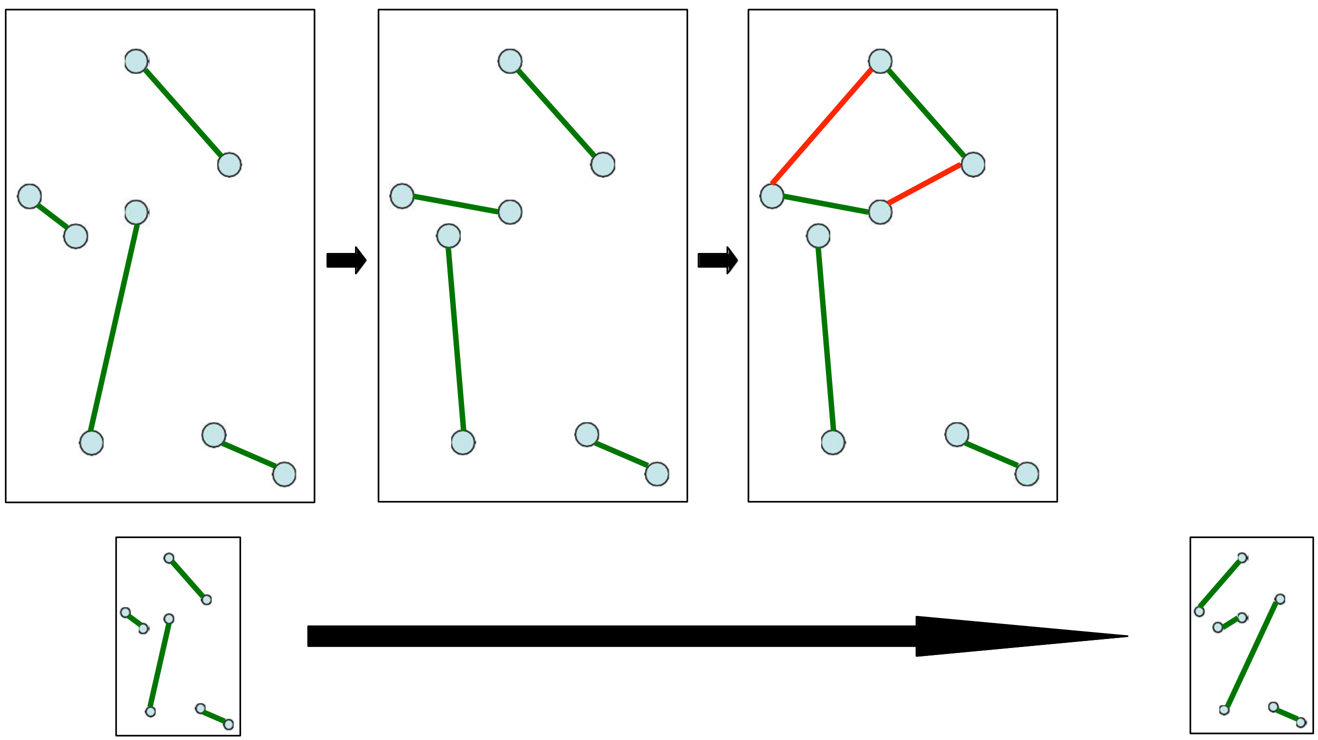

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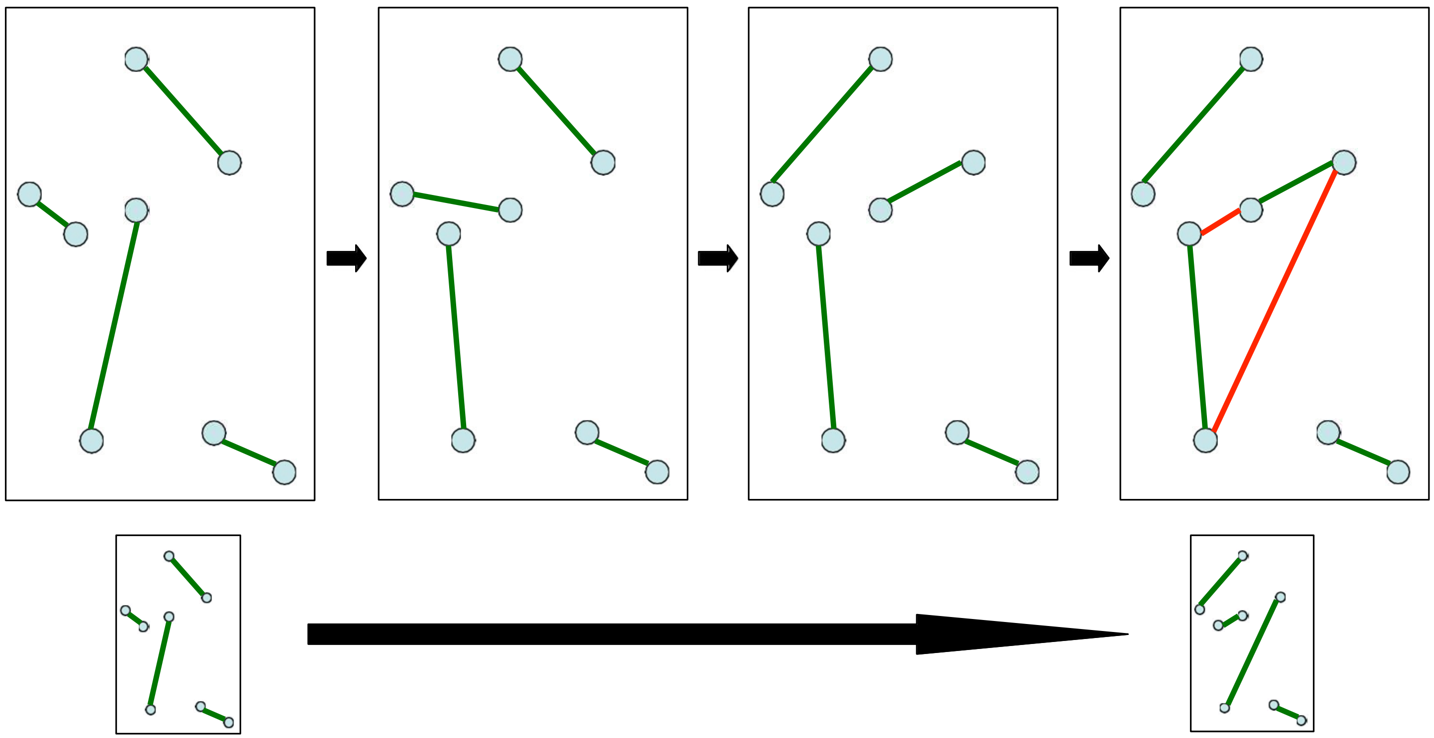

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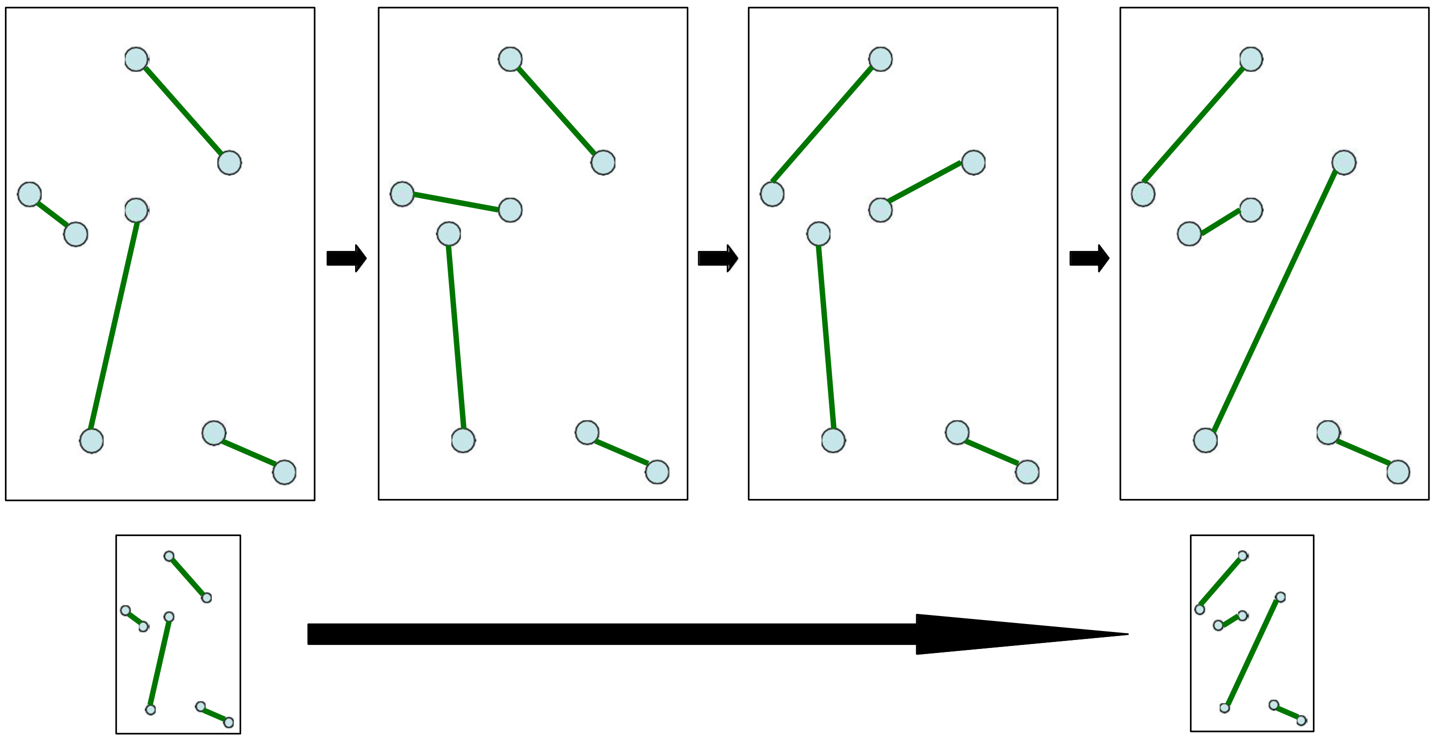
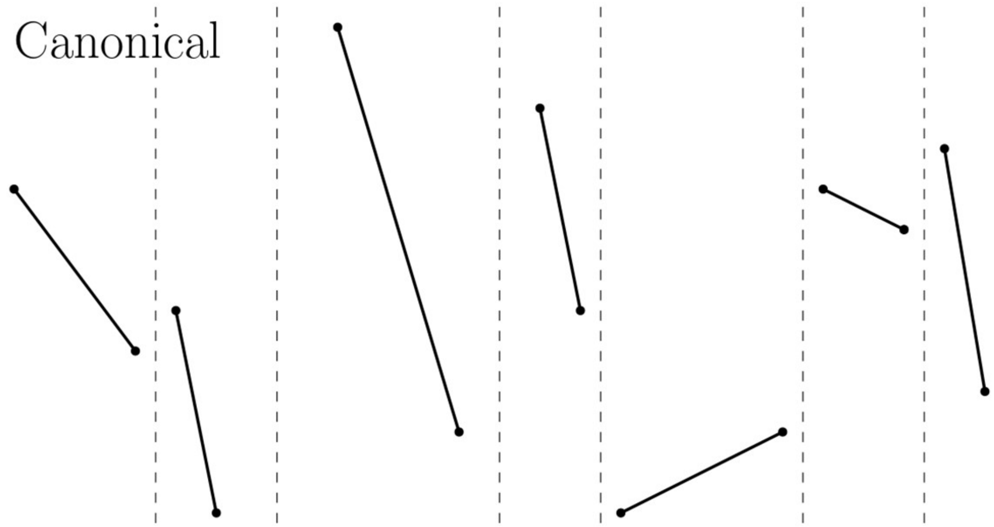
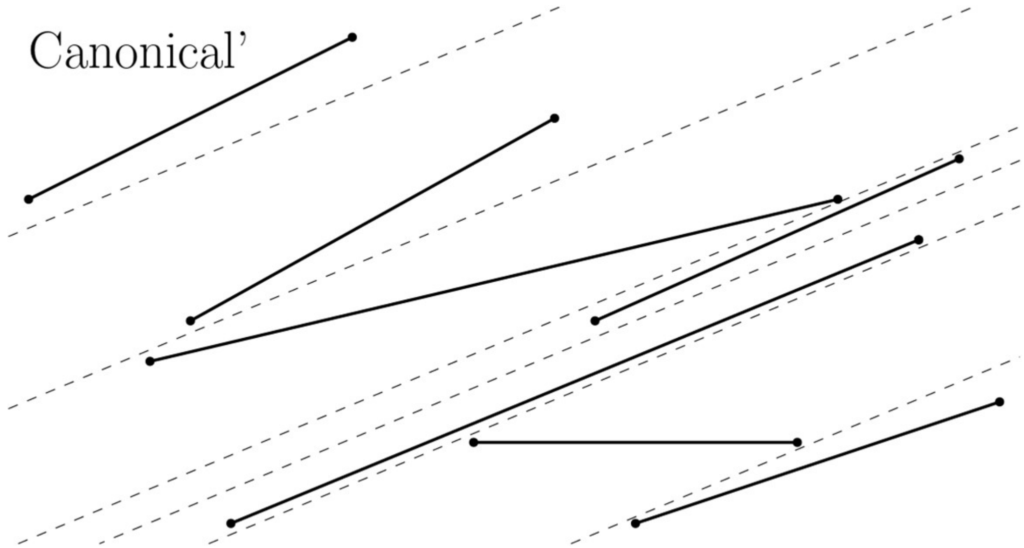


An intermediate canonical configuration



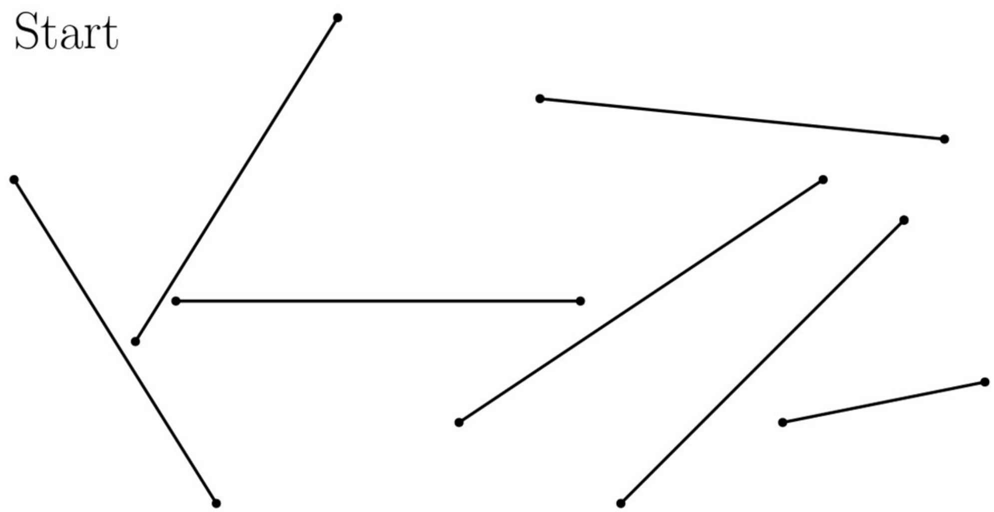
Canonical can be any fixed configuration.

An intermediate canonical configuration



Canonical can be any fixed configuration.

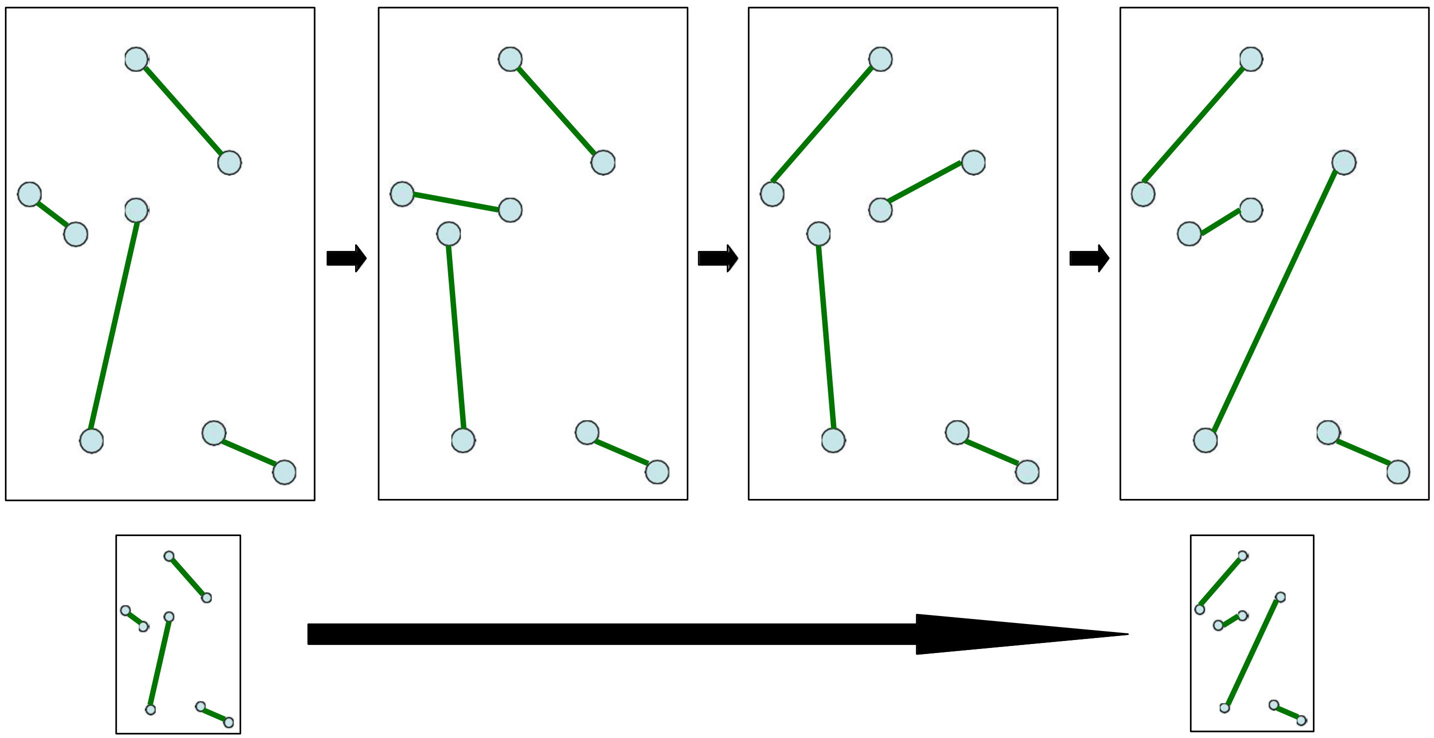
An intermediate canonical configuration



Reconfiguring matchings

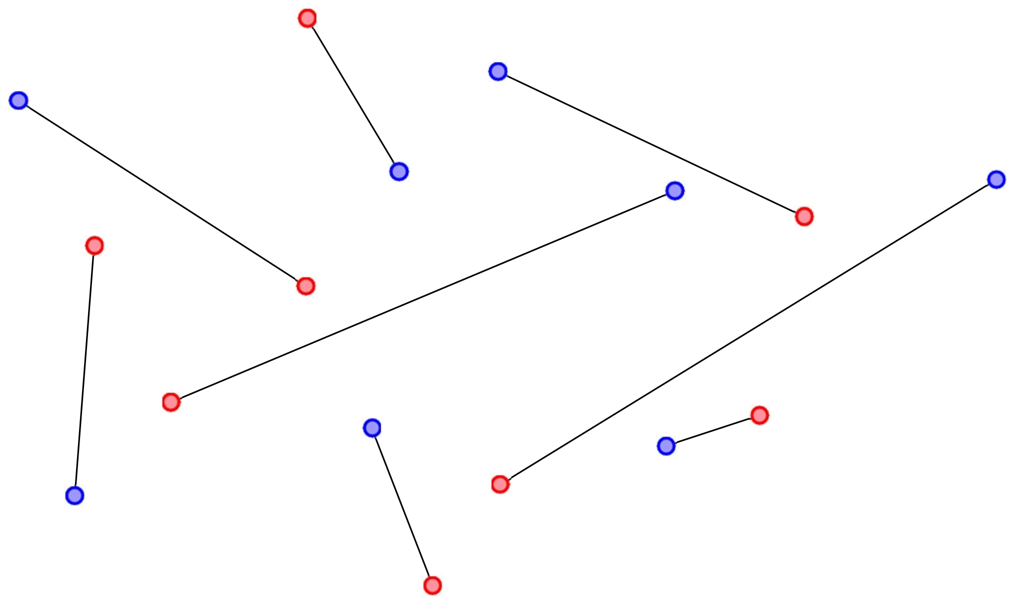
For every set of *2n* points in general position, the diameter of the matching's configuration space is *O(log n).*

Aichholzer, Bereg, Dumitrescu, Garcia, Huemer, Hurtado, Kano, Márquez, Rappaport, Smorodinsky, Souvaine, Urrutia, Wood, 2007.

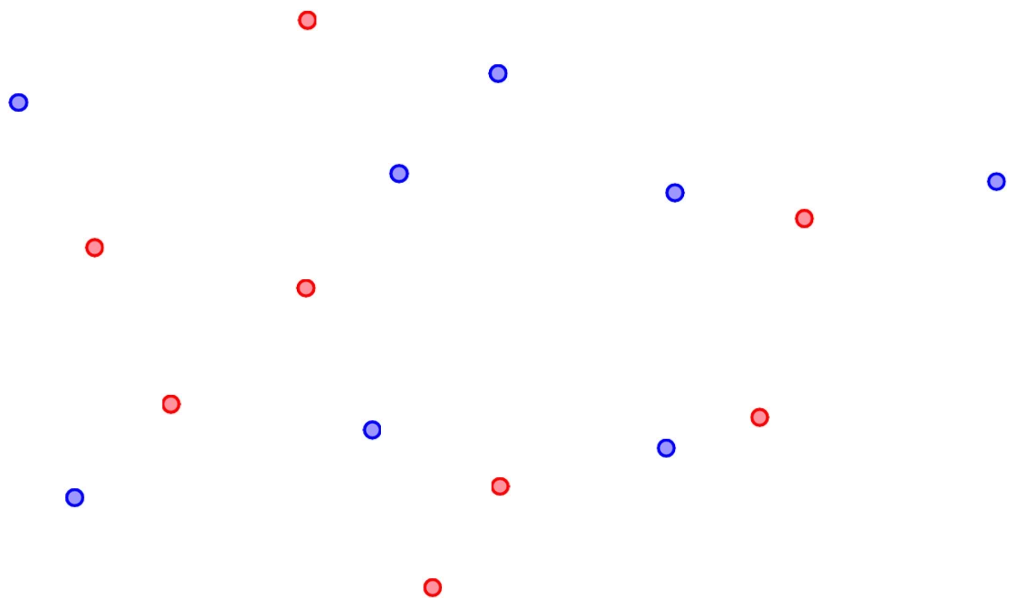


Bichromatic Matchings

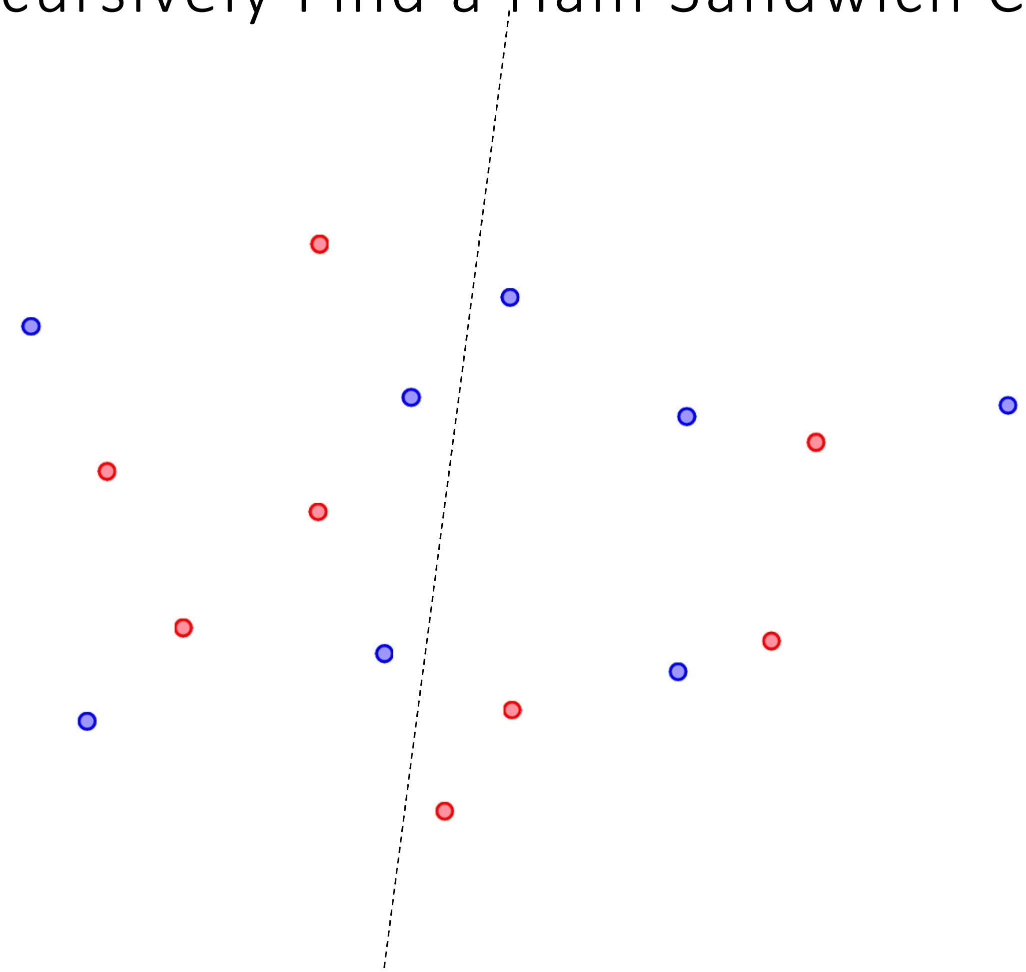
A bichromatic matching (*BR*-matching) is a perfect planar bichromatic matching.



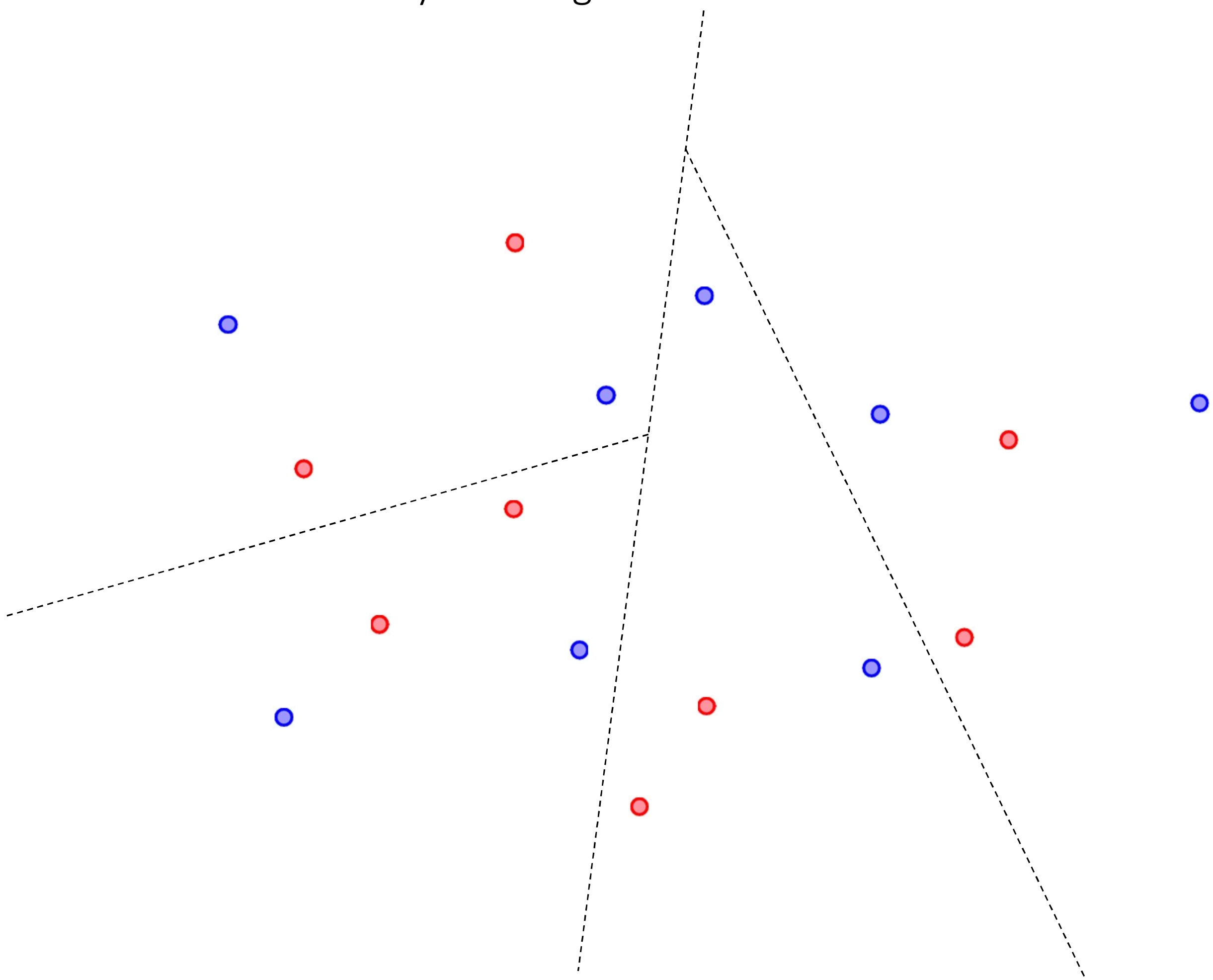
Bichromatic Matching – Does it always exist?



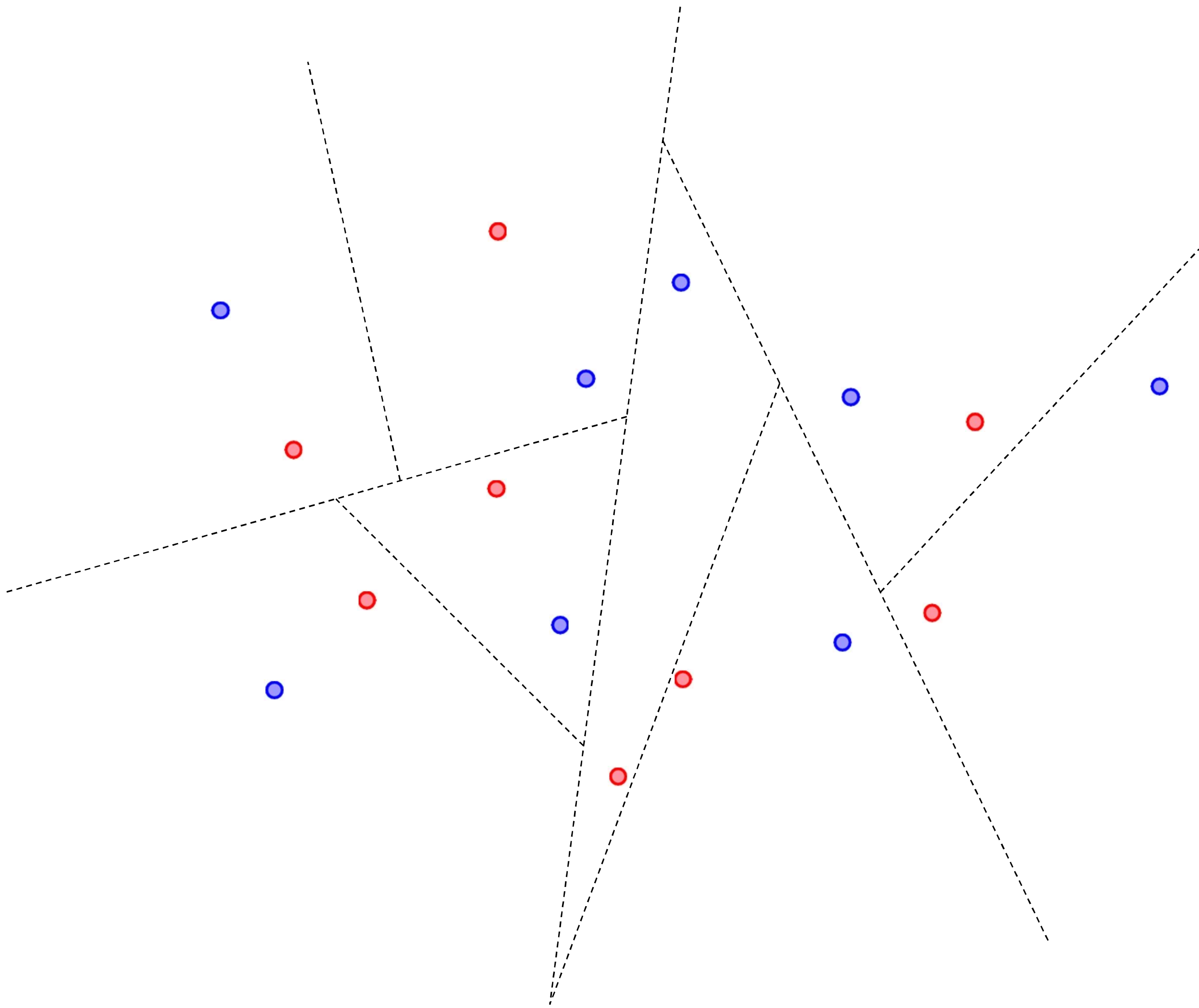
Yes!! Recursively Find a Ham Sandwich Cut



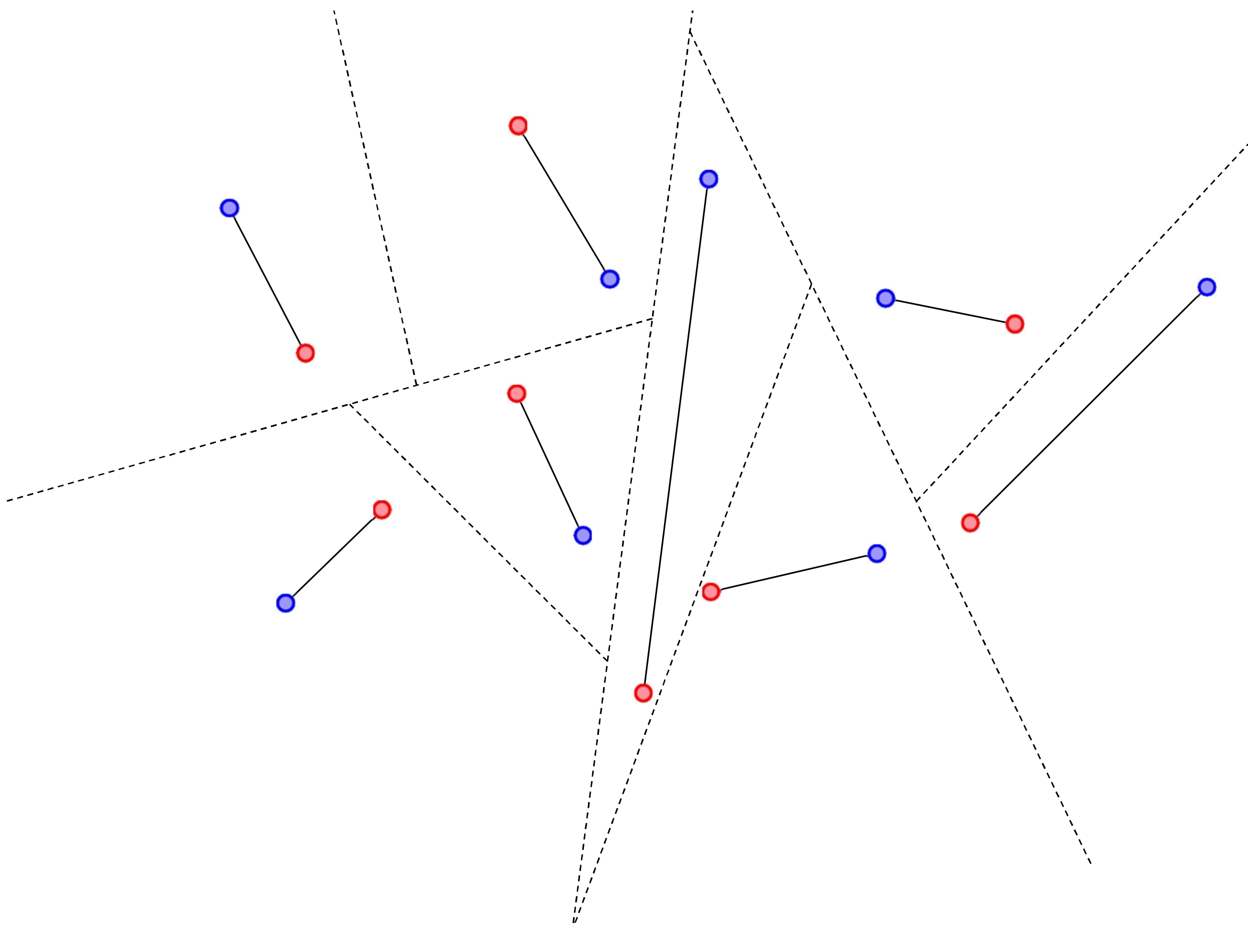
Recursively finding a Ham-Sandwich Cut



Completing a sequence of Ham Sandwich Cuts

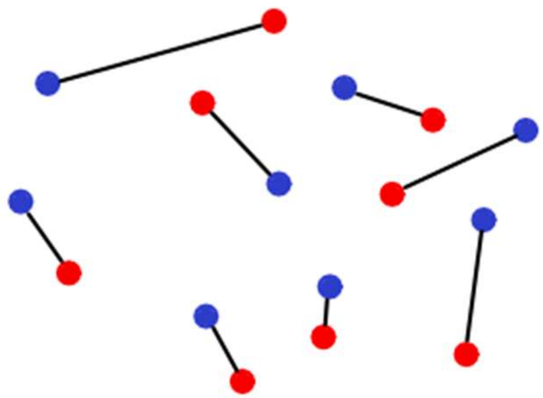


A ham-sandwich matching is a BR-matching defined by recursively applying ham-sandwich cuts.

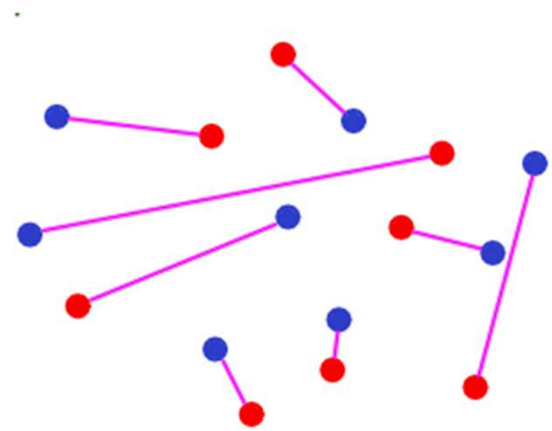


Reconfiguring bichromatic matchings

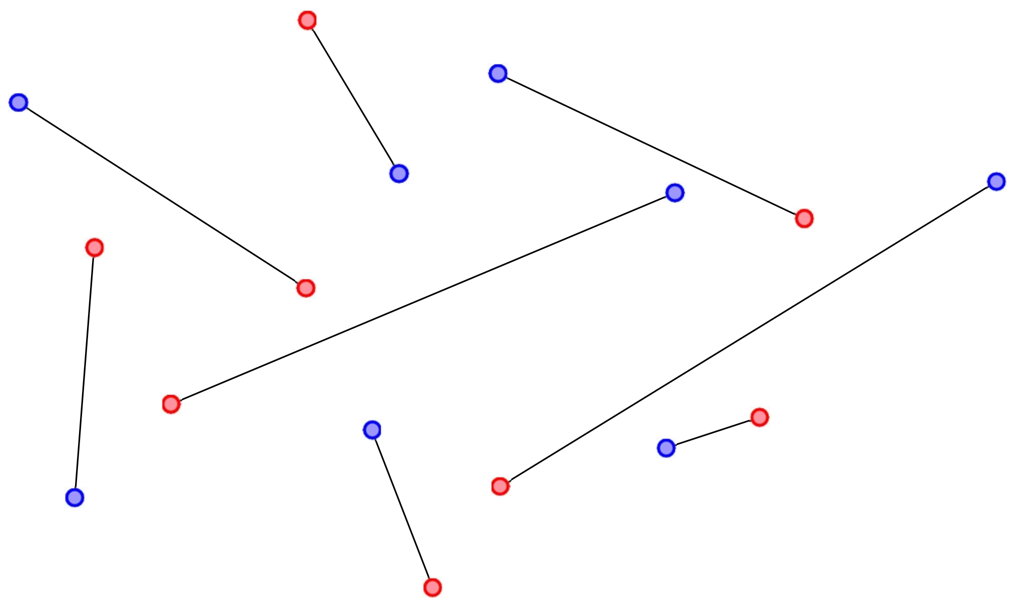
Transforming a bichromatic perfect matching M into another bichromatic perfect matching.



➡



Two *BR*-matchings are compatible if their union is non crossing



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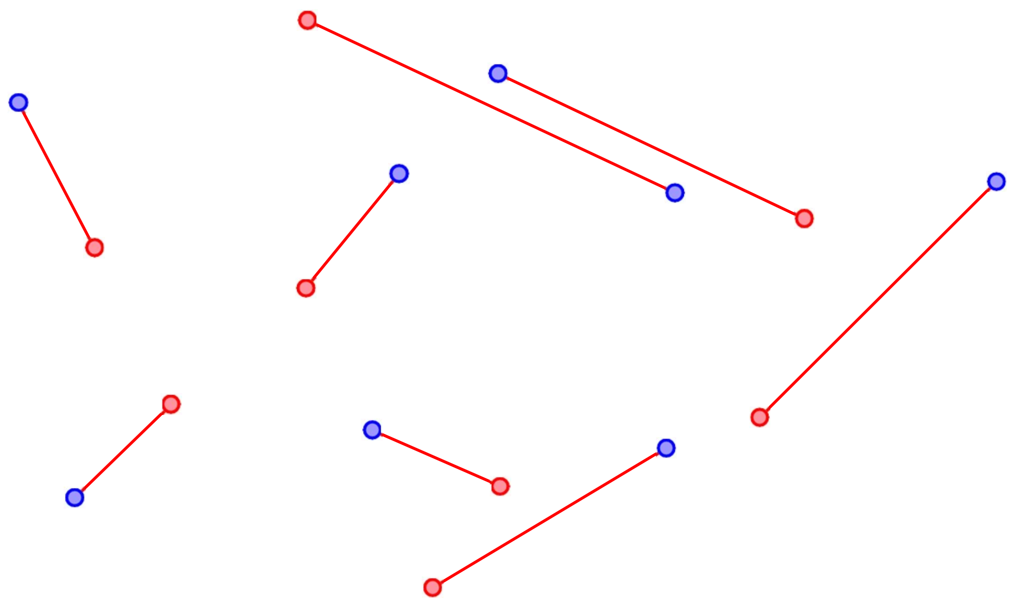

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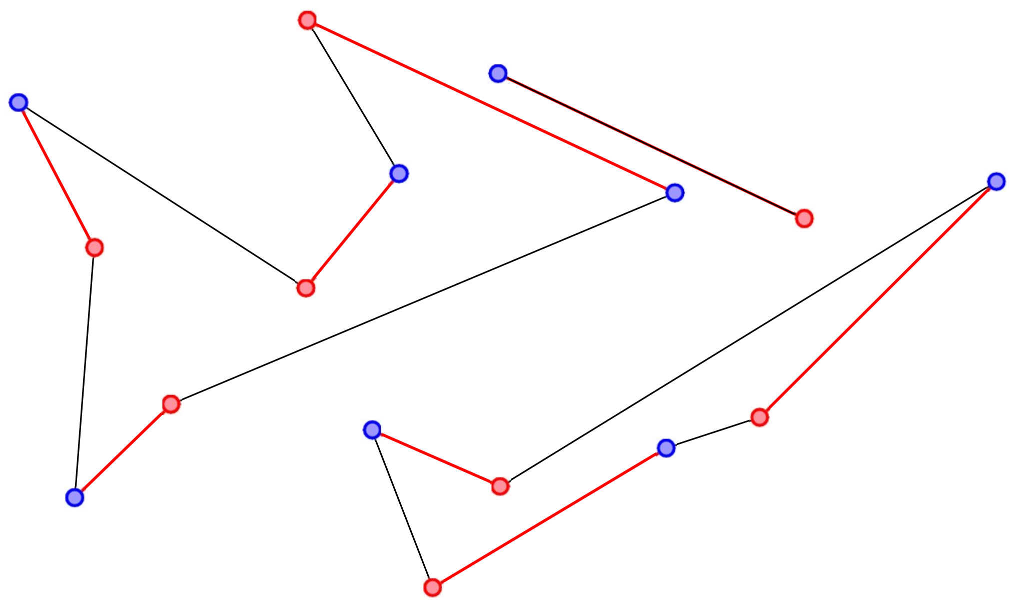
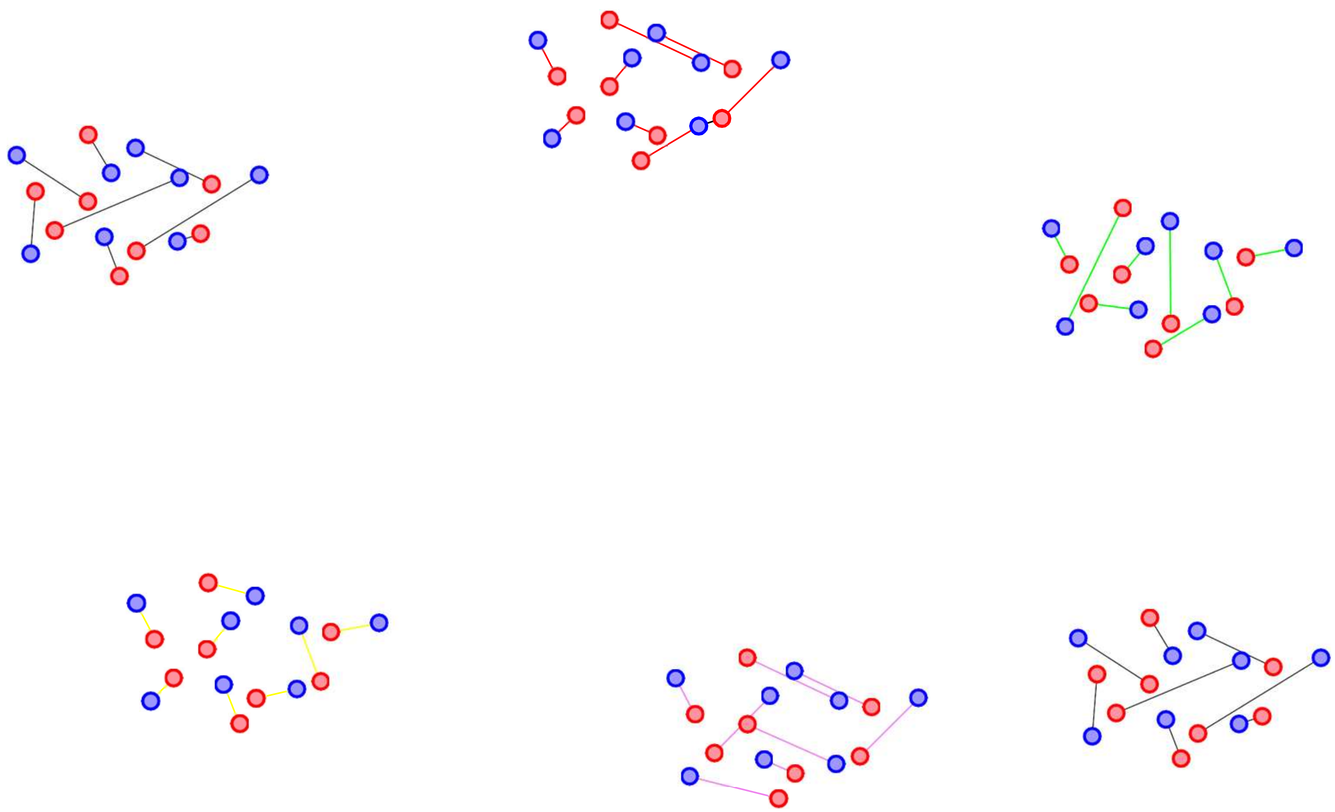
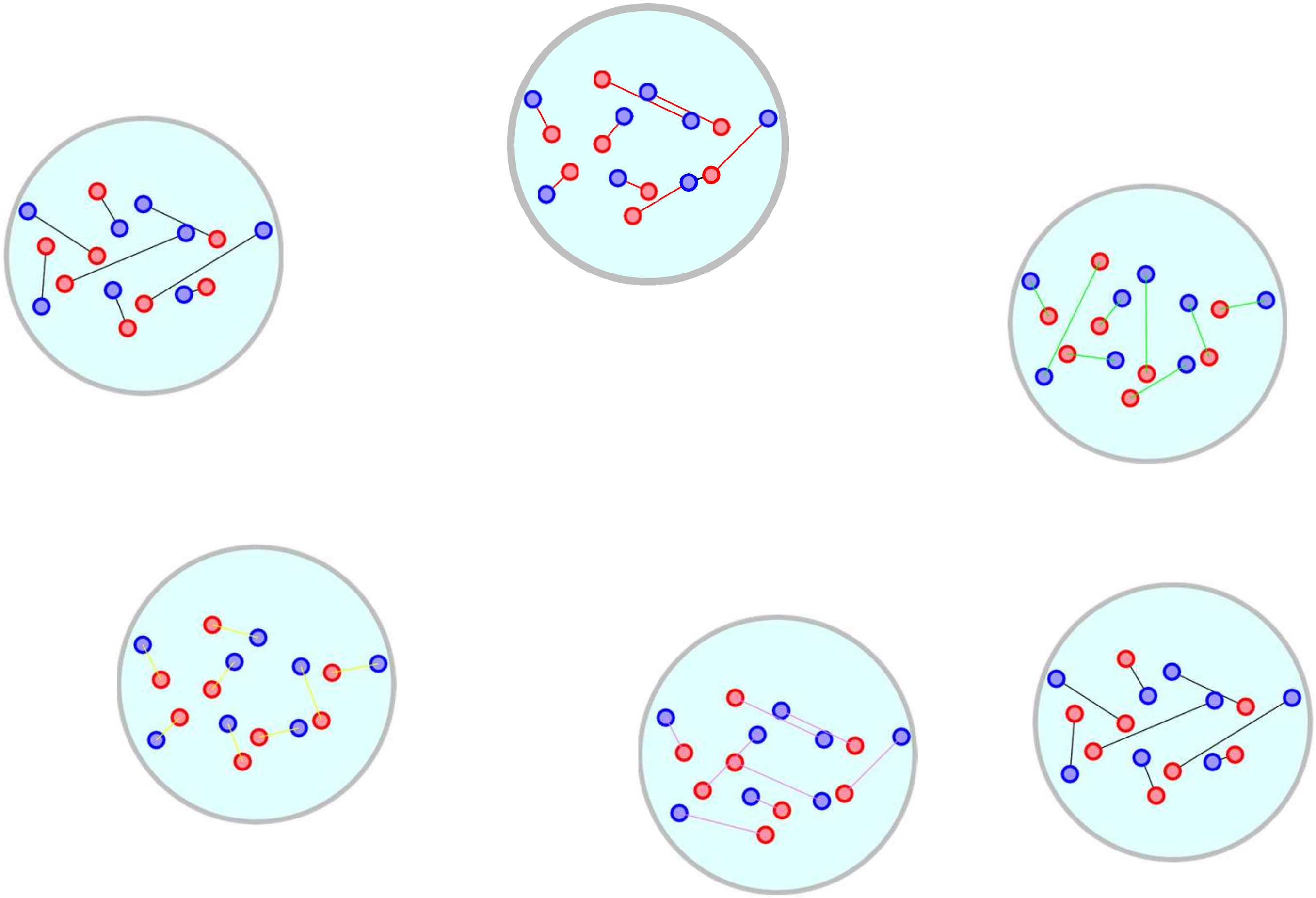


Some segments may be part of both compatible *BR*-matchings

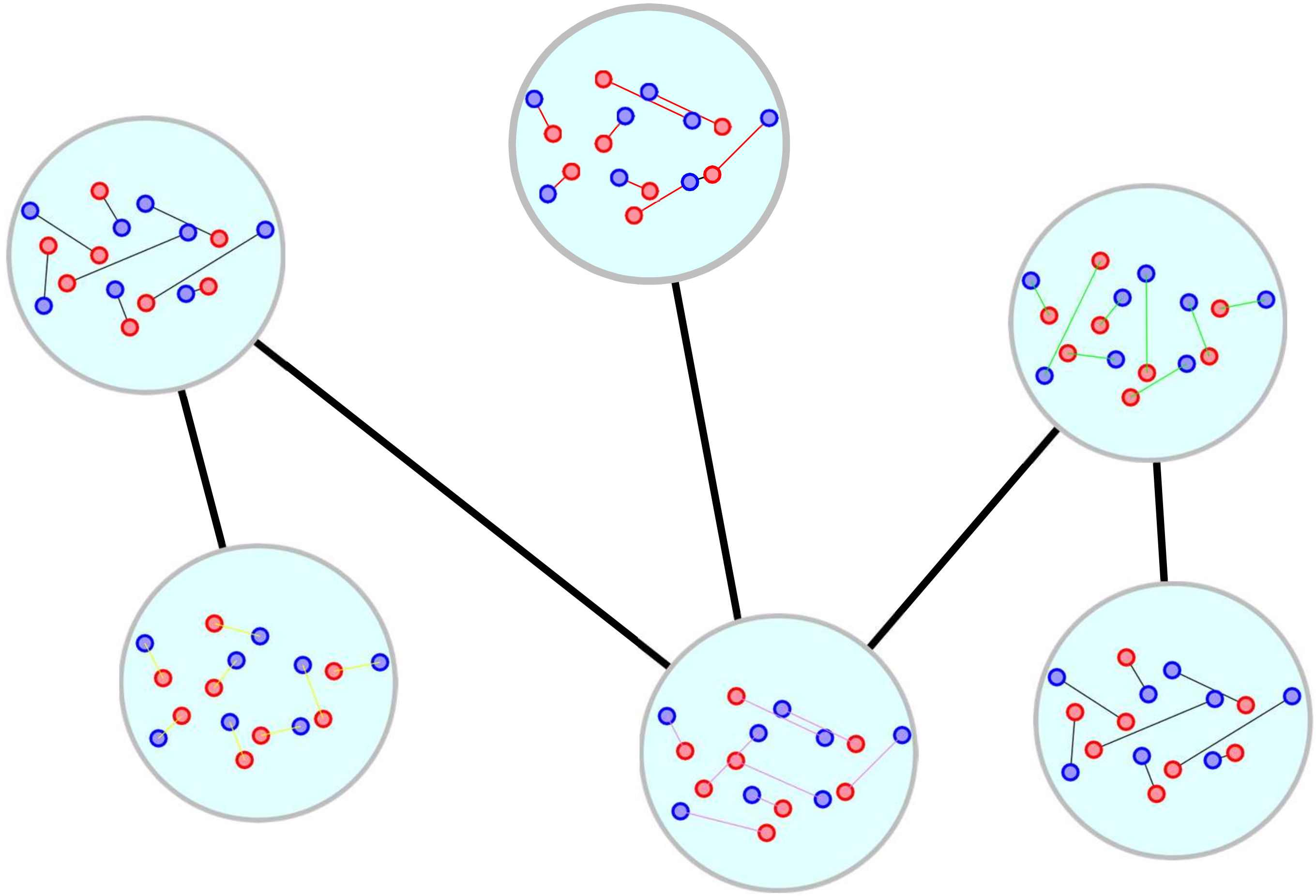
Bichromatic Matchings



Configuration Space of the Matches

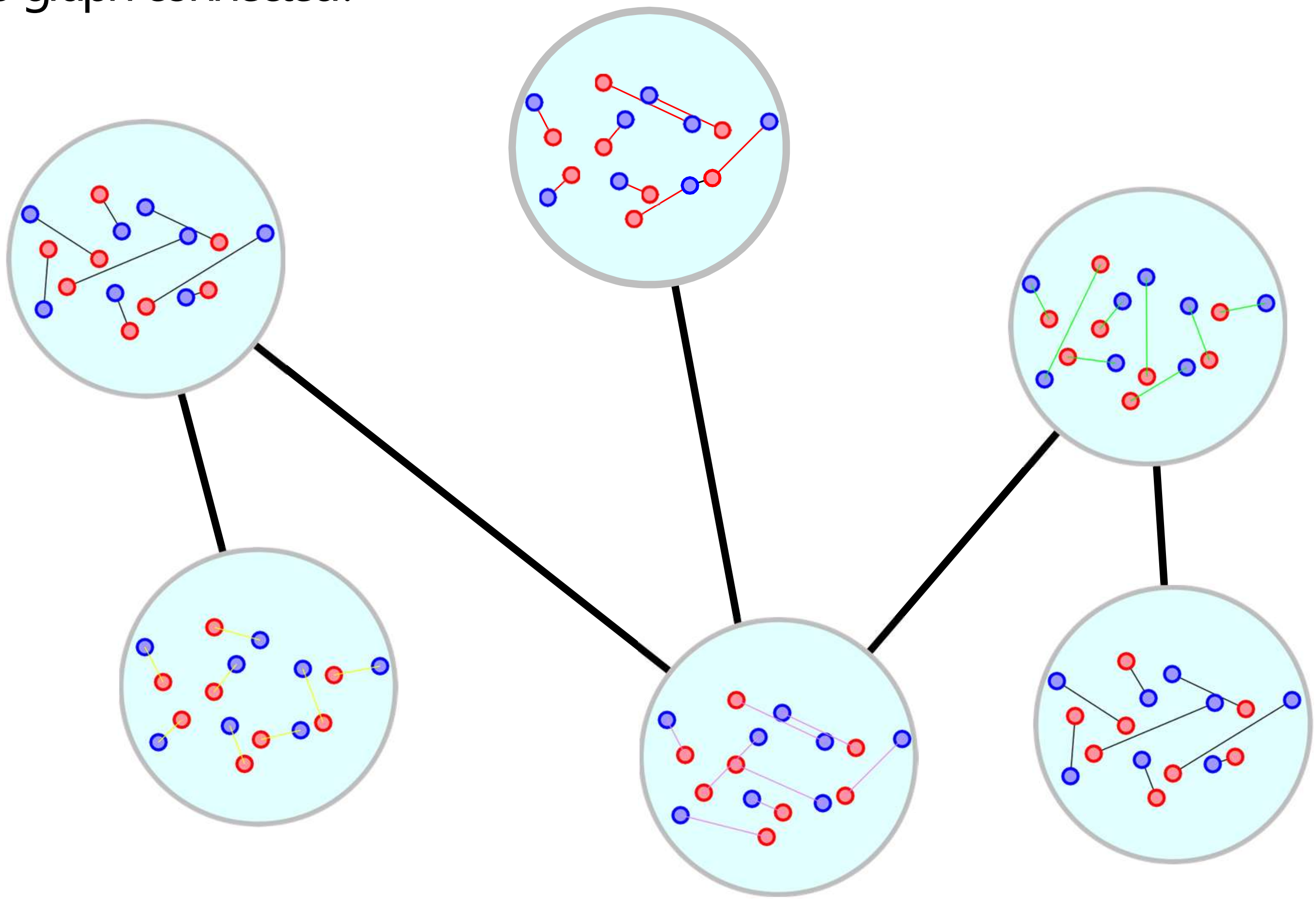


Configuration Space of the Matches



Bichromatic Compatible Matchings

Is this graph connected?



Bichromatic Compatible Matchings

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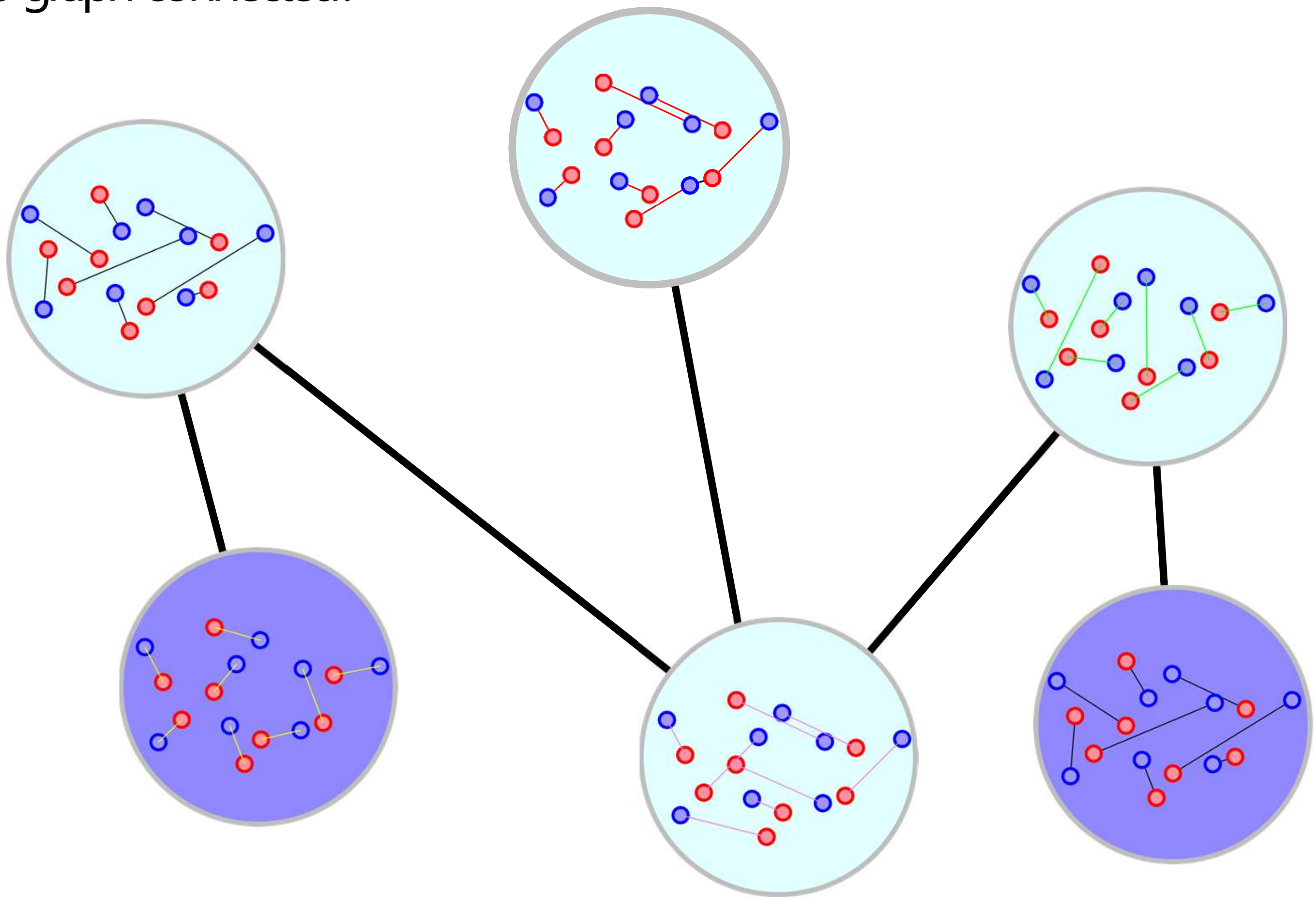
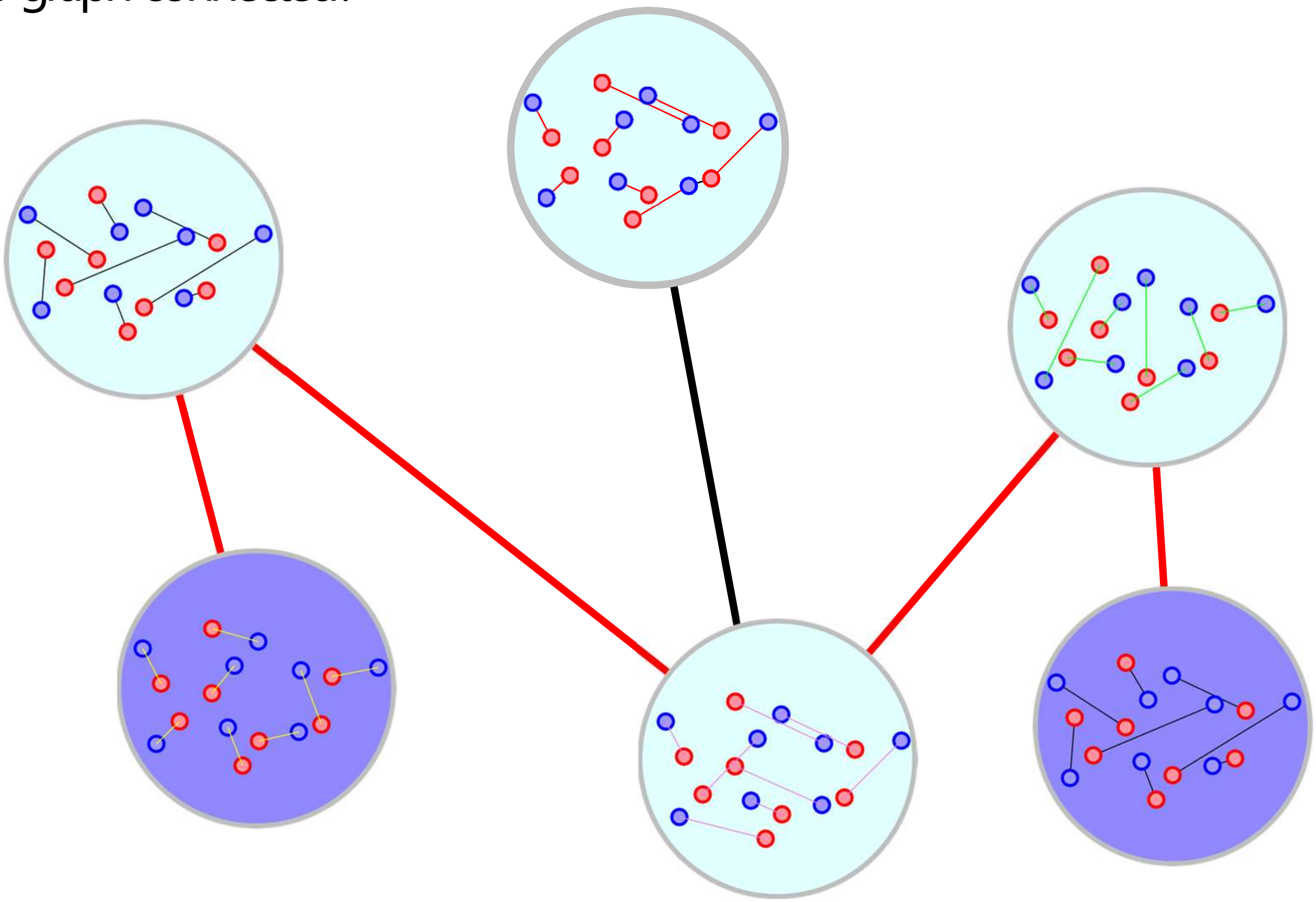


Bichromatic Compatible Matchings

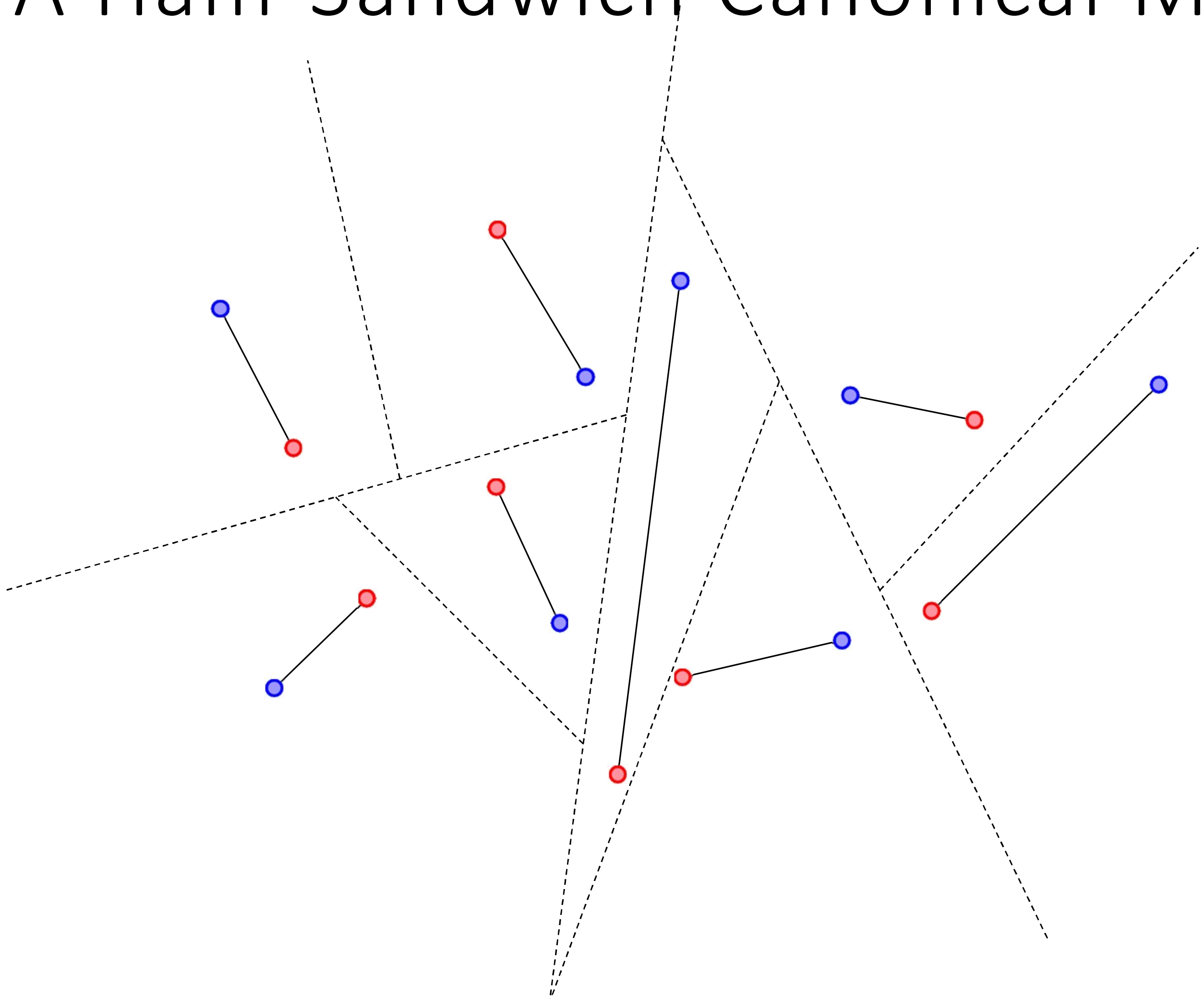
Is this graph connected?



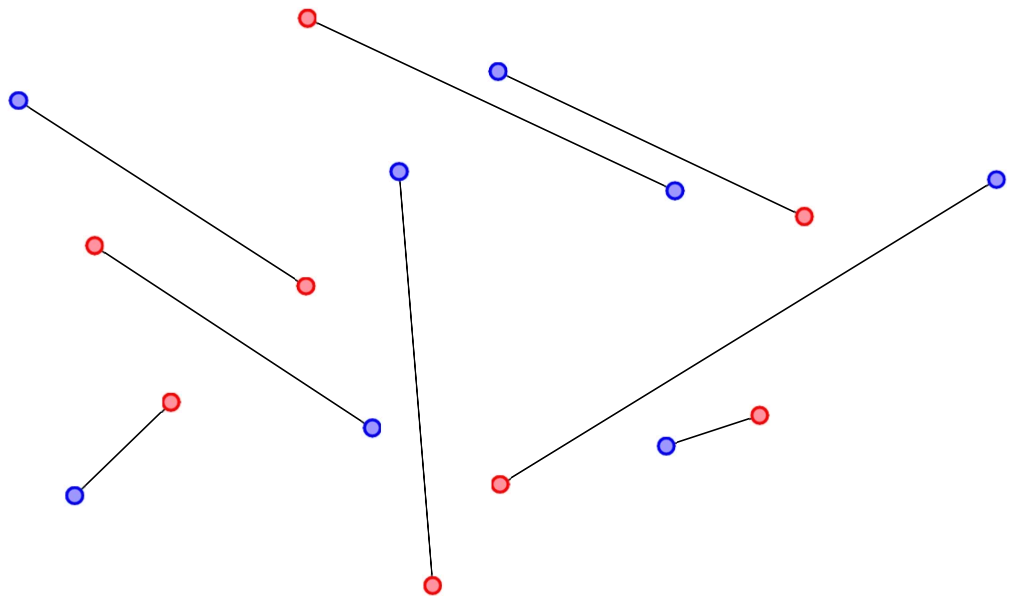
Bichromatic Compatible Matchings

It is possible to show that one can get from any BR-matching to the canonical Ham-Sandwich BR-matching.

A Ham-Sandwich Canonical Match

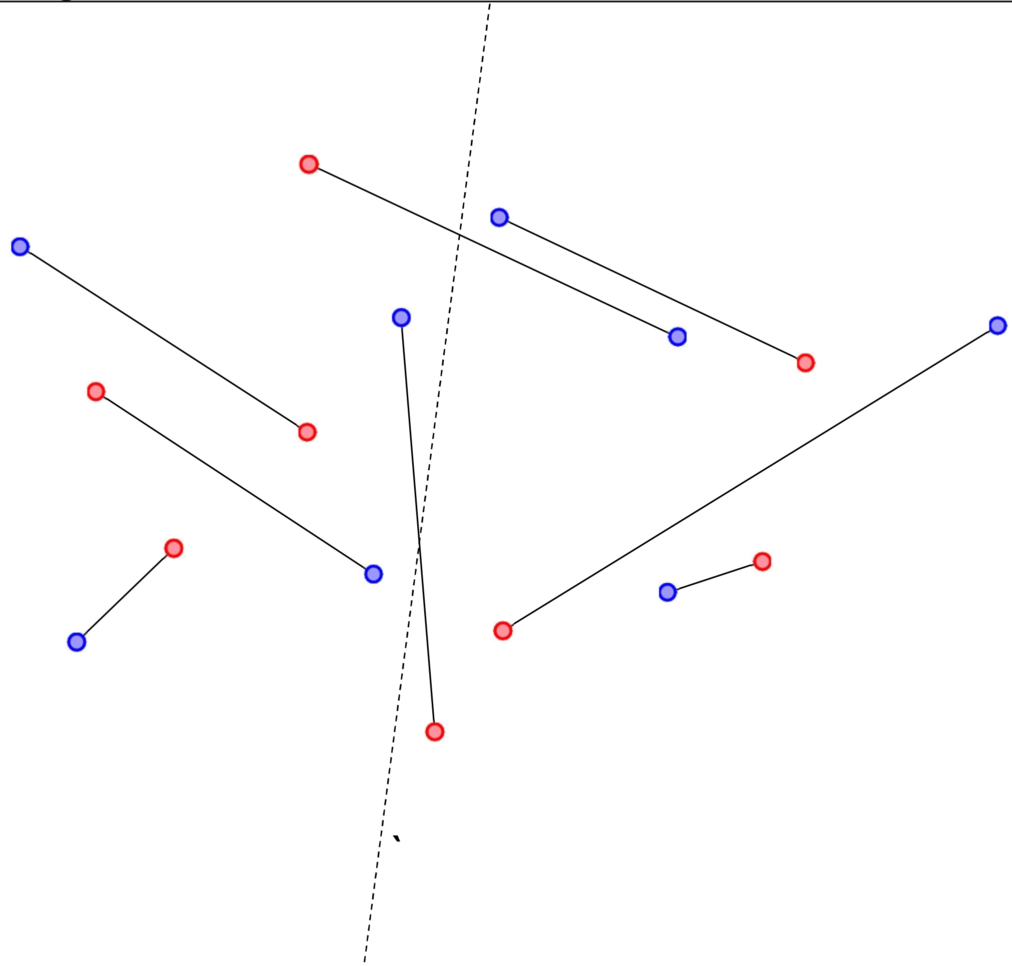


Original Match



One step:

Given a *BR*-matching M and a single ham-sandwich cut *C*, reconfigure M into a *BR*-matching that does not intersect *C*.



One Step:

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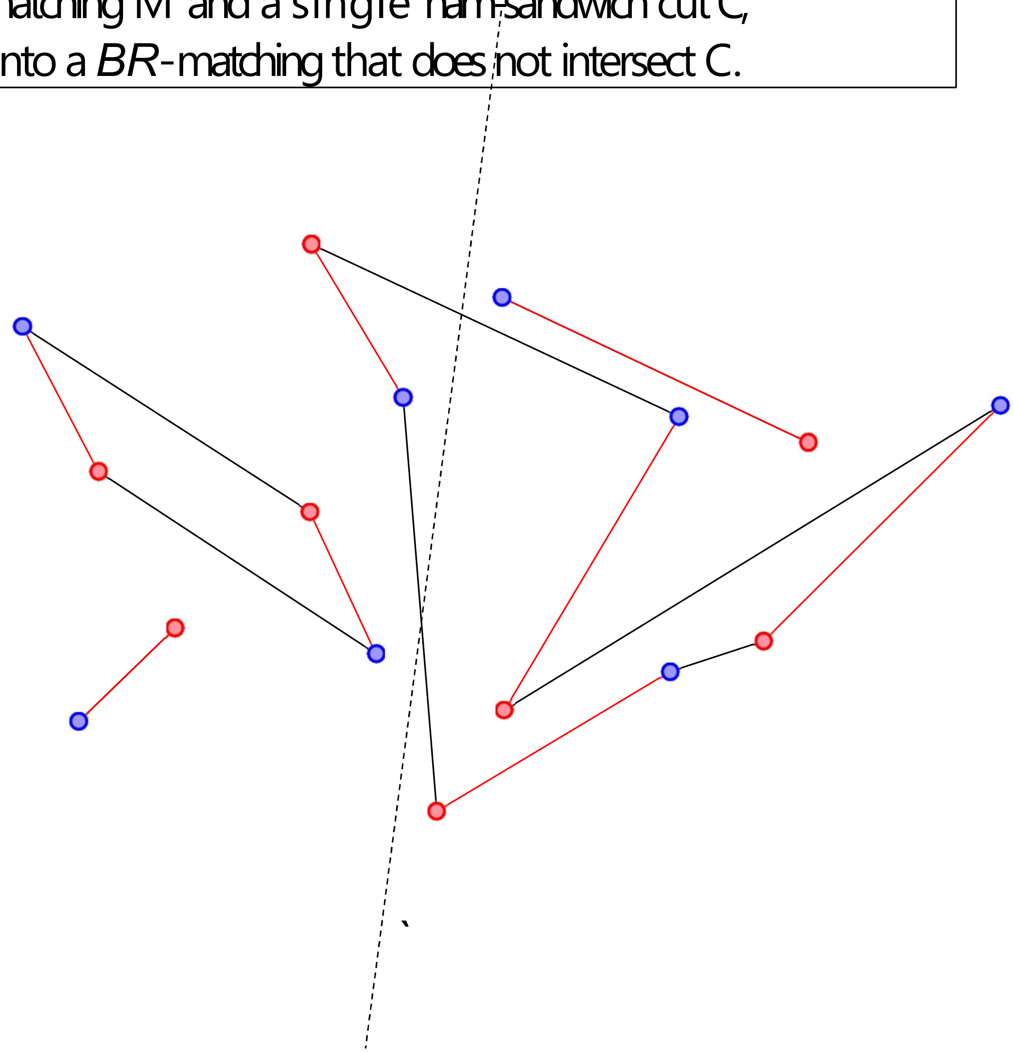
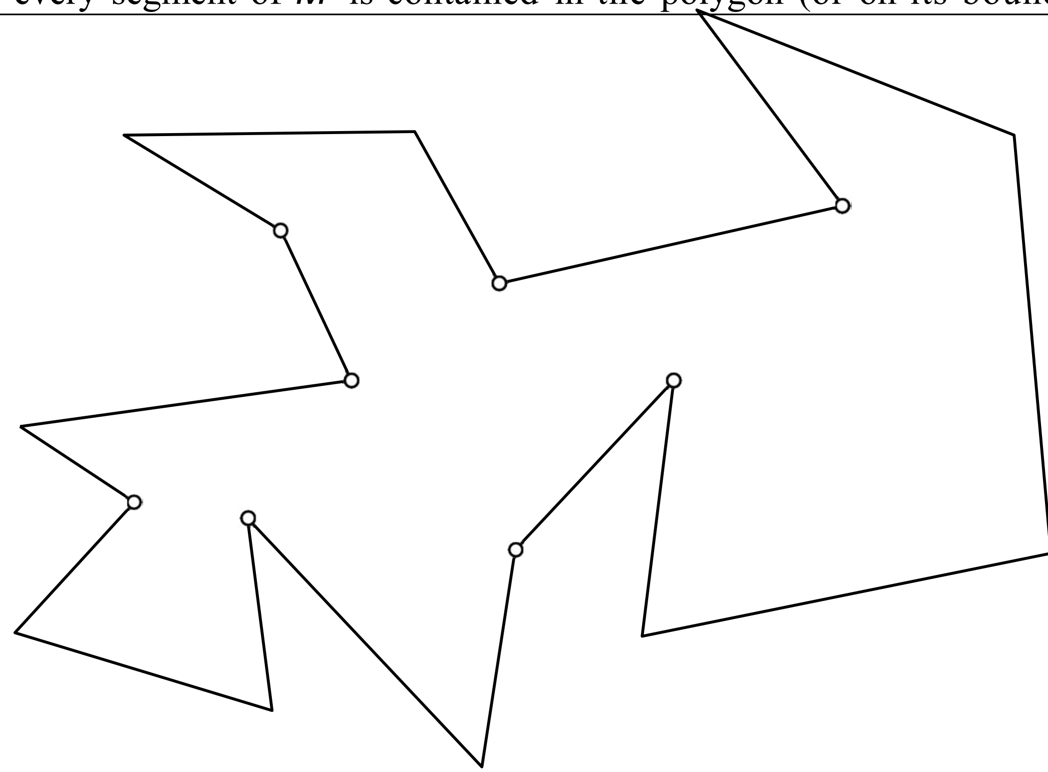


The Key Tool for doing this:

Theorem. (Abellanas *et. al* '08) Given a simple polygon with an even number of reflex vertices, there exists a perfect planar matching M of its reflex vertices where every segment of M is contained in the polygon (or on its boundary).



The Key Tool:

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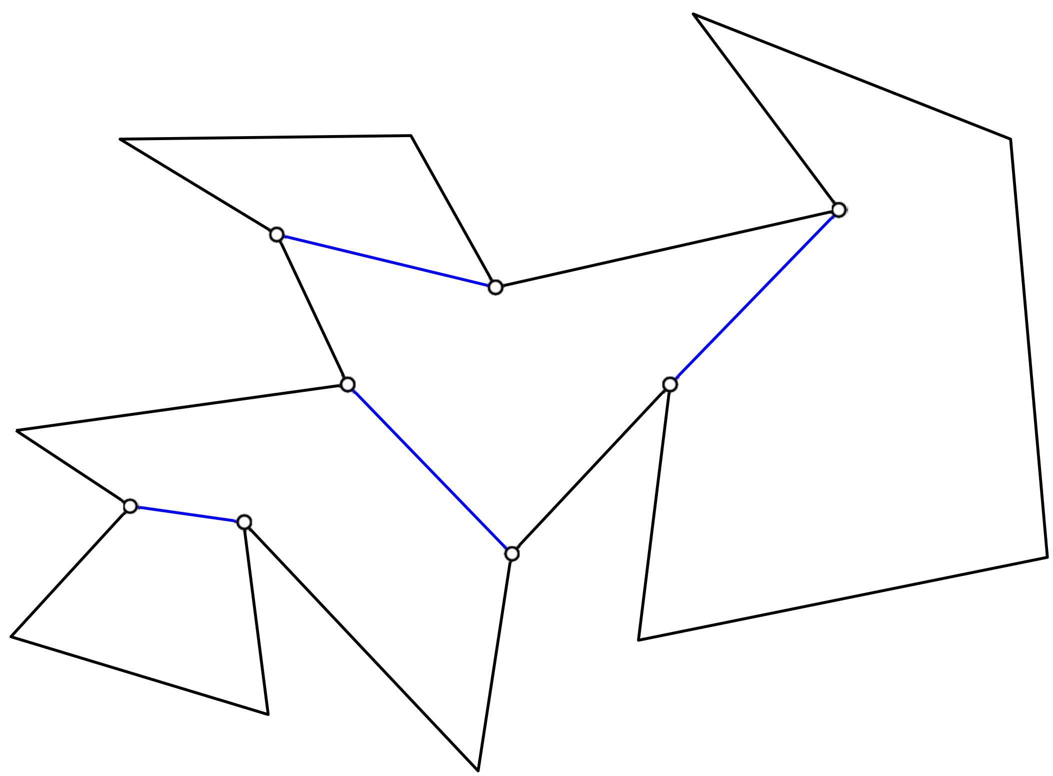
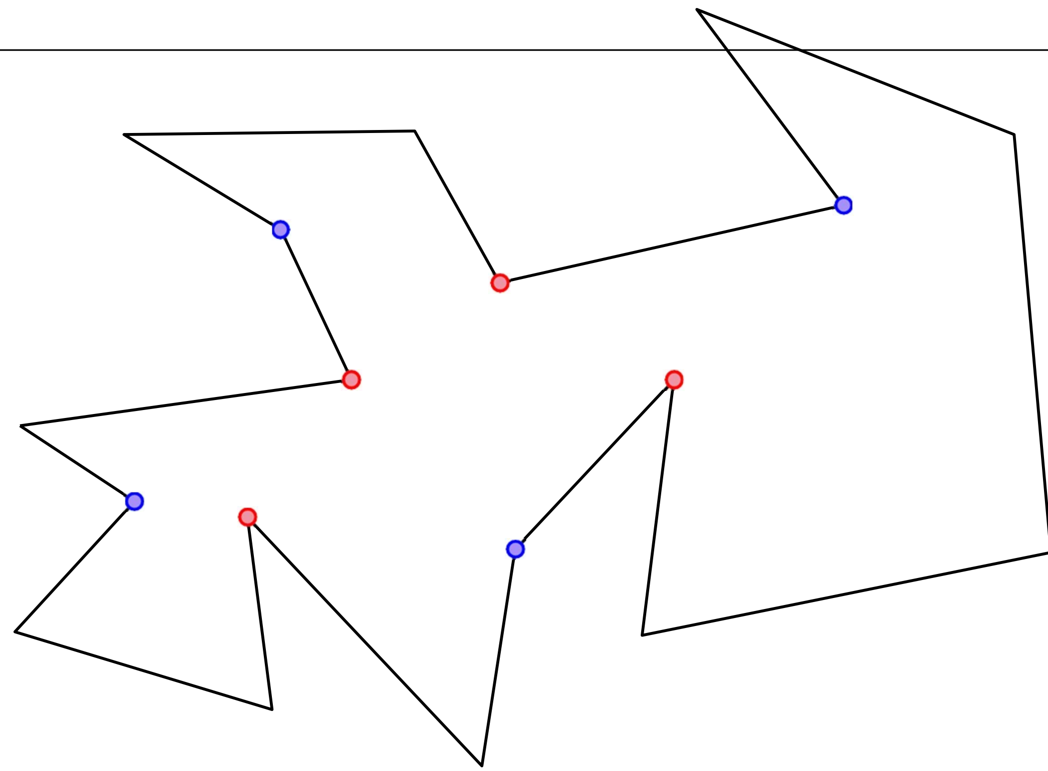


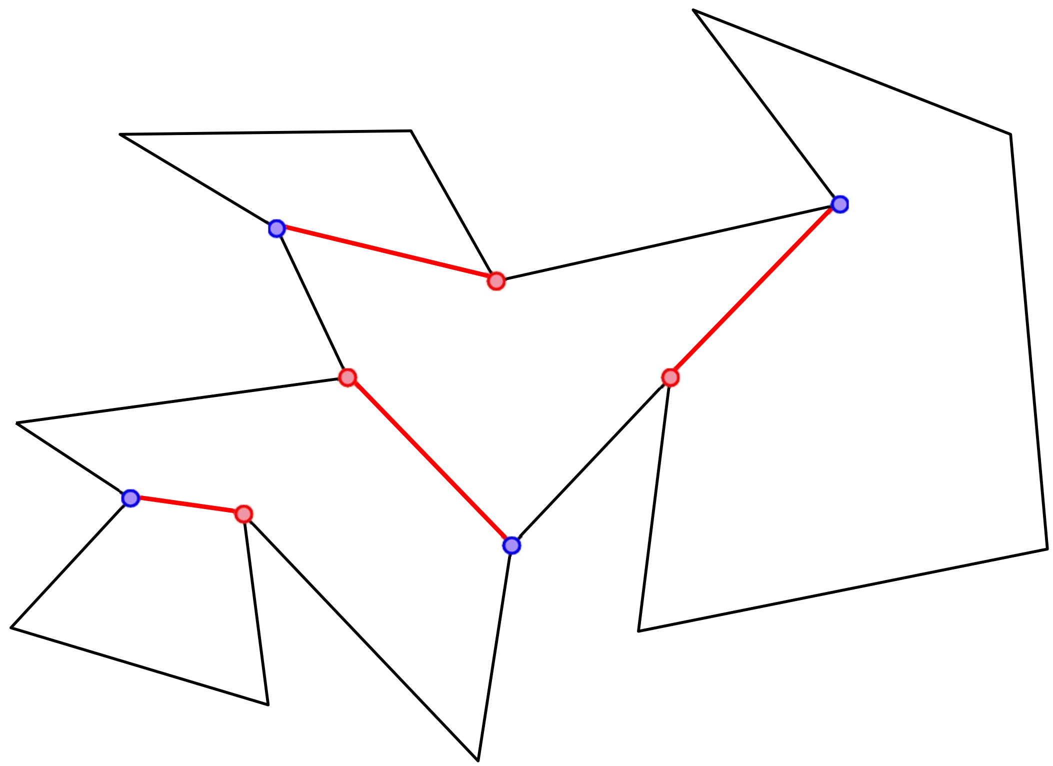
A Key Tool:

A simple polygon whose reflex vertices are colored either red or blue is *well-colored* if the sequence of reflex vertices along its boundary alternates in color.



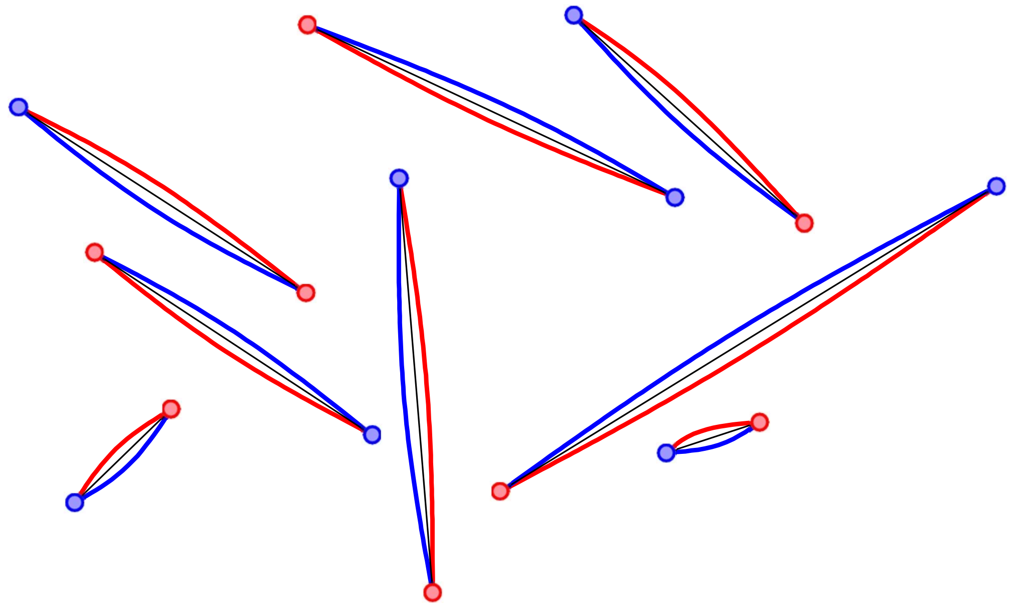
A Key Tool:

Polygon Matching Theorem. Given a well-colored polygon with an even number of reflex vertices, there exists a *BR*-matching *M* of its reflex vertices where every segment of *M* is contained in the polygon (or on its boundary).



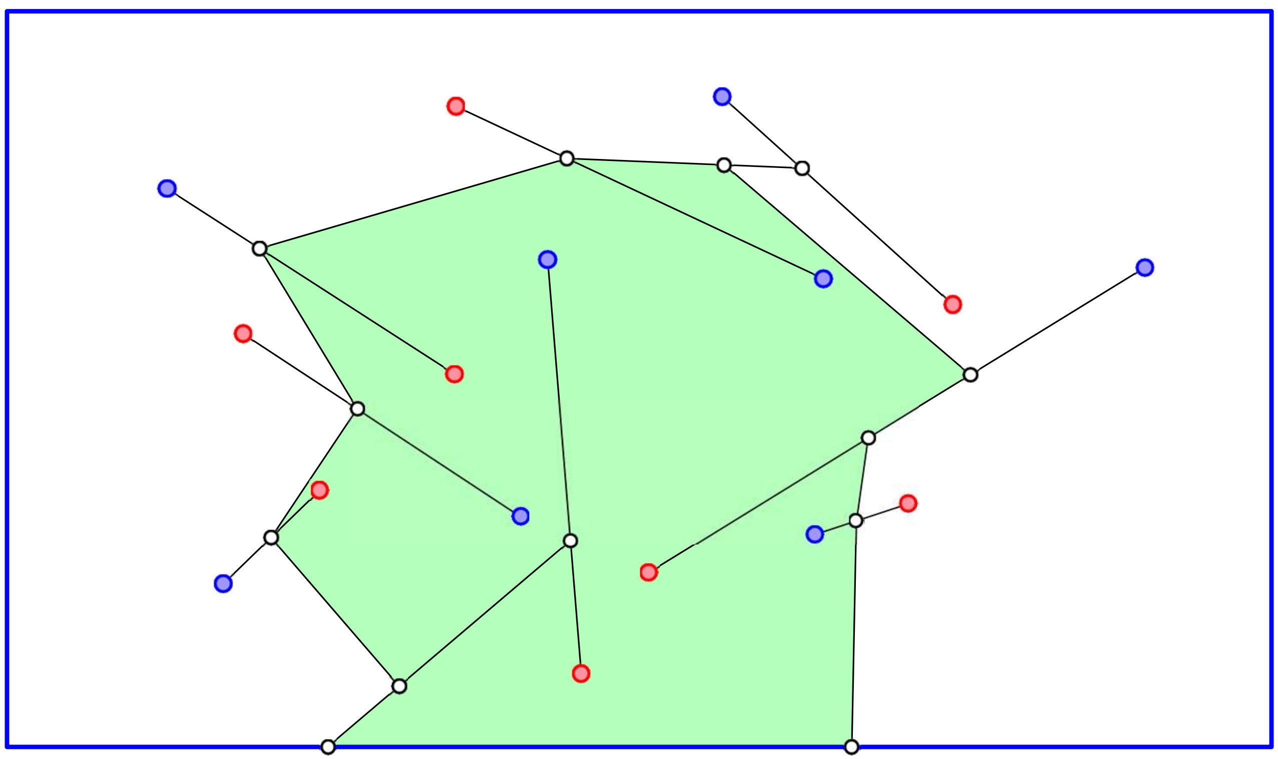
So how do we invoke that tool?

We consider that very segment has two sides, one colored blue and one colored red depending on the orientation of its endpoints. We connect the edges (red-red & blue-blue) in order to create a *well-colored* simple polygon.



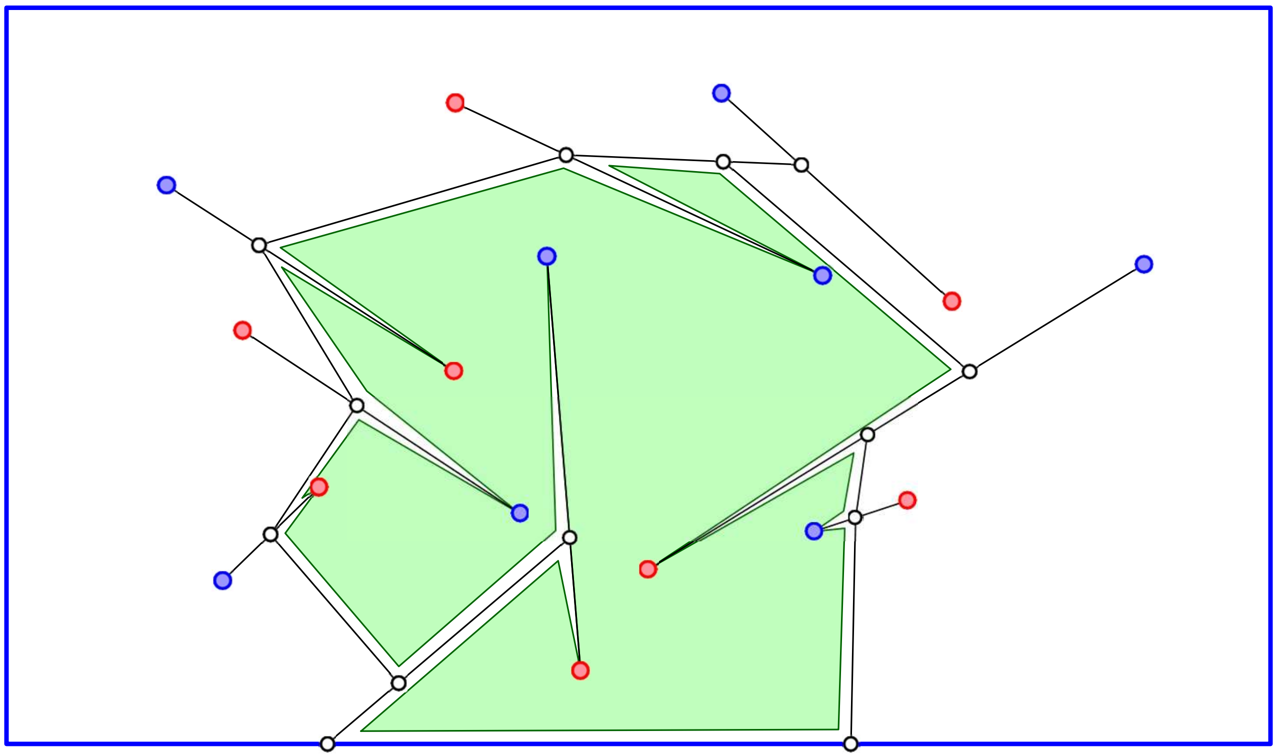
Creating a *well-colored* simple polygon

Each face of the obtained planar straight line graph is well-colored, i.e., the sequence of reflex vertices alternates in color along its boundary.



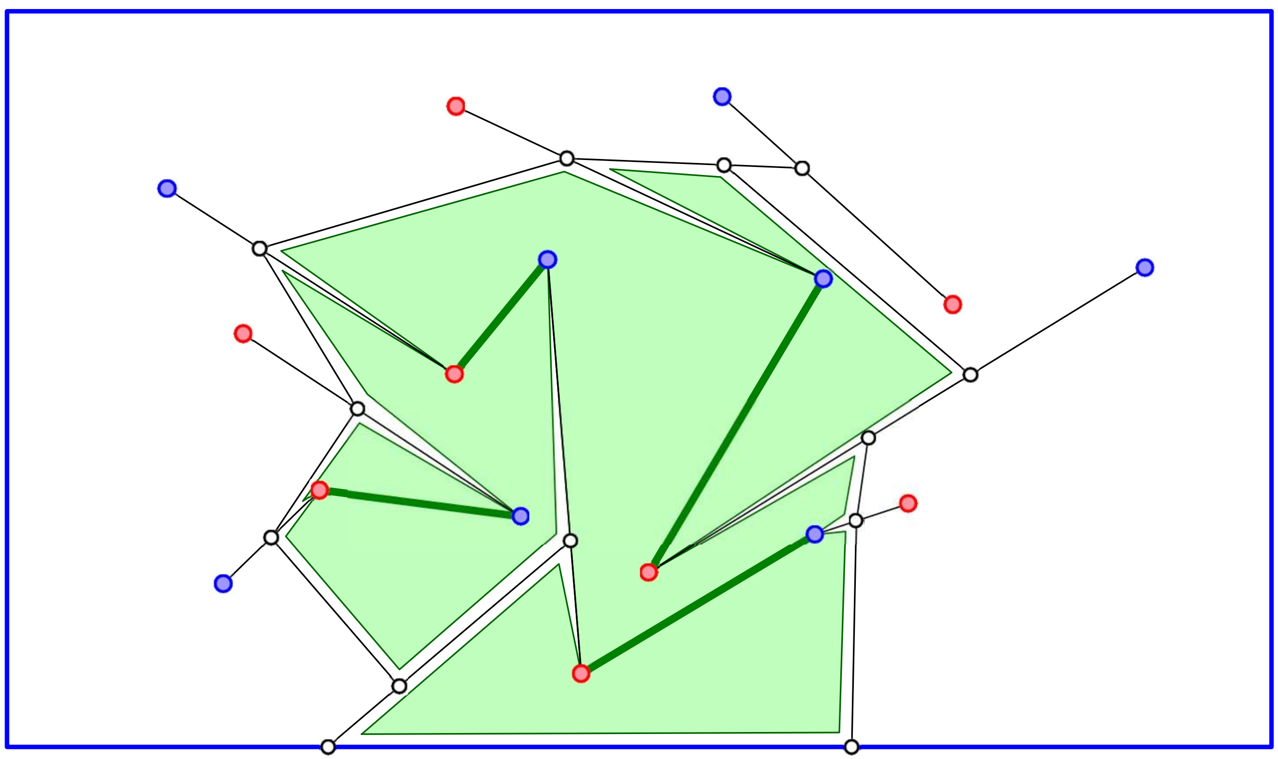
Creating a *well-colored* polygon

We can shrink every face (or open up the edges of the original match) to obtain a simple polygon that has the sameset of reflex vertices.



Creating the new match

We use the Polygon Matching Theorem to obtain a matching of the reflex vertices inside this polygon.



Bichromatic Compatible Matchings

- The configuration graph is connected: using the tool described above, one can move from any BR-matching to another BR-matching through a sequence of compatible matches.
  - Aloupis, Barba, Langerman, Souvaine, *Computational Geometry*, 2015
- The diameter of the configuration graph is linear: subsequently, a different method verified that one can move from any BR-matching to another other BR-matching through a sequence of compatible matches of length at most $2n$.
  - Aichholzer, Barba, Hack, Pilz, Vogtenhuber, *Computational Geometry,* 2018

Summary

- Across the examples so far:

  --- concept of canonical configuaration

  --- basic steps for getting there

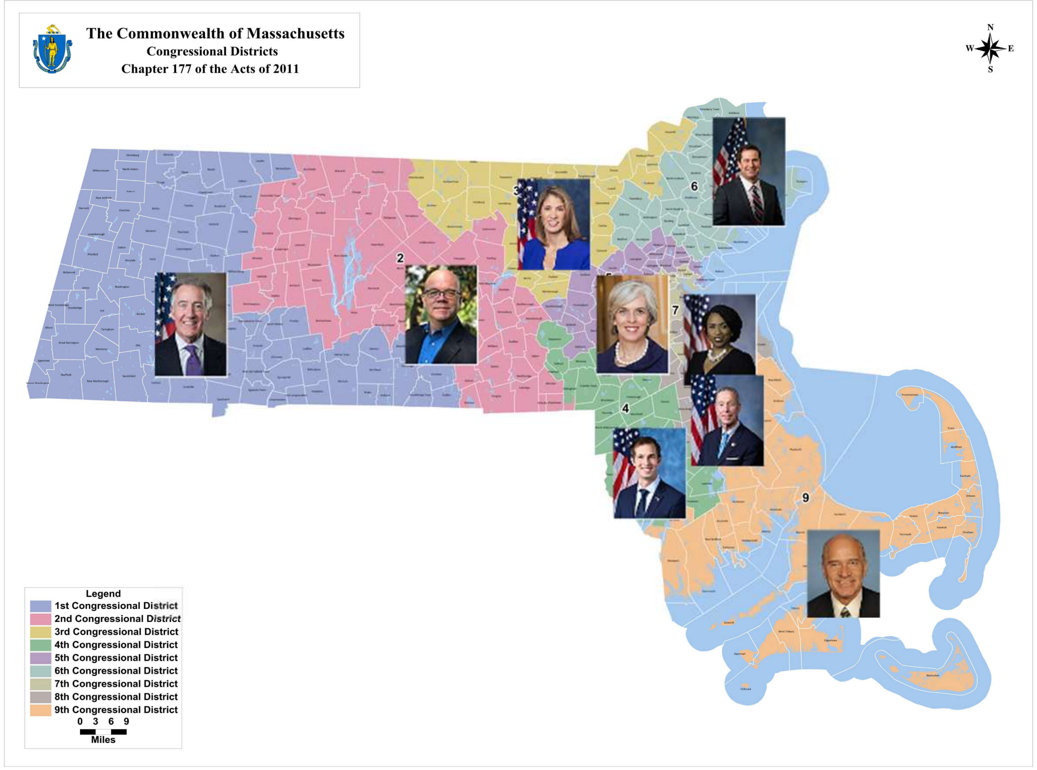
  --- concept of using “thin channels”
- Here are a few references for some of the material we have seen

Selected References

1. C. L. Lawson. Transforming triangulations. *Discrete Mathematics*, 3(4):365–372, 1972.
2. F. Hurtado, M. Noy, and J. Urrutia. Flipping Edges in Triangulations. *Discrete & Computational Geometry,* 22:333–346, 1999.
3. J. Galtier, F. Hurtado, M. Noy, S. Perennes, and J. Urrutia. Simultaneous Edge Flipping in Triangulations. *Int. J. Comput. Geometry Appl.,* 13:113-133, 2003.
4. D. Souvaine, C. Tóth, and A. Winslow. Simultaneously Flippable Edges in Triangulations. C*omputational Geometry, Lecture Notes in Computer Science*, 7579:138-145, 2011.
5. M. Ishaque, D. Souvaine, and C. Tóth. Disjoint Compatible Geometric Matchings. *Discrete & Computational Geometry*, 49(1):125-134, 2011.
6. G. Aloupis, L. Barba, S. Langerman, D. Souvaine. Bichromatic compatible matchings. *Symposium on Computational Geometry*, 29:267-276, 2013.
7. O. Aichholzer, L. Barba, T. Hackl, A. Pilz, B. Vogtenhuber. Linear transformation distance for bichromatic matchings. *Symposium on Computational Geometry*, 30:154-162, 2014.
8. A. Lubiw, Pathak. Flip distance between two triangulations of a point set is NP-complete. *Computational Geometry 11 (4), 52.* 2015
9. N. Mishimura. Introduction to Reconfiguration. *Algorithms* 49*,* 17–23*.* 2018

Gerrymandering

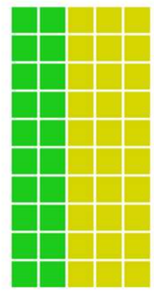
Analysis



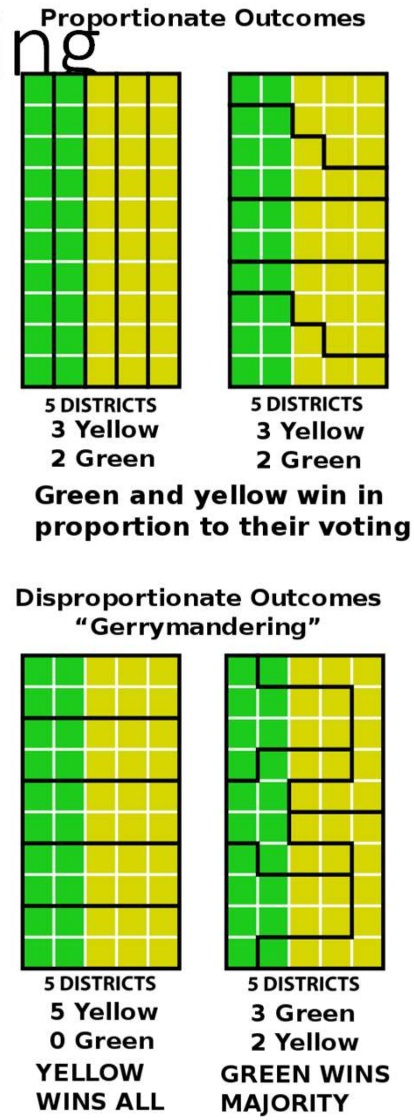
"Gerrymandering is a practice intended to establish an unfair political advantage for a particular party or group by manipulating district boundaries"

Analysis of gerrymandering

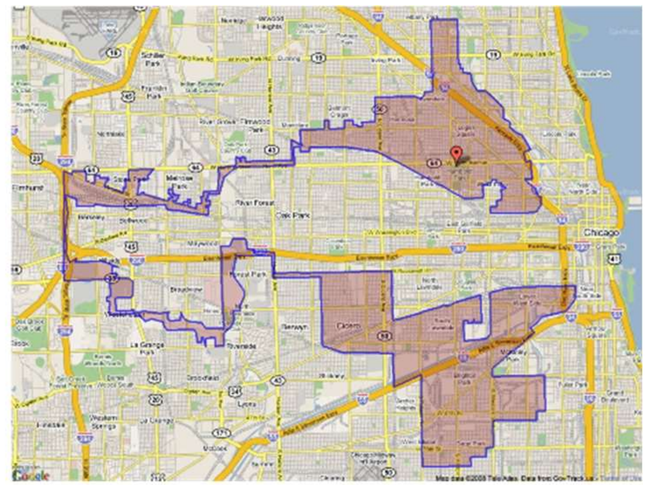
50 Precincts
60% Yellow
40% Green



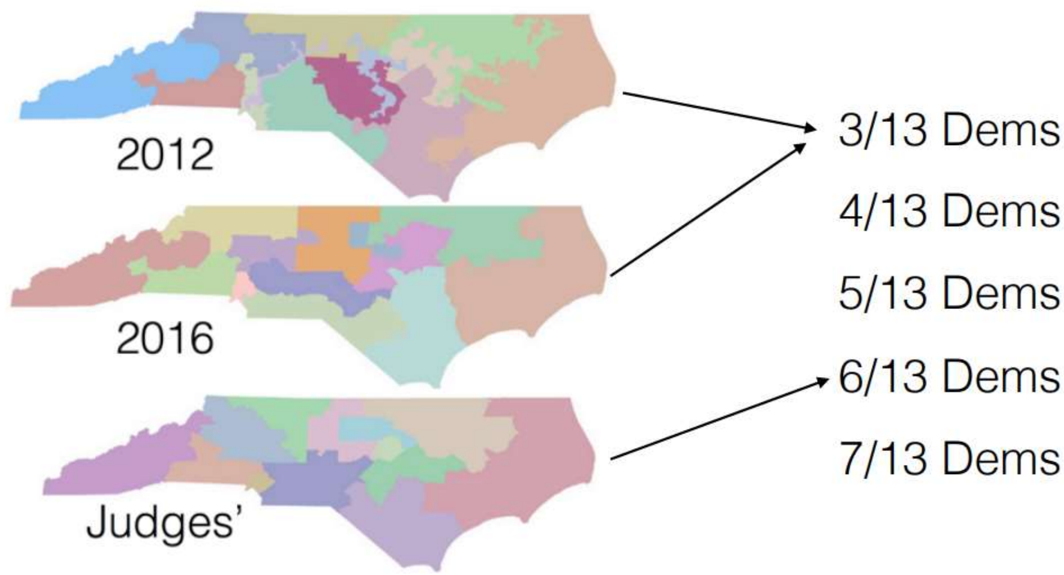
50 Precincts are to be apportioned into 5 districts, 10 precincts each district.



Analysis of gerrymandering



Analysis of gerrymandering

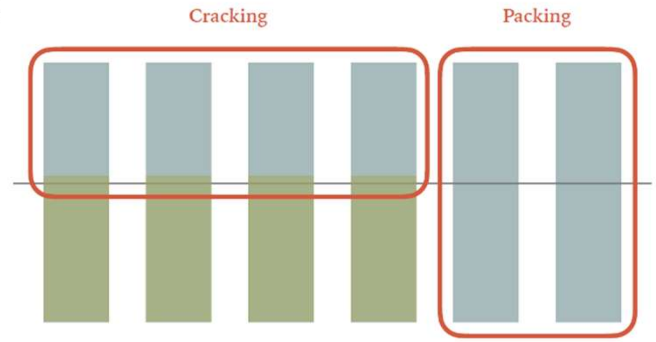


Credit: Mattingly et al

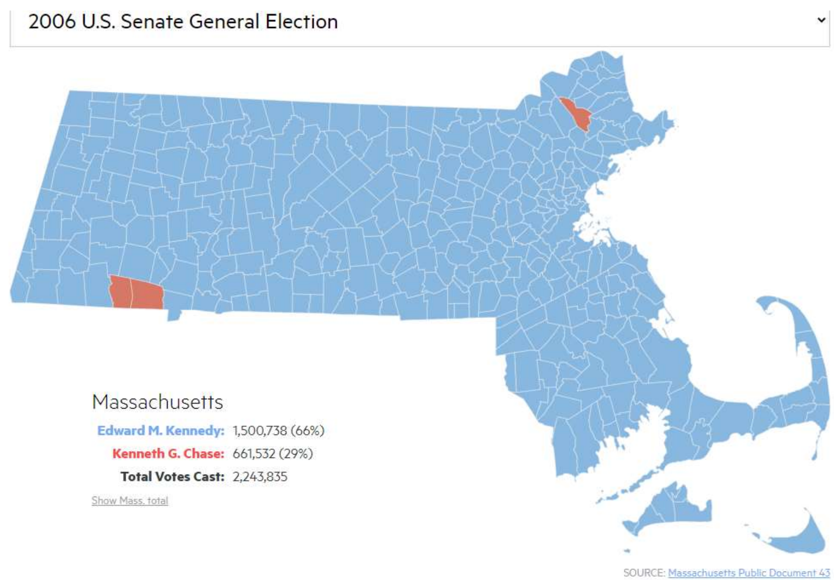
Analysis of gerrymandering

Efficiency gap – based on "wasted" votes, w_A, and w_B

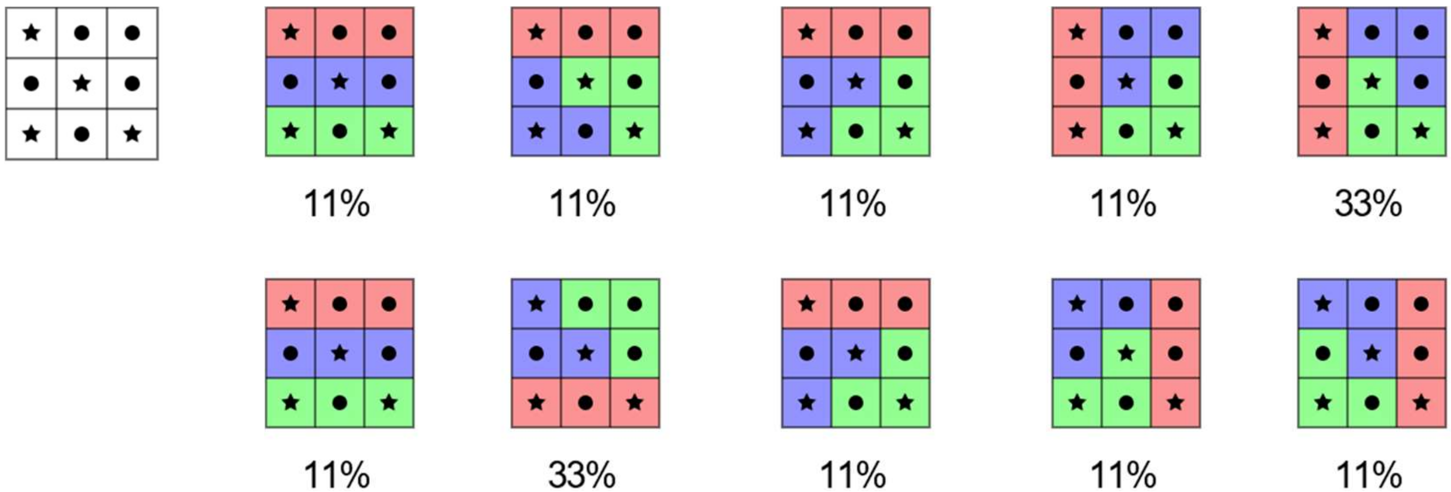
$$\frac{w_A - w_B}{total}$$

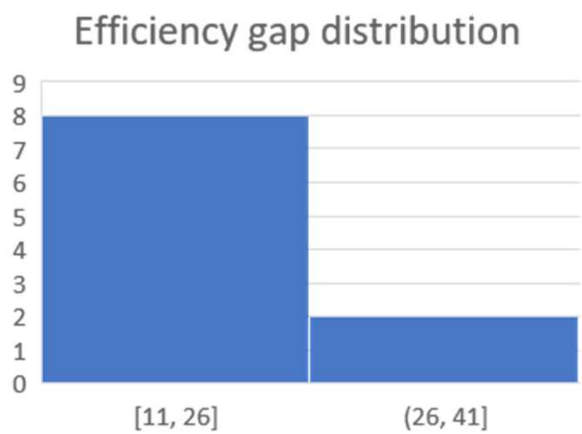


Efficient majorities for you = Packing and Cracking for your opponents



Analysis of gerrymandering

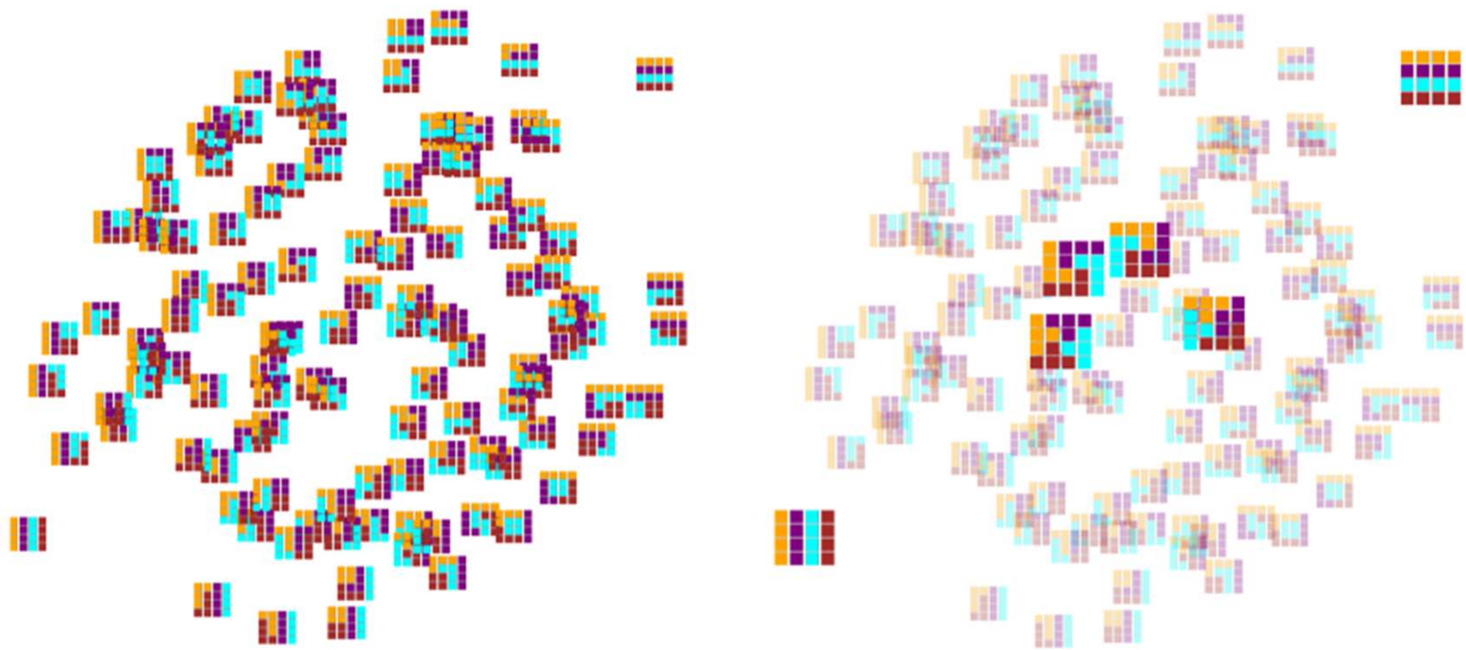




Analysis of gerrymandering

Ensemble-based approach to districting plan analysis
M. Duchin, “Geometry versus Gerrymandering”, Scientific American(2018)

- Outlier analysis
- Sample the space using Markov chains



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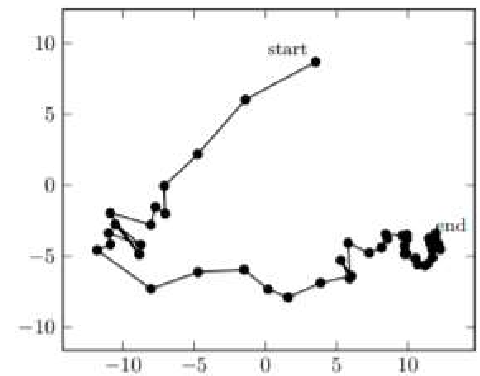


Fig. 12: A Markov chain-generated walk in the space of partitions with simulated annealing.

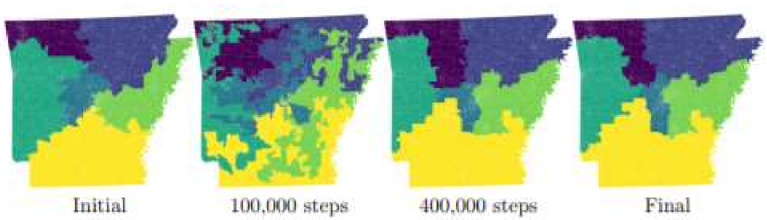
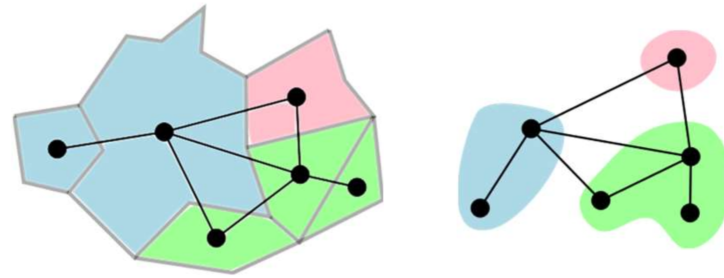
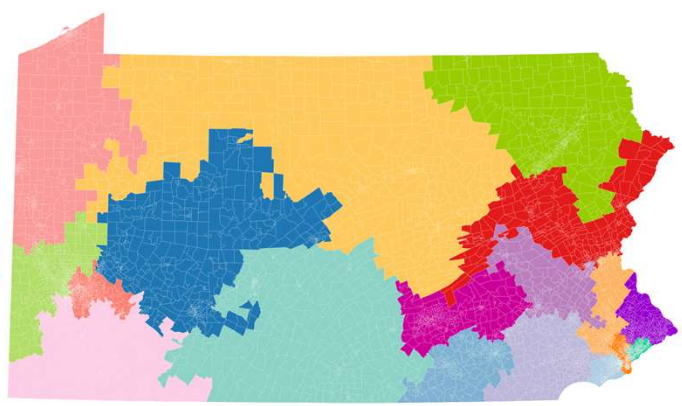


Fig. 11: Snapshots of a simulated annealing Markov chain on Arkansas districts.

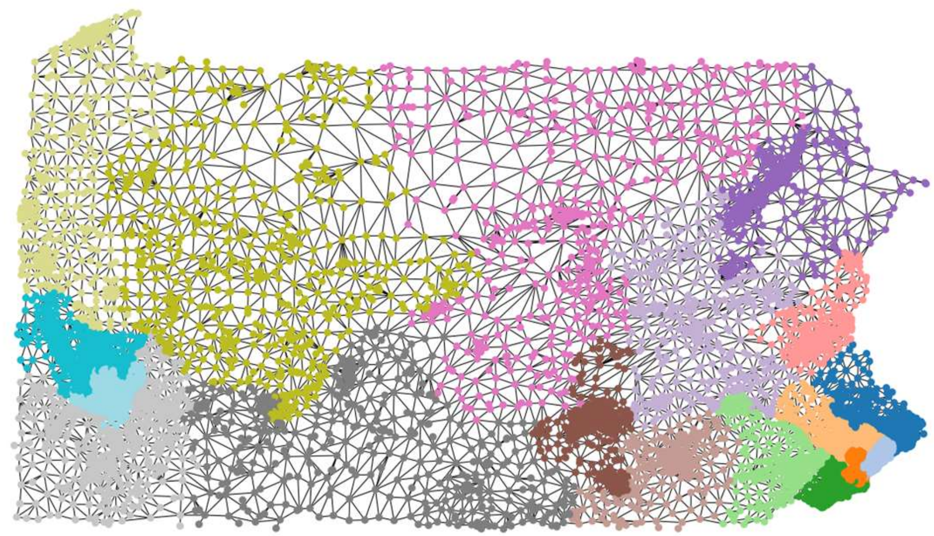
Graph Setting



- k: Number of **districts** (connected subgraphs)
- District map = Connected k-partition of adjacency graph

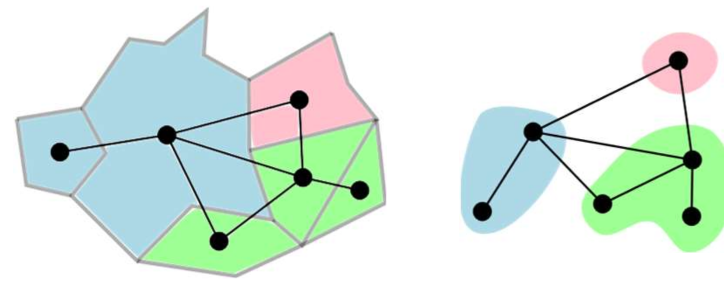


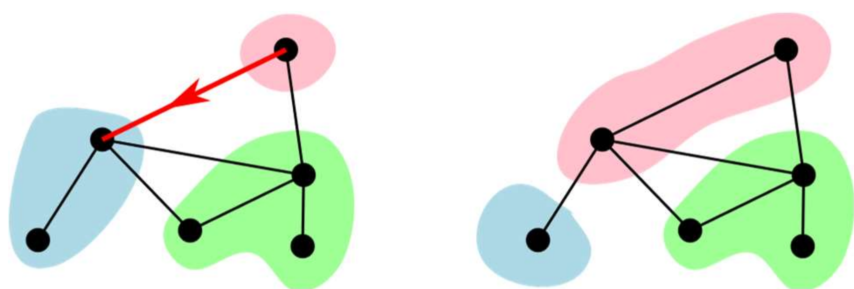
Pennsylvania Congressional districts



Definitions

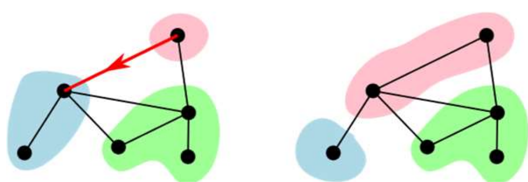
- Taking a step:
  - District map = Connected graph partition
  - Step = switch/flip



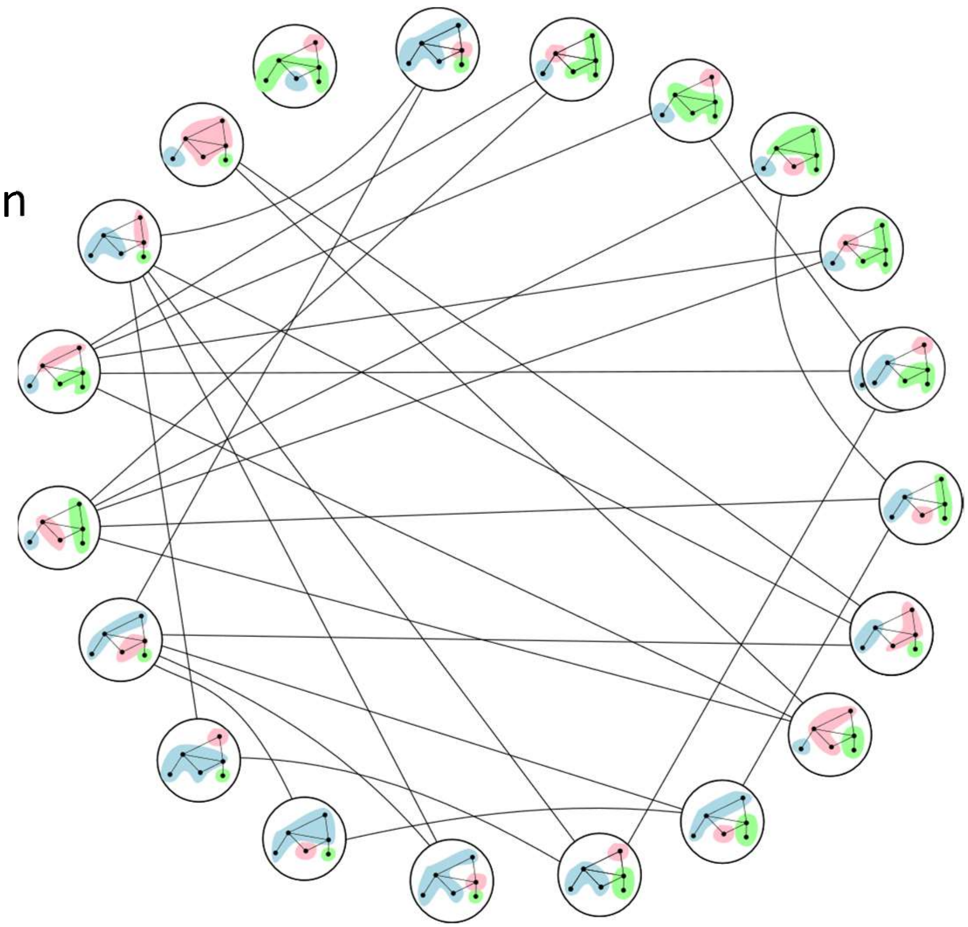


Definitions

- Taking a step:
  - District map = Connected graph partition
  - Step = flip

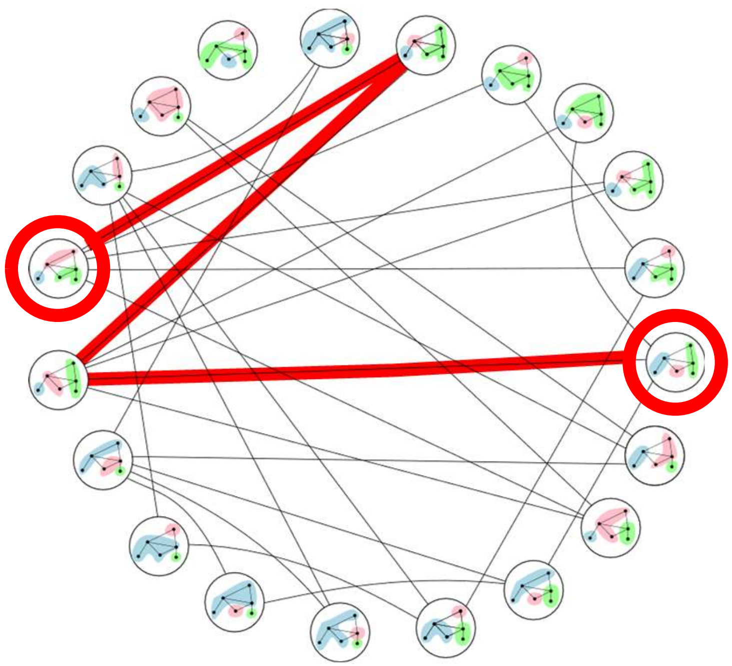


- Configuration space



Problem definition

- Given two k-district maps A and B, find a path between A and B in the configuration space via flips.



Reconfiguration of Connected Graph Partitions*

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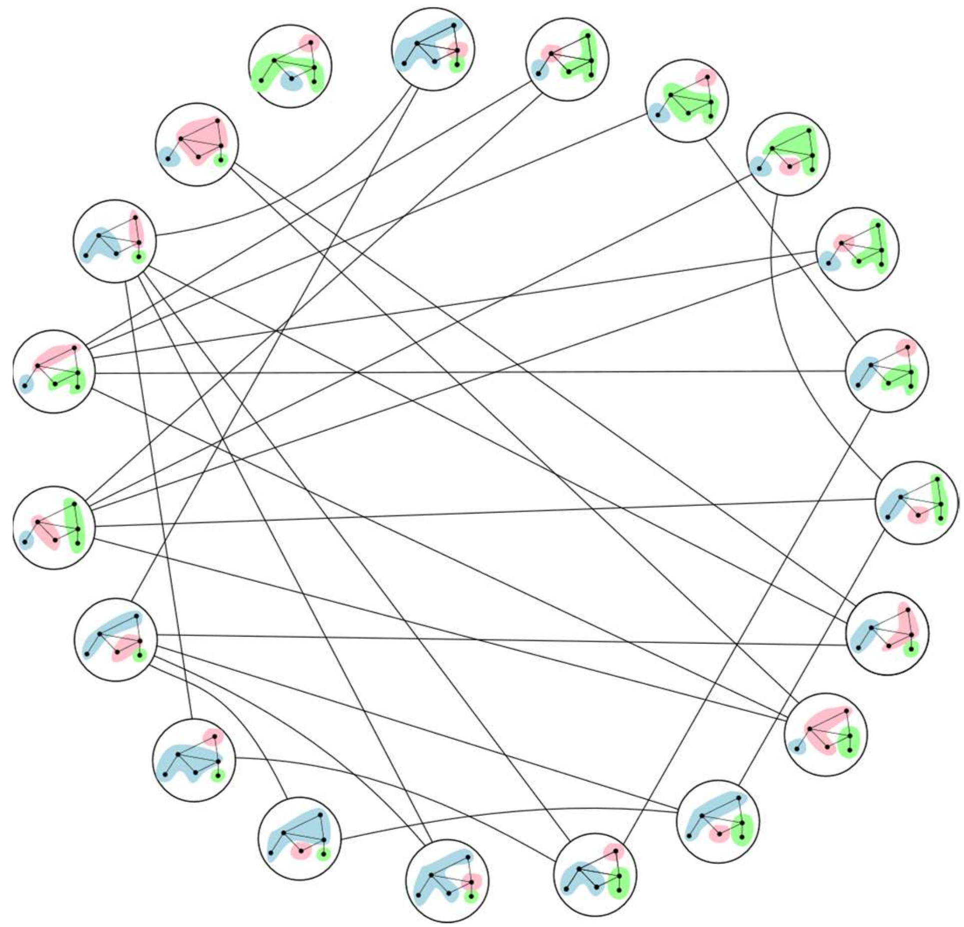
Christopher Meierfrankenfeld[†] Michael J. Munje[‡] Diane L. Souvaine[†]
Michael Thramann[†] Csaba D. Tóth[§†]

March 1, 2019

Abstract

Motivated by recent computational models for redistricting and detection of gerrymandering, we study the following problem on graph partitions. Given a graph G and an integer $k \geq 1$, a **k-district map** of G is a partition of $V(G)$ into k nonempty subsets, called **districts**, each of which induces a connected subgraph of G. A **switch** is an operation that modifies a k-district map by reassigning a subset of vertices from one district to an adjacent district; a **1-switch** is a switch that moves a single vertex. We study the connectivity of the configuration space of all k-district maps of a graph G under 1-switch operations. We give a combinatorial characterization for the connectedness of this space that can be tested efficiently. We prove that it is NP-complete to decide whether there exists a sequence of 1-switches that takes a given k-district map into another; and NP-hard to find the shortest such sequence (even if a sequence of polynomial length is known to exist). We also present efficient algorithms for computing a sequence of 1-switches that takes a given k-district map into another when the space is connected, and show that these algorithms perform a worst-case optimal number of switches up to constant factors.



*Research supported in part by NSF CCF-1422311, CCF-1423615, and the Science Without Borders program.
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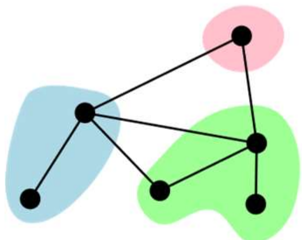
Our Results

- Combinatorial characterization for the connectedness of this space that can be tested efficiently.
  - Constructive proof: worst-case optimal algorithm if connected.

- Hardness results:
  - PSPACE-complete to decide whether there exists a sequence of switches that takes a given k-district map into another; and
  - NP-hard to find the shortest such sequence (even if a sequence of polynomial length is known to exist)

Definitions

- Taking a step:
  - District map = Connected graph partition
  - Step = ~~flip~~ Recombination



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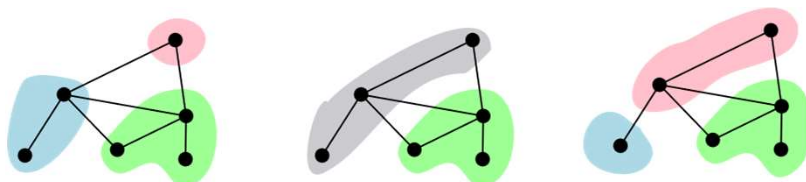
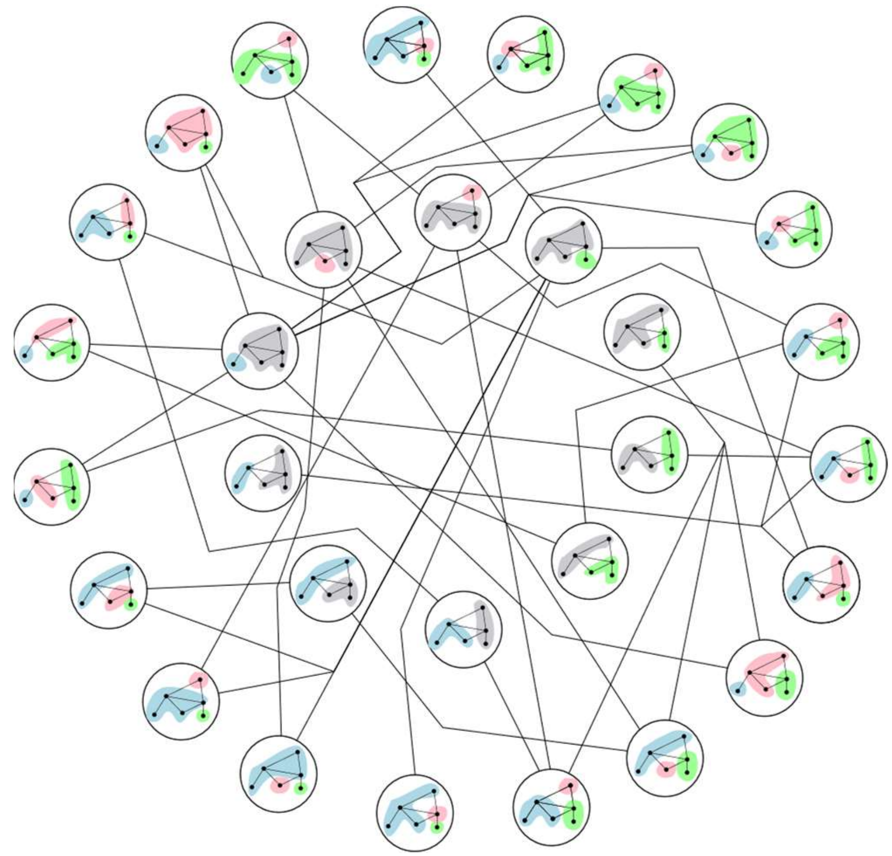
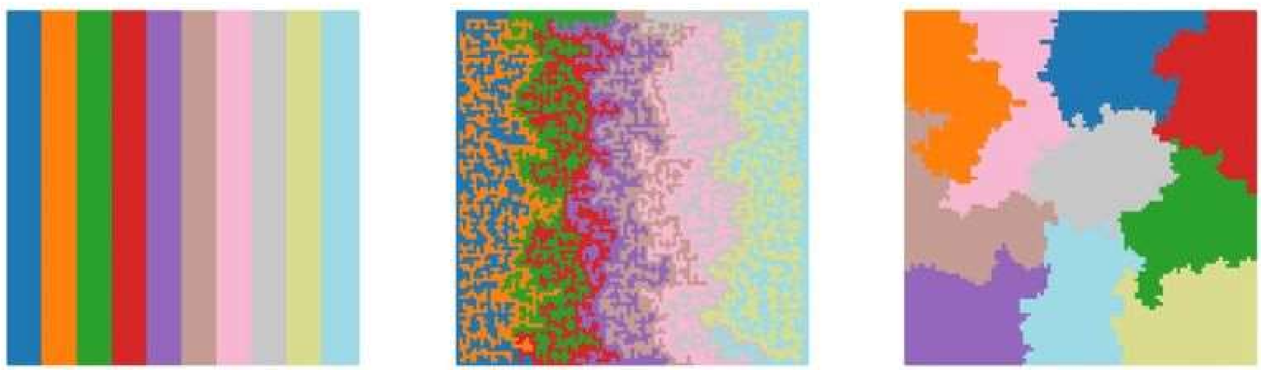


- Configuration space:
  - Supergraph of switch graph



Flip/Recomb comparison



Initial Partition | 1,000,000 Flip steps | 100 ReCom steps

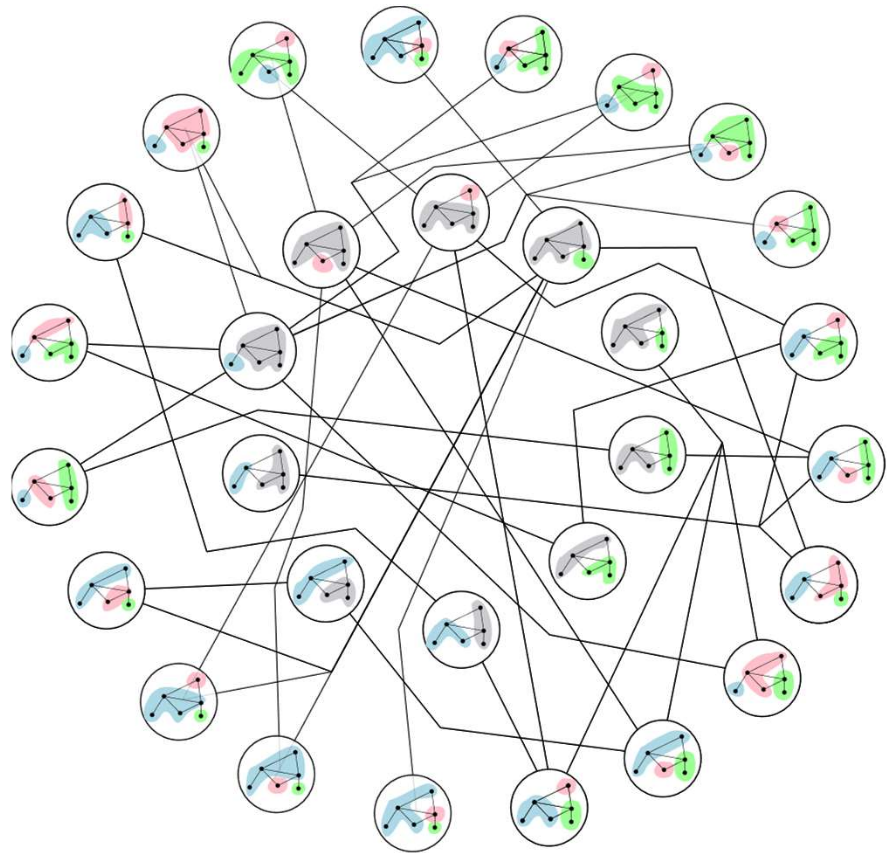
Definitions

- Slack: deviation from balance

Size of a district is in $\left[\max\left\{1, \frac{n}{k} - s\right\}, \frac{n}{k} + s\right]$

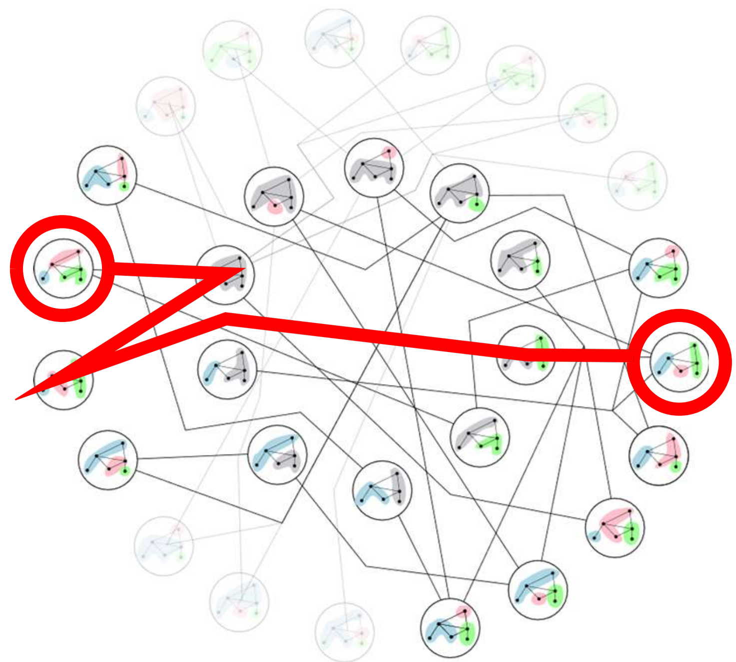
$$s = 1$$

(k, s)-BCP: Balanced Connected
k-Partition with slack s
(known to be NP-hard even for grid graphs)



Problem definition

- Given two k-BCPs A and B, and slack s, find a path between A and B in the (k, s)-BCP configuration space via recombinations.



Our Results

- For unbounded slack ($s = n - k$), the configuration space is connected, and its diameter is at most $6(k-1)$.

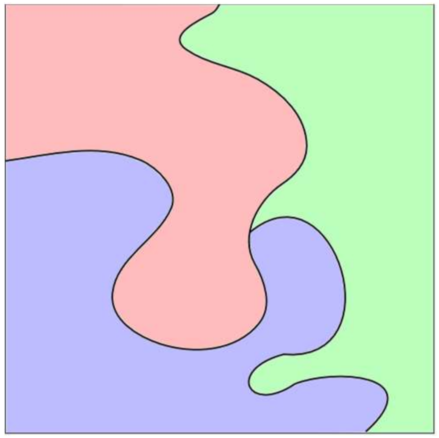
- If the underlying graph is Hamiltonian, and $s \geq \frac{n}{k}$ then the configuration space is connected, and its diameter is $O(kn)$.

- Infinite family of negative instances (also Hamiltonian) for $s \leq \frac{n}{3k}$.

- Various PSPACE-completeness results for "small" k, even when $s \in O(n^{1-\varepsilon})$, for any constant $0 < \varepsilon \leq 1$.

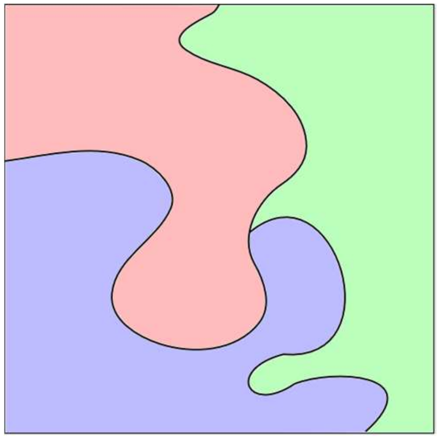
Continuous domain (motivation)

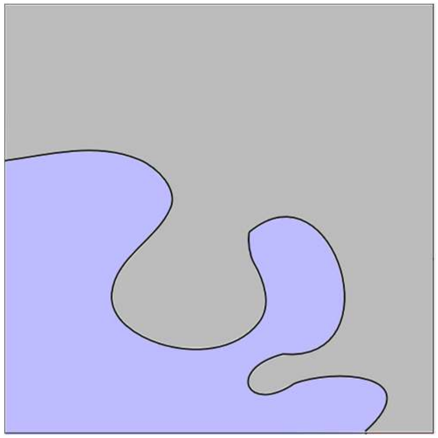
- We need special structure in the graph, else not ergodic or rapid mixing
- Experimental results suggest that real precinct maps are rapid mixing
- Intuitively, refining the underlying graph (increasing "resolution") makes reconfiguration easier
- Instead of a graph -> topological disk (simple polygon)
- Zero slack (balanced districts)



Continuous domain

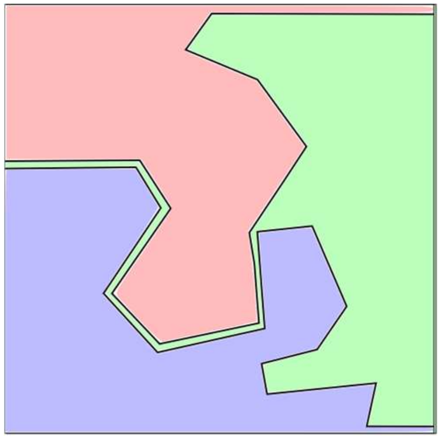
- Domain: square (area preserving homeomorphism)
  - Possibly with a density function
- Districts:
  - Topological disks (polygonal)
  - Prescribed “area” (integral of density function)
- Recomb: merge two adjacent distr. and re-split

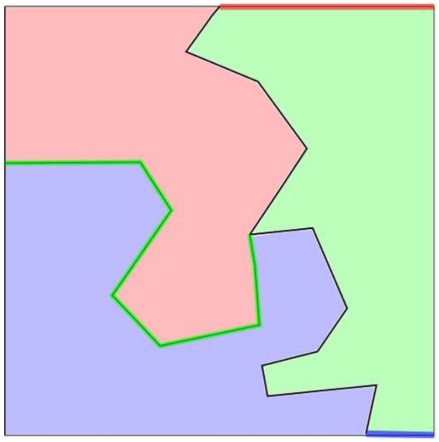




Continuous domain

- Polygonal boundaries: n complexity
- **Corridor:** infinitesimal area neighborhoods of arc
- Vertical ordering: **ordering property**
  - Easy to achieve by propagating corridors along boundaries





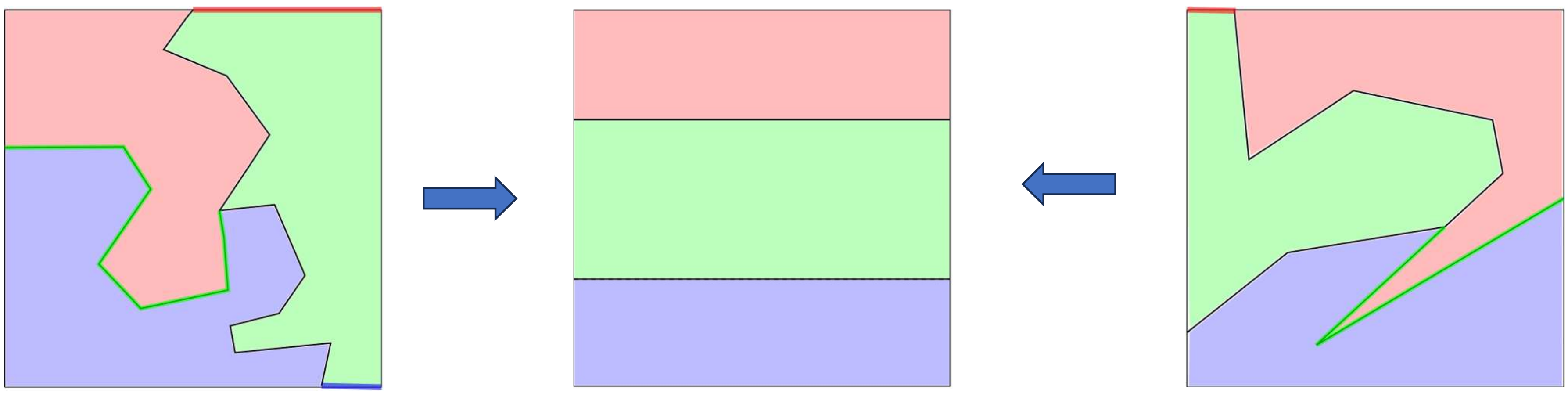
Our Results

- Universal reconfiguration (connected configuration space)
- n: complexity of the map
- k: number of districts
  - For $k = 3$ districts, $O(\log n)$ moves suffice
    - Intermediate maps have $O(n)$ complexity
  - For general k, $(\log n)^{O(\log k)}$ moves suffice
    - Intermediate maps have $n^{k^{O(1)}}$ complexity

- Lower bound (first result of this type):
  - Even for $k = 3$ districts, $\Omega(\log n)$ moves are necessary

Algorithm ideas

- Focus on case with 3 districts
- Transform map into canonical form (flag) $O(\log n)$ moves
- Transform canonical into desired



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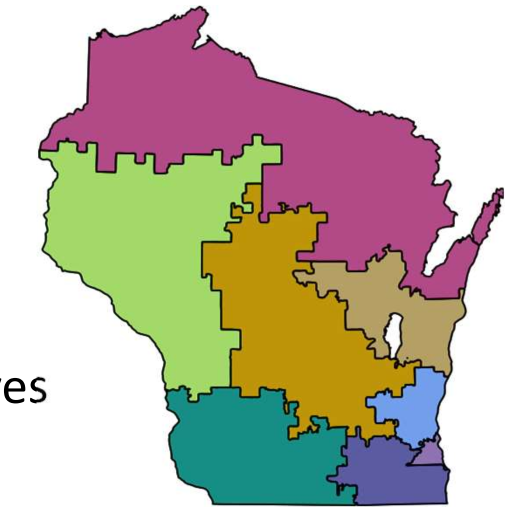
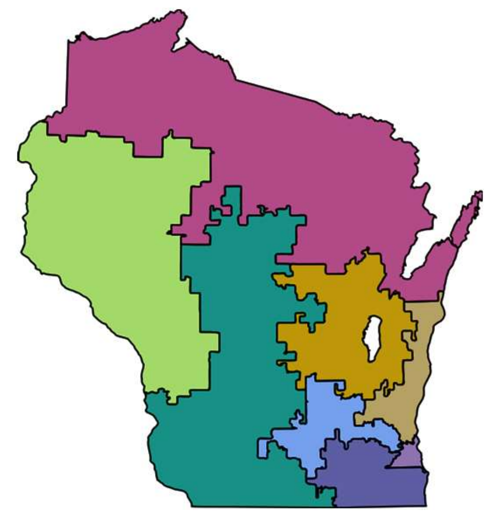


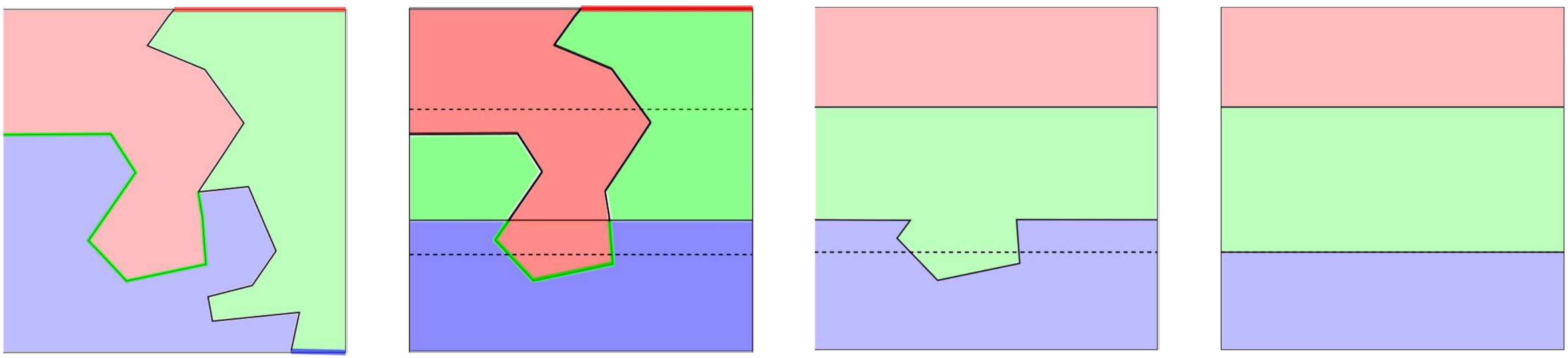
- Generalize 3-district algorithm to k districts
- Group districts together so they "move" as one
- Base case is 3 districts
- $O(\log n)$ moves



Potential districts in Wisconsin

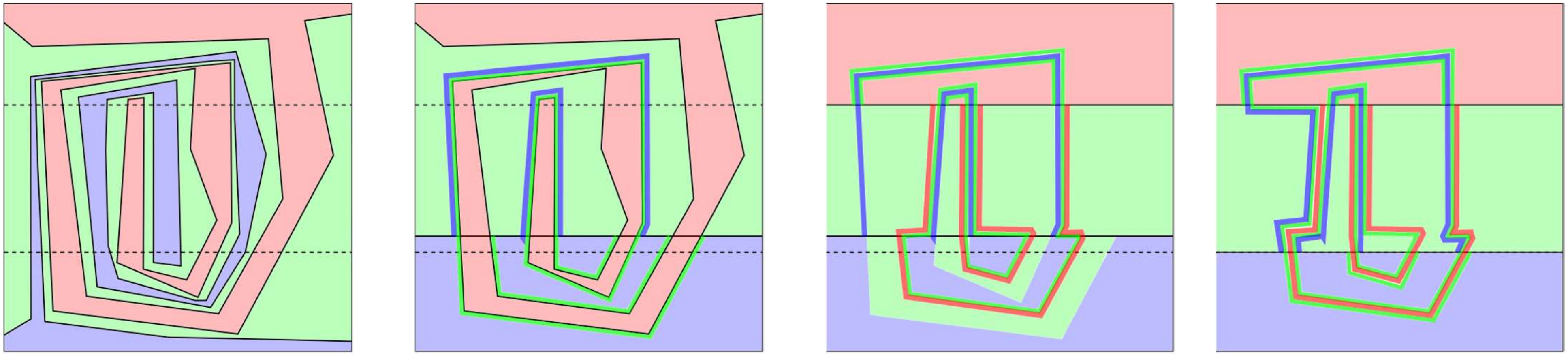
Gravity move

- First, reorder districts so that $area(D_2) \geq \max\{area(D_1), area(D_3)\}$
- Then, after a gravity move, D_1 has 0 area in the canonical region of D_3



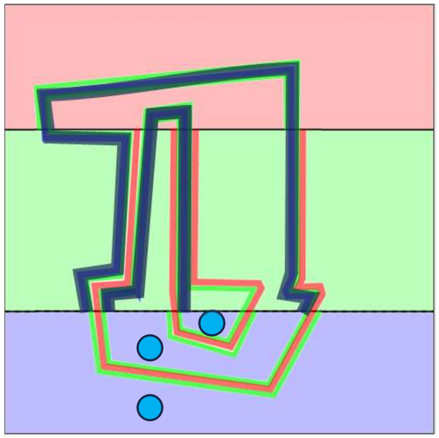
Gravity phase

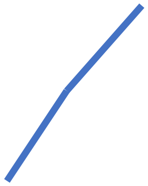
- After 3 gravity moves, the noninfinitesimal areas are in the right place
- Exchange phase eliminates corridors



Corridor Graph

- Adjacency of “fat” components
- Since each district is topological disk, the corridor graph is a tree
- The corridor graph of the union of two adjacent districts is also a tree (ordering property)
- Topology of the map enforces a combinatorial embedding of the tree





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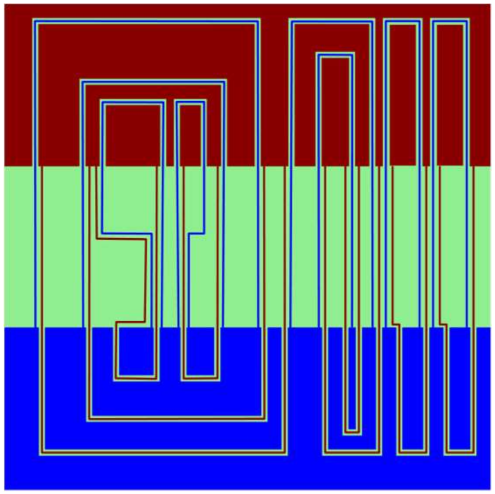

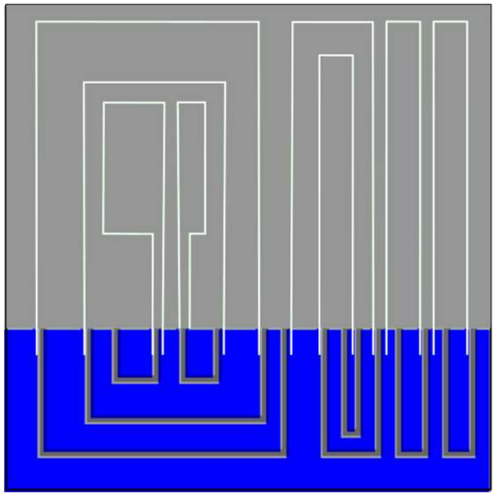

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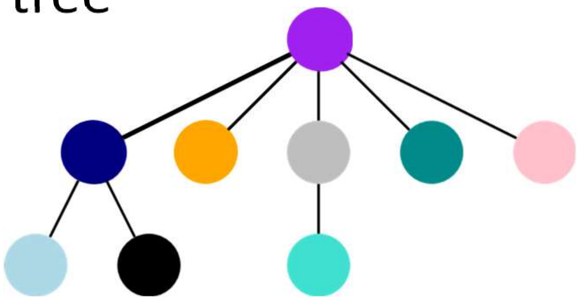
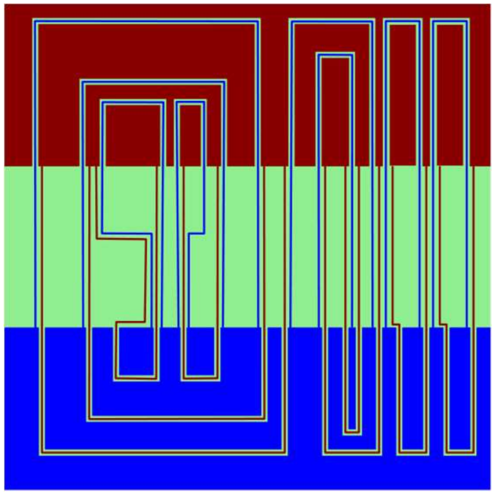
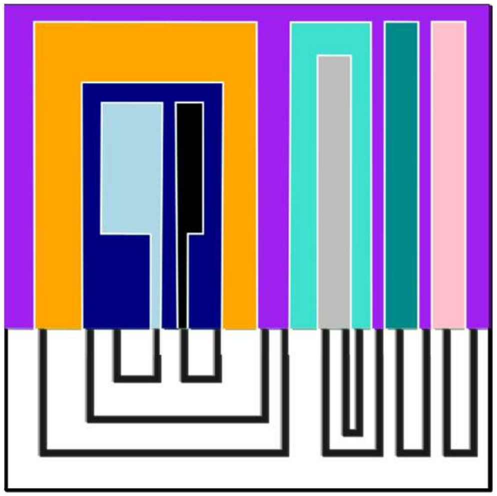


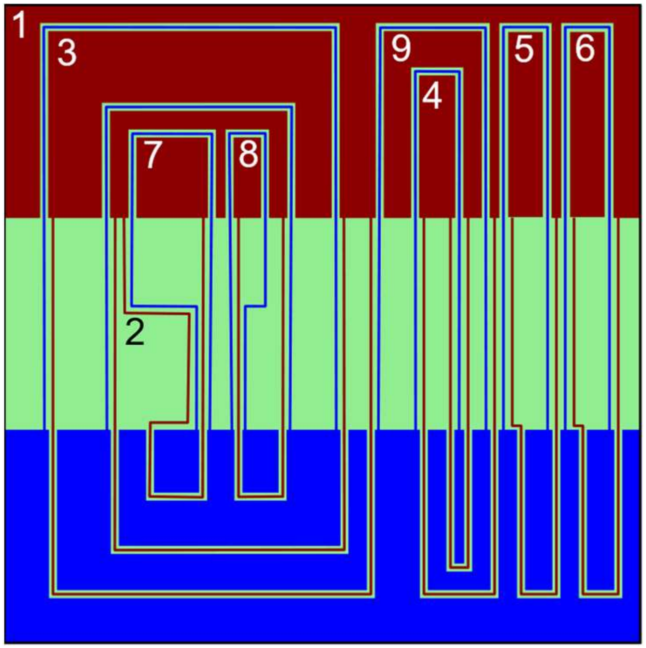


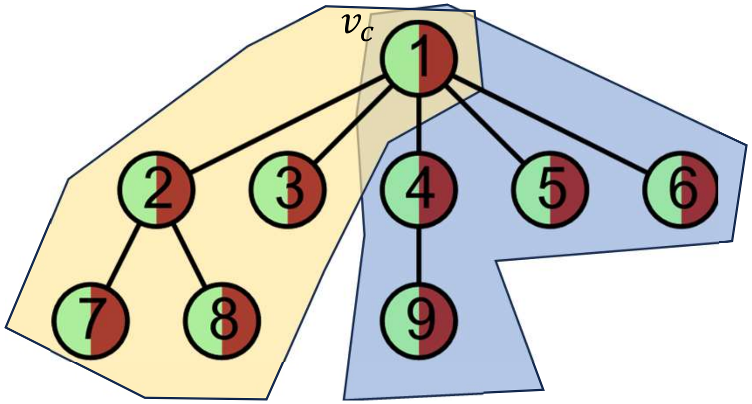


Exchange phase

- Consider the union of two districts ($D_2 \cup D_3$)
- Find "center of mass" v_c of tree G_P
- There are up to three "contiguous" subtrees T rooted at v_c each with weight less than half (without the weight of v_c)
- By pigeonhole, we can choose T so that

$$|E(T)| \geq \frac{1}{3}|E(G_P)|$$





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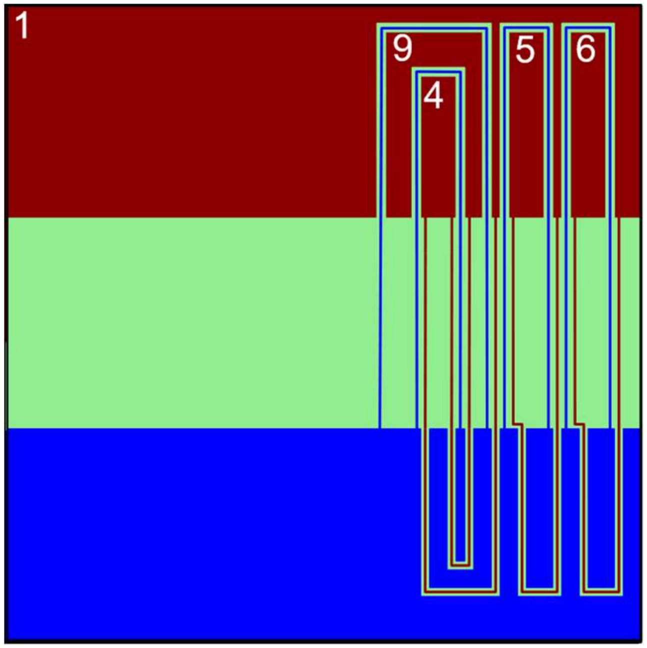
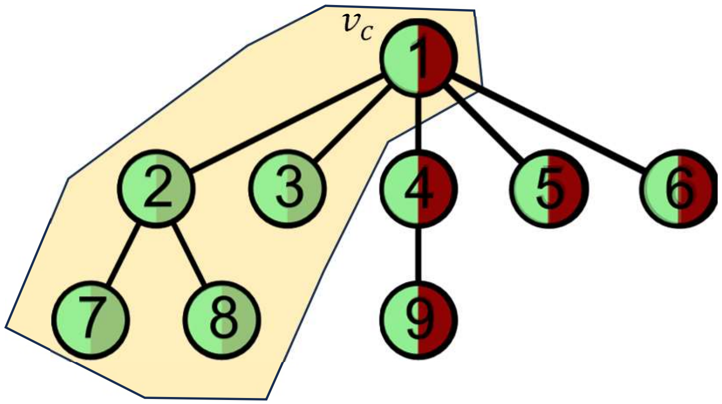
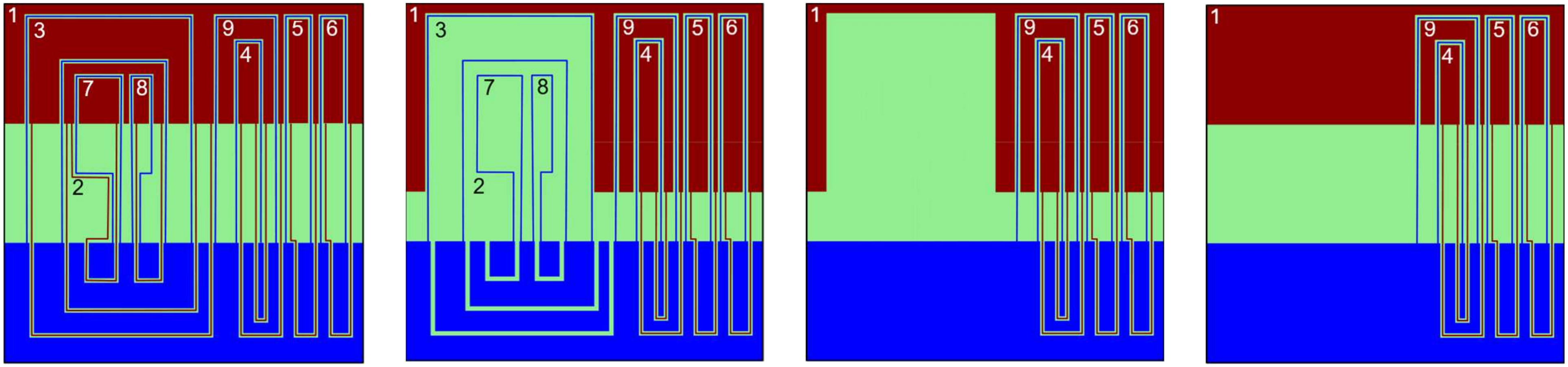
- Recombine $D_2 \cup D_3$ giving T entirely to D_2
- Kill entirely green corridors (ReCom $D_1 \cup D_2$)
- More gravity moves to restore ordering property





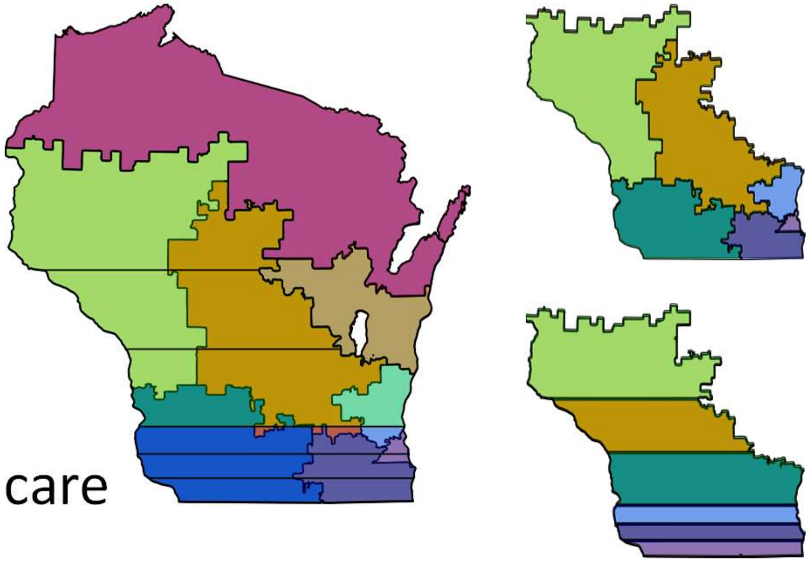
Analysis

- Each round "kills" at least $1/3$ of corridors
- Initial # of corridors is bounded by $O(n)$
- $O(\log n)$ rounds reaches canonical



Generalizing for k

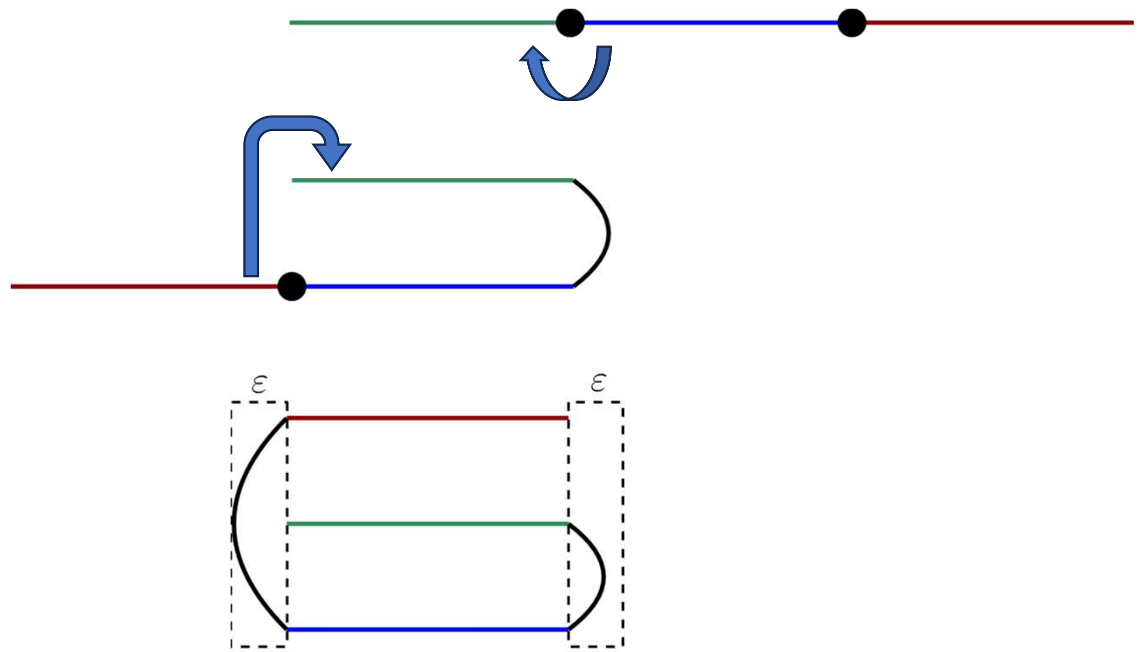
- Partition into superdistricts of roughly same size ($k/3$).
- Each recombination of a superdistrict is solved recursively (involving $\frac{2k}{3}$ smaller districts)
- $T(k,n) \leq O(\log n) \cdot T\left(\frac{2k}{3}, n\right)$
- $T(k,n) = k^{O(\log \log n)} = (\log n)^{O(\log k)}$
- Simple idea, but execution needs lots of care



Lower bound

- Construction

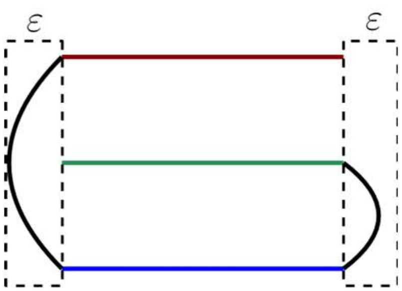
$\ell = 0$



$\ell = 1$

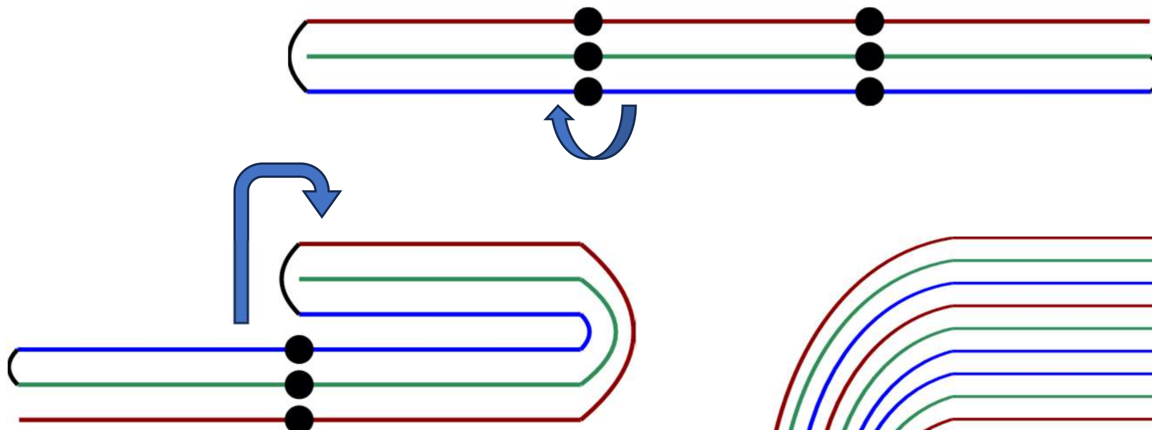
Lower bound

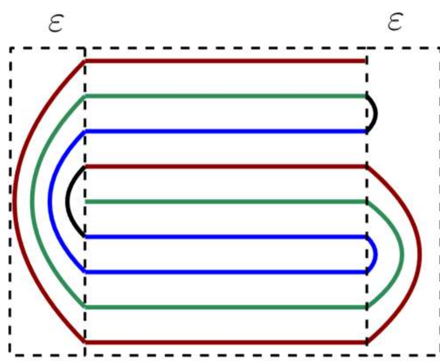
- Construction



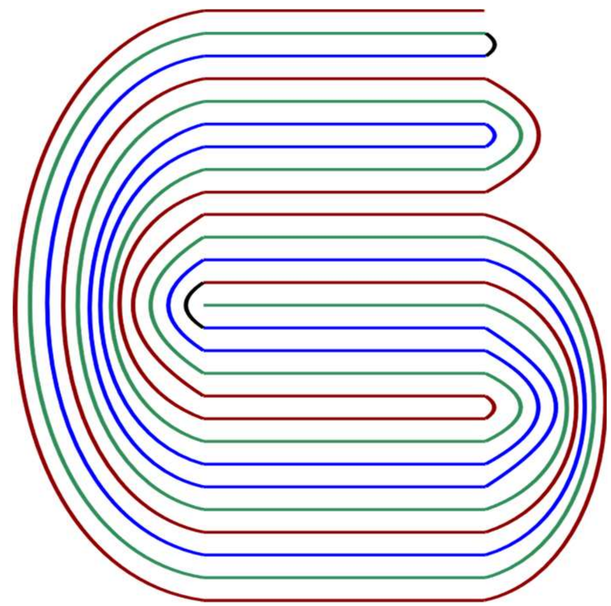
$\ell = 1$

From: https://sallysbakingaddiction.com/



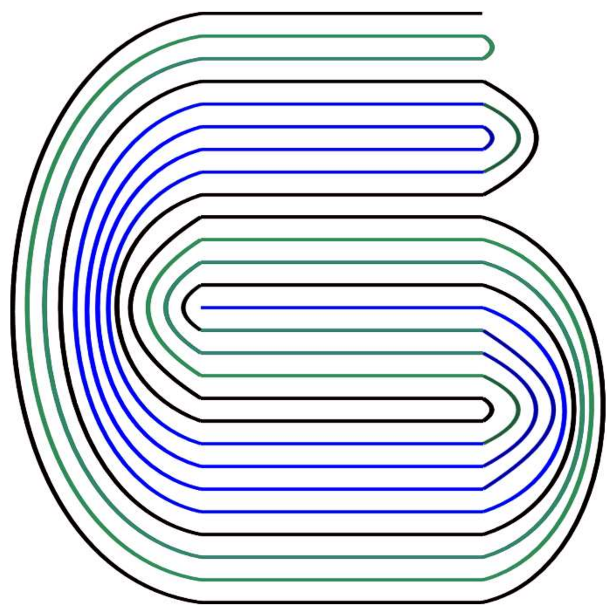


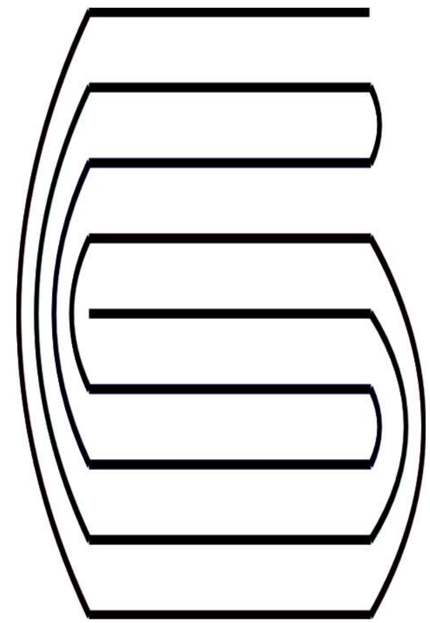
$\ell = 2$



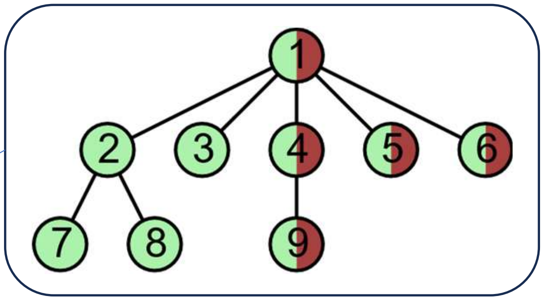
Lower bound

- Analysis
  - Shape of a district in level ℓ is the same as level $\ell - 1$
  - **Invariant**: "gaps" between horizontal bars of a district are "small"
  - Because of a level-ℓ obstacle, any move produces districts of level at least $\ell - 4$
  - It takes $\Omega(\log n)$ moves to reach canonical



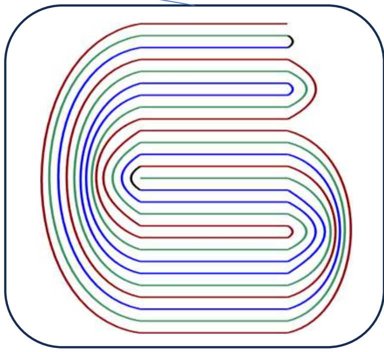


Conclusion and Open Questions

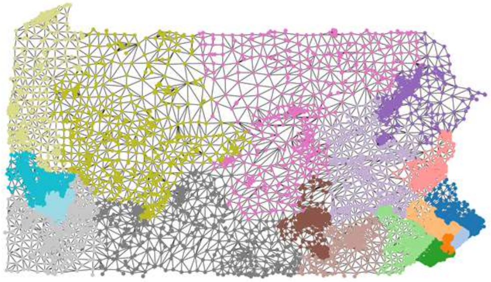


We have a tight $\Theta(\log n)$ bound for $k = 3$.

In general, the upper bound is $(\log n)^{O(\log k)}$ and lower bound is $\Omega(\log n)$.



- Can the upper bound be reduced for $k \geq 4$?
  - Maybe polynomial in both k and n?
- Reduce complexity of intermediate maps (currently $n^{k^{O(1)}}$)
- Can the lower bound be generalized to arbitrary k?
- In the graph model, how do we “refine" a given precinct map to guarantee a connected configuration space?



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- Akitaya, H.A., Korman, M., Korten, O., Souvaine, D.L. and Tóth, C.D., 2022. Reconfiguration of connected graph partitions via recombination. *Theoretical Computer Science*, 923, pp.13-26.
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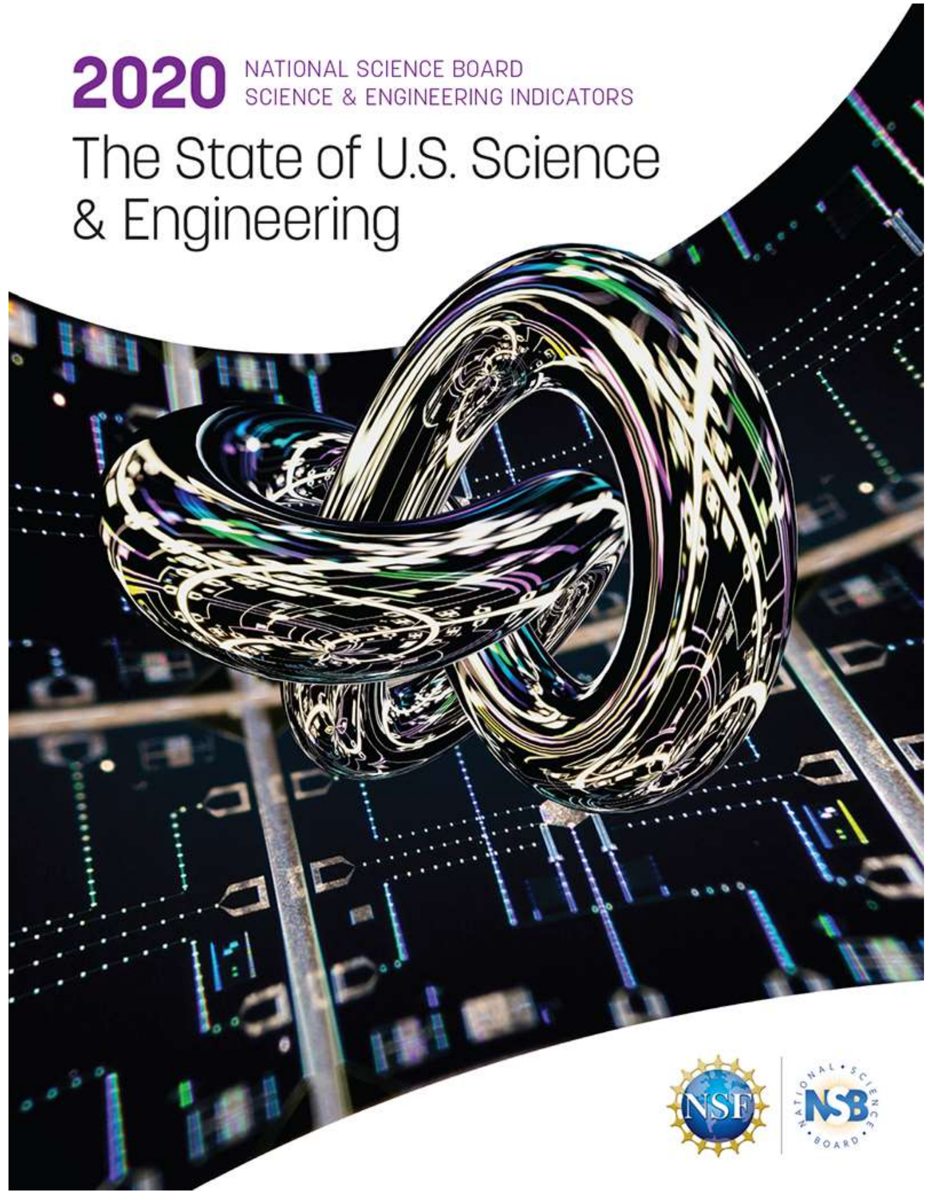
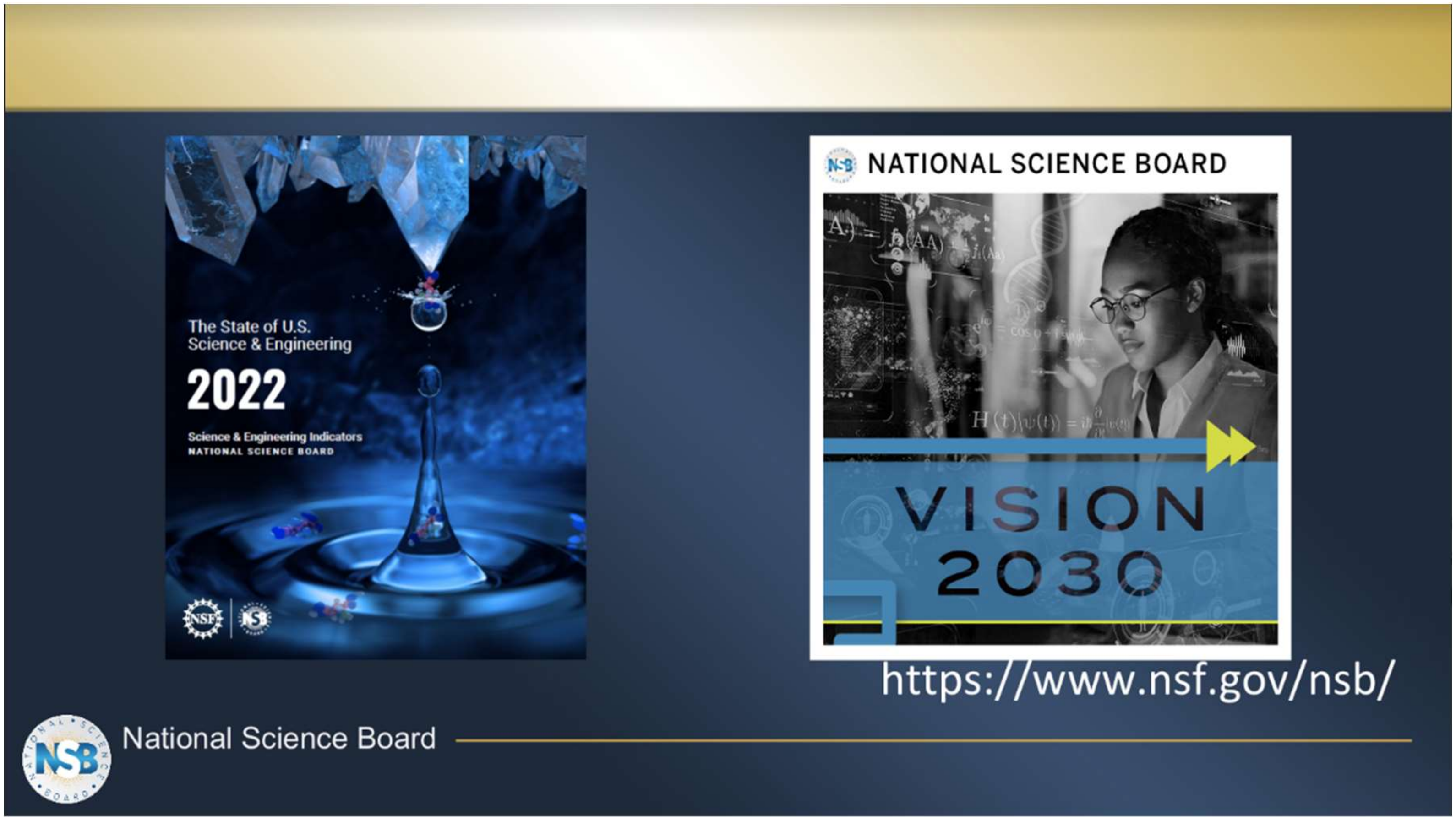
Science & STEM



Source: https://www.nsf.gov/nsb/publications/2020/nsb202015.pdf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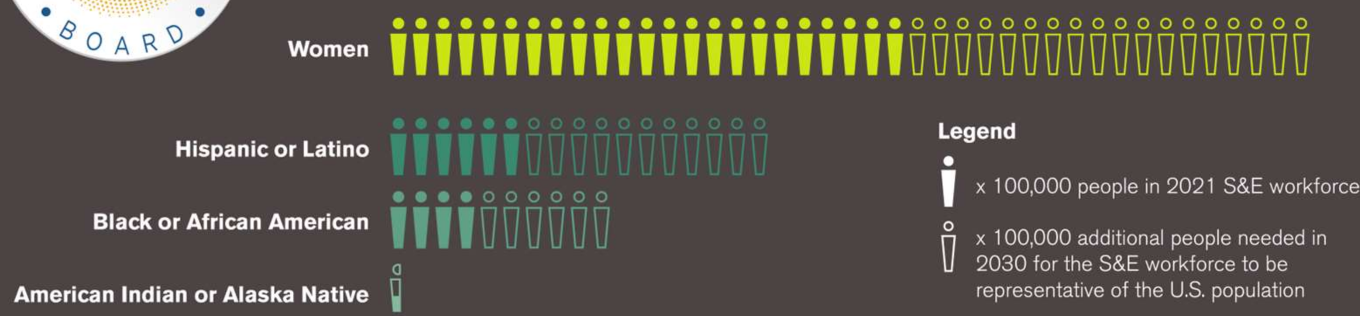
Missing Millions: Faster Progress in Increasing Diversity Needed to Reduce Significant Talent Gap



While the number of people from under-represented groups in the S&E workforce has grown over the past decade, much faster increases will be needed for the S&E workforce to be representative of the U.S. population in 2030. To achieve that goal, the NSB estimates that the number of women must nearly double, Hispanic or Latinos must triple, Black or African Americans must more than double, and the number of American Indian or Alaska Native S&E workers needs to quadruple (from 15,000 to 60,000). The NSB estimates that the number of Native Hawaiian or Other Pacific Islanders will be slightly overrepresented in the S&E workforce in 2030.

These estimates are based on projections from the U.S. Census and Bureau of Labor Statistics, together with data from the 2021 Women, Minorities, and Persons with Disabilities in Science and Engineering report published by the National Center for Science and Engineering Statistics and assume that participation of these groups in the S&E workforce increases at current rates.

Consider the trajectory of Computational Geometry since the founding of SoCG 1985…
In those early days, collaboration was not always easy….
Which makes me particularly grateful for the fabulous collaborations that I have had, many of which are included here: http://www.cs.tufts.edu/r/geometry/people.php

- Many markers of progress.
  - CCCG in 1989 addressed a perceived narrowing of scope of SoCG in 1988 and has continued to welcome a broad audience and foster community.
  - SoCG has become a full week with various targeted workshops, many of the form “CG *and* …”, and the young researchers forum.
  - SoCG 2023 featured double-blind reviewing.
  - SoCG 2023 actively encouraged the submission of applied papers.
  - Increased mentoring being offered.
- And yet ….
  - The size and the diversity of the community …..
  - And questions like “Is Computational Geometry really Computer Science?”

- What can we do actively to
  - Further enhance inclusivity in the community
  - Use CG to draw many more students both to CG and more broadly to STEM
  - Accelerate the impact of CG.

Not clear how to accomplish this “Reconfiguration”

No obvious “Canonical Configurations”

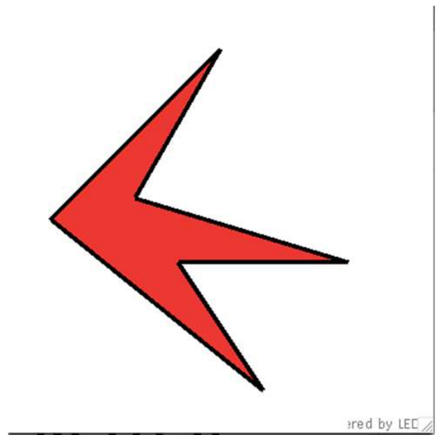
And yet geometry can be accessible!

- Create courses and outreach
  - Precollege students
  - Precollege teachers
  - First-year college non-majors
- Many in this room have developed and participated in programs across these categories!!
- A personal example: Exploring Computer Science
  - C/C++, since the CS 1 class is in C.
  - Link to a simple subset of LEDA so that every program is visual
  - All code reused in the final project on medical diagnosis using live data from BME
  - Only prerequisite being *no* background in programming.

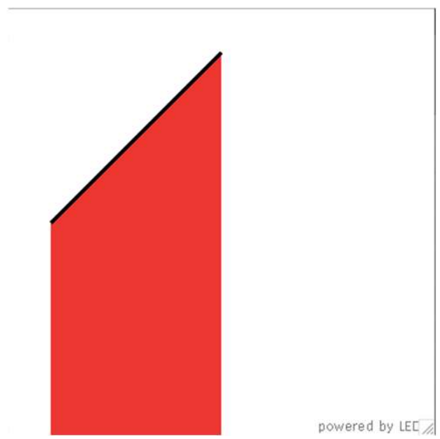
Simple "first" coding assignment with visualization:

Find the area of a polygon.

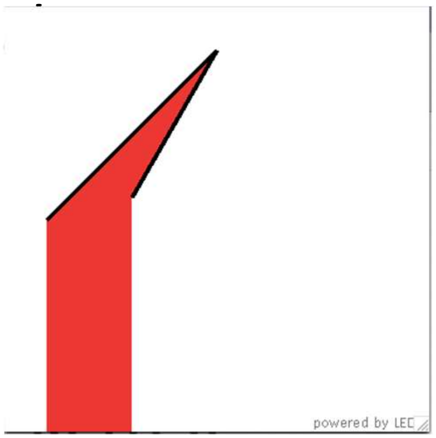
Shamos & Hoey, 1975



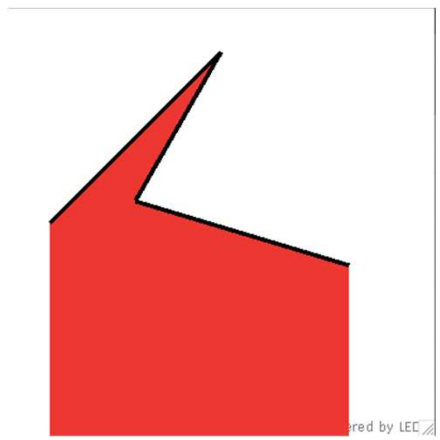
Area of Trapezoid: $.5\ (y_i + y_{i+1})(x_{i+1} - x_i)$



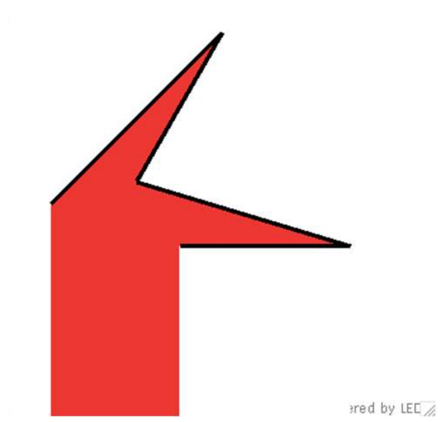
Advance the value of *i*.
The next trapezoid has "negative area".
It is subtracte



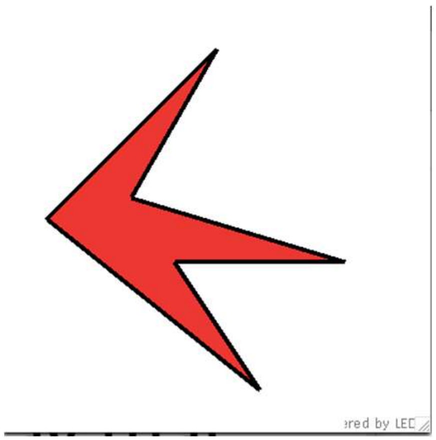
Another “positive” area trapezoid



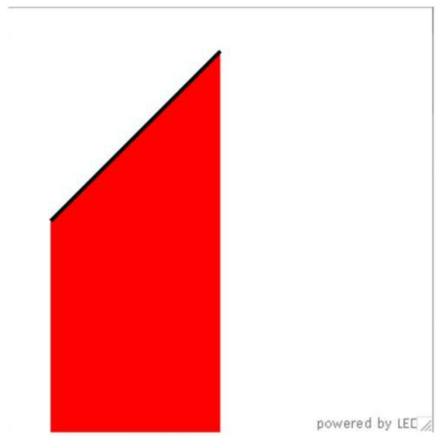
Another negative trapezoid.



Followed by one positive and one negative trapezoid.

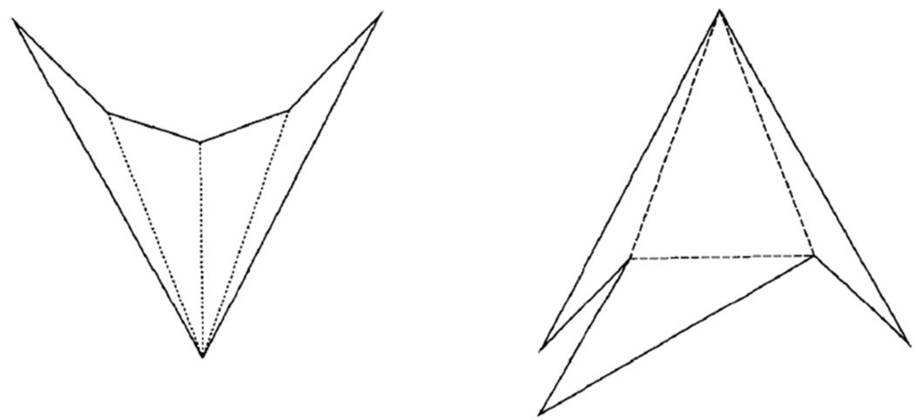


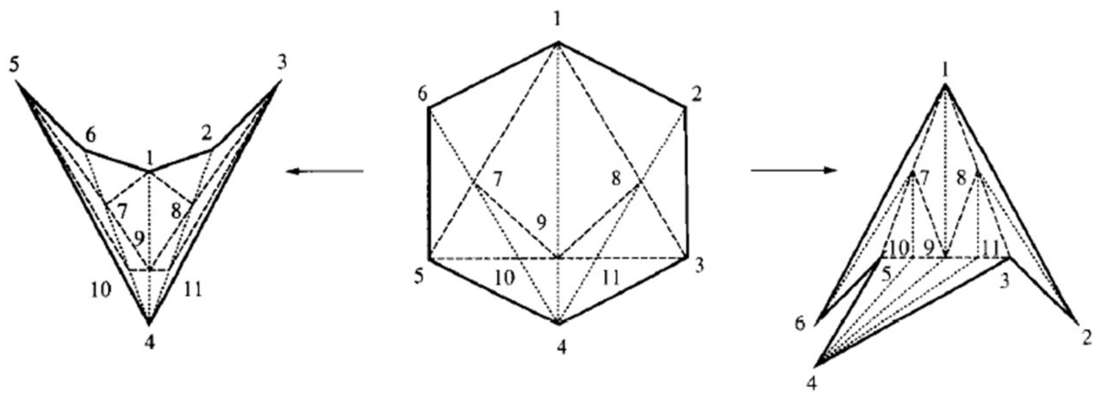
Animation



On Compatible Triangulations of Simple Polygons

Aronov, Seidel, Souvaine. *Computational Geometry.* 1993. [Used in a section of EN 1 for first-year students]





On Compatible Triangulations of Simple Polygons

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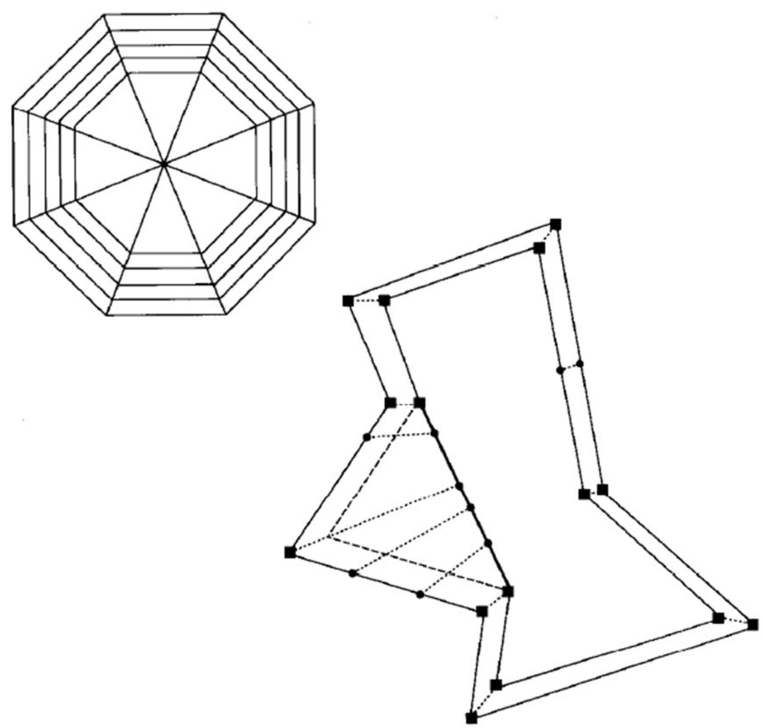
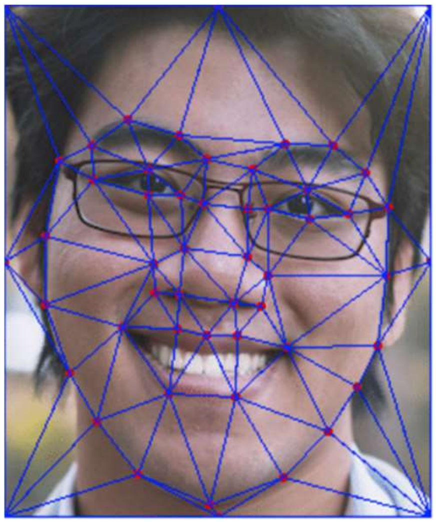
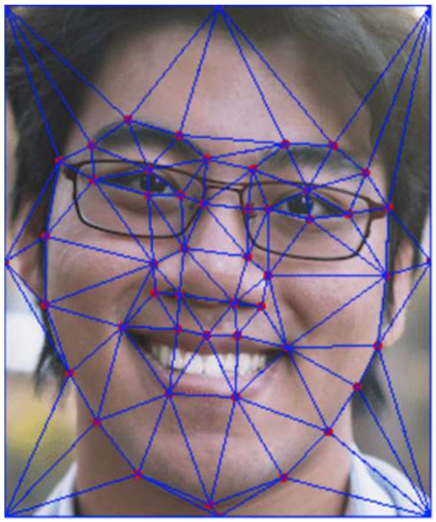


Compatible triangulations



Compatible triangulations



Michael Jackson – Black or White Face morphing

DIMACS

Center for Discrete Mathematics & Theoretical Computer Science
Founded as a National Science Foundation Science and Technology Center

The DIMACS RESEARCH AND EDUCATION INSTITUTE (DREI) took the approach that research and education should work hand-in-hand. Collaborations between researchers and educators were forged by having separate programs]focused on each group's interests, along with plenary sessions and evening lectures aimed at meshing the two groups with the planned outcome of stimulating discussion and problem-solving..

Research Program:
- Software and Mathematical Visualization, June 24 – 28, 1996
- Computational Geometry Problems in Aerodynamics, July 1-3, 1996
- Hot Topics in Computational Geometry, July 8-12, 1996

Teacher Program, June 24-July 12, 1996
- A hands-on computing and internet laboratory
- Workshop focusing on computational geometry and discrete mathematics

Questions

- How can we collectively contribute to further "reconfiguration":
  - Increase prompt impact of Computational Geometry results?
  - Contribute to allowing pre-college teachers to gain versatility?
  - Use Computational Geometry to help create more on-ramps broadly into science for students?
  - Foster greater diversity in our own research groups?
  - Share successful strategies with each other?

Hoping to attend CCCG and SoCG in 2033 with a far greater number and diversity of participants and a greater balance between theoretical and applied/translational.